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Emission Taxes and Standards for an Asymmetric Oligopoly*

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Résumé / Abstract

On décrit les propriétés des taux de taxes pigouviennes optimales pour les firmes polluantes, ainsi que les standards d'émission optimaux. On démontre qu'il y a un cadre unifié pour analyser ces deux instruments d'intervention. Pour atteindre l'optimum social dans le cas où les firmes sont identiques, il faut les traiter de manière différente. On explique le traitement inégal des égaux en terme du motif d'influencer la concentration de l'industrie. Notre méthode de solution est de nature géométrique, ce qui nous permet d'obtenir les conditions d'optimalité globales.

We characterize optimal firm-specific emission tax rates, and optimal firm-specific emission standards, and provide intuitive explanation on differential treatments. We show that there is a unified framework for deriving firm-specific policy measures. When firms are identical, the optimal policy may call for "unequal treatments of equals". When firms are not identical, we characterize the optimal degree of dispersion of tax rates. The "unequal treatments of unequals" is explained in terms of the motive of the government to affect industry concentration. Our new approach is geometric in nature and enables us to give optimality conditions in global terms.

Mots Clés : Pollution, oligopole

Keywords: Pollution, oligopoly

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1 Introduction

When production generates pollution as a by-product, competitive firms over-produce in the sense that marginal social cost exceeds price. Under perfect competition, a Pigouvian tax equal to marginal damage cost is called for. When the market is not competitive, however, there is another source of distortion. For example, under monopoly, output is restricted in order to raise price. Because of this, a polluting monopoly may overproduce, or underproduce, by comparison with the socially optimal output. Buchanan (1969) and Barnett (1980) have shown that the optimal tax per unit of e-mission under monopoly is *less than* the marginal damage cost (and it can be negative). The case of oligopoly is more complicated. Katsoulacos and Xepapadeas (1995), consider the case of a *symmetric* polluting oligopoly (i.e., they assume that firms are identical) and show that if the number of firms is endogenous and if there are fixed costs, the optimal Pigouvian tax could *exceed* the marginal damage cost, because free entry may result in an excessive number of firms.

In this paper, we consider an *asymmetric* polluting oligopoly: firms have different production costs, and their pollution characteristics may also be different. Asymmetry is important, because it is a prevalent real world feature, and because it introduces another source of distortion: in a Cournot equilibrium, marginal production costs are not equalized across firms, resulting in production inefficiency at any given total output. In this context, pollution taxes or pollution standards must seek to remedy both the environmental problem and the intra-industry production inefficiency problem. With asymmetric oligopoly, the regulator would want to be able to correct distortion on a *firm-specific* basis. While *de jure* differential treatments to firms in the same industry (in the sense that different nominal tax rates apply to different firms), may be politically unacceptable in most economies, *de facto* differential treatments (e.g., different degrees of enforcement and verification, that have the same effects as different tax rates) may be feasible. In what follows, whenever the terms firm-specific tax rates, or firm-specific pollution standards are used, they should be interpreted in the de facto sense.

The objective of this paper two-fold. Our first aim is to characterize optimal firm-specific emission tax rates, and optimal firm-specific emission standards, and to provide intuitive explanation of our results on differential treatments. We would like to make clear that we *do not* seek to compare taxes to standards, because there exists already a large literature on that subject.

In fact, the model we develop to analyze optimal standards (in section 4) is quite different (in terms of the pollution generation process) from the model we use to analyze optimal taxes, and therefore it would not make sense to discuss, using our results, the relative attractiveness of these two policy measures. Our reason for presenting results for taxes and standards in the same paper is of a methodological nature: we wish to show that there is a unified framework for deriving optimal firm-specific policy measures. This is the second objective of our paper. Furthermore, it will become clear that our unified approach can be applied to related problems such as tradeble pollution permits, and delocation in response to environmental regulations.(See Long and Soubeyran, 2001b,c).

In the models we present below, we use a two-stage game framework. In the first stage, the regulator sets firm-specific taxes (in section 2), or firm-specific standards (in section 4). In the second stage, firms compete in the final good market. To fix ideas, we take the case where firms produce a homogenous good, and compete à la Cournot. However, our analysis can easily be adapted to deal with other cases, such as Bertrand competition with differentiated products, spatial competition (as in the Hotelling model), and even markets in which some firms are Stackelberg leaders. (See Long and Soubeyran, 1997a,1997b.)

The games considered in this paper belong to the class of games called "cost manipulation games with costs of manipulating" (see Long and Soubeyran, 2001a). The regulator uses the chosen policy instrument to affect, on a discriminatory basis, the marginal costs of individual firms. This in turn affects their equilibrium outputs and market shares. The costs of manipulating can take different forms, depending on the chosen policy instrument. In the case of emission taxes, there is a transfer of funds from the private sector to the public sector (or vice-versa, if the taxes are negative) and such a transfer is not costless because a dollar in the hand of owners of firms does not have the same worth as a dollar in the hand of the government: as is well known, the marginal cost of public fund is not unity, in general[1]. In the case of standards, firms are induced to acquire costly equipment to reduce the pollution generated by their production process. Such equipment alters the marginal production costs.

[1]As Brander (1995, p 1410) puts it, "'raising subsidy revenue imposes distortionary costs on the economy, implying that the opportunity cost of a dollar of of public fund would exceed one". According to Ballard et al. (1985), this cost lies in the range of 1.17 to 1.56.

A distinctive feature of our paper is that firm-specific treatments are permitted. While firm-specific sales taxes or tariffs are generally not acceptable, firm-specific emission taxes or standards are more acceptable because it is widely recognized that environmental problems vary with local conditions. Our general formulation of the model of heterogenous firms contains, as a special case, a scenario where firms are ex ante identical. We show that in this case it is possible that the optimal policy calls for the "unequal treatment of equals"[2]. For the more realistic case where firms are ex ante heterogenous, we characterize the optimal degree of dispersion of tax rates. The optimal "unequal treatment of unequals" is explained in terms of the motive of the government to affect the industry concentration. This is formulated as a "concentration motive theorem", see section 3. Our new approach, which is essentially geometric in nature, enables us to give optimality conditions in global terms, in marked constrast to the usual local characterization.

We are able to provide a unified treatment of firm-specific pollution policies because we transform variables in such a way that all modes of intervention (in distinct models of emission generation) can be seen to have the same basic structure. We show that maximizing the first stage objective with respect to one of the environmental instruments (such as Pigouvian taxes, specific pollution standards, tradable pollution permits) is equivalent to choosing the Cournot equilibrium quantities. This is because the discriminatory use of policy instruments in the first stage amounts to the same thing as manipulating the marginal costs of production which in turn affect second-stage equilibrium outputs.

Formally, the first stage optimization has the structure of an inf-convolution problem[3]:

$$\min_{x_i} \sum_{i \in I} f_i(x_i, X)$$

subject to

$$\sum_{i \in I} x_i = X$$

$$x_i \geq 0, \quad i \in I$$

[2]This possiblity has been recognized in Long and Soubeyran (1997a,b), and Salant and Shafer (1999).

[3]See Rockafellar (1970).

For the special case where marginal costs are independent of output level and emissions are linear in output, we show that the above inf-convolution problem is equivalent to minimizing the distance of a point in an $n-$simplex to a given point outside the simplex. This geometric interpretation allows us to obtain the optimal tax vector without computing derivatives. This is our Projection Theorem, given in section 3.

The rest of our paper is organized as follows. In section 2, we characterize optimal firm-specific Pigouvian taxes. The results obtained in section 2 is given further interpretation in section 3, where we present a Projection Theorem and a Concentration Motive Theorem. In sections 4 we present a model of optimal firm-specific pollution standards when choice of technology is possible. In section 5, we examine optimal firm-specific pollution standards in the context of abatement costs. Section 6 offers some concluding remarks. The Appendix deals with some technical matters.

2 Optimal Firm-specific Pigouvian Taxes

We now present our basic model of an asymmetric polluting oligopoly, and derive the optimal firm-specific Pigouvian taxes.

2.1 The basic model

We consider a polluting oligopoly consisting of n non-identical firms producing a homogenous final good, using an intermediate good that it also produces. Thus, within each firm, there is an upstream process and a downstream process. For the upstream process, firm i has u_i identical plants. The cost of maintaining each plant is $h_i \geq 0$. The cost of producing z_i units of this intermediate input in plant i is $c(z_i)$ where $c(0) = 0$ and $c' > 0$ and $c'' \geq 0$. If firm i wants to produce x_i units of the intermediate input, then it instructs each plant to produce $z_i = x_i/u_i$ and thus the total cost is $u_i c(x_i/u_i)$. The production of the intermediate input generates pollutants. Producing z_i at each plant yields $e_i(z_i)$ units of pollutants. Total emission by firm i is $u_i e_i(z_i)$.

Turning now to the downstream process, we assume that to produce one unit of the final good, firm i needs one unit of the intermediate input, and d_i units of a primary factor (say labor hours), whose price is unity. (The downstream process may simply be transportation to the final good market.)

Let q_i denote firm i's output of the final good.

Let $I = \{1, 2, ..., n\}$. The total output of the final good is $Q = \sum_{i \in I} q_i$. The inverse demand function for the final good is $P = P(Q)$ where $P'(Q) < 0$, $P(0) > c'(0) + d_i$. Industry marginal revenue is assumed to be decreasing in Q. We assume that firm i must pay a tax t_i *per unit of its emission.* Its profit function is

$$\pi_i = P(Q)q_i - d_i q_i - u_i c_i\left(q_i/u_i\right) - t_i u_i e_i(q_i/u_i) - u_i h_i$$

where h_i is the overhead cost at each plant.

We focus on the case where $e_i(z_i)$ is linear, $e_i(z_i) = \varepsilon_i z_i$. In this case, total emission by firm i is $u_i \varepsilon_i z_i = \varepsilon_i q_i$. We define

$$\tau_i \equiv t_i \varepsilon_i \tag{1}$$

so that τ_i is the tax *per unit of output* of firm i, and let $\omega_i \equiv d_i + \tau_i$. The profit function becomes

$$\pi_i = P(Q)q_i - C_i(q_i, \omega_i, u_i)$$

where $C_i(q_i, \omega_i, u_i)$ is the (tax-inclusive) total cost function:

$$C_i(q_i, \omega_i, u_i) \equiv \omega_i q_i + u_i c_i\left(q_i/u_i\right) + h_i u_i$$

The (tax-inclusive) average variable cost function is:

$$\rho_i(q_i, \omega_i, u_i) = \omega_i + \frac{c_i\left(q_i/u_i\right)}{q_i/u_i} = \omega_i + \frac{c_i(z_i)}{z_i} \tag{2}$$

and the marginal cost function is

$$\theta_i(q_i, \omega_i, u_i) = \omega_i + c_i'(z_i) \tag{3}$$

The difference between θ_i and ρ_i , defined as r_i, measures the degree of convexity of the cost function. We have

$$r_i = \theta_i - \rho_i = c_i'(z_i) - \frac{c_i(z_i)}{z_i}$$

If $c_i(.)$ is linear, then r_i equals zero identically. If $c_i(.)$ is strictly convex, then r_i is positive for all $z_i > 0$.

We will show how the government can optimally manipulate the tax-inclusive costs of the firms so as to maximize social welfare. To do this, we set up the problem as a two-stage game. In the first stage, the government sets firm-specific taxes, and in the second stage, firms compete as Cournot rivals, taking tax rates as given. As usual, to solve for the optimal taxes, we must first analyse the equilibrium of the game in stage two.

2.2 Stage two: Cournot equilibrium given tax rates

The first order condition for an interior equilibrium for firm i is

$$\frac{\partial \pi_i}{\partial q_i} = P'(Q)q_i + P(Q) - \theta_i = 0, \qquad i \in I \tag{4}$$

We assume that these conditions determine a unique[4] Cournot equilibrium $(\widehat{Q}, \widehat{q}_i, i \in I)$, where the hat over a symbol indicates that it is the Cournot equilibrium value. It is convenient to express the *equilibrium* output of firm i as a function of the *equilibrium* output of the industry, and of the parameters of firm i's (tax-inclusive) cost function:

$$\widehat{q}_i = \widehat{q}_i(\widehat{Q}, \omega_i, u_i) \tag{5}$$

Inserting (3) and (5) into (4), we obtain

$$P'(\widehat{Q})\widehat{q}_i(\widehat{Q}, \omega_i, u_i) + P(\widehat{Q}) = \omega_i + c_i' \left[\frac{\widehat{q}_i(\widehat{Q}, \omega_i, u_i)}{u_i} \right] \tag{6}$$

Summing (6) over all i, we obtain the identity

$$P'(\widehat{Q})\widehat{Q} + nP(\widehat{Q}) = n\omega_I + \sum_{i \in I} c_i' \left[\frac{\widehat{q}_i(\widehat{Q}, \omega_i, u_i)}{u_i} \right] \tag{7}$$

where $\omega_I \equiv (1/n) \sum_{i \in I} \omega_i$. Equation (7) indicates that, given the u_i's, the industry equilibrium output can be determined from the knowledge of the ω_i's. Given the ω_i's, we assume that there exists a unique $\widehat{Q}$ that satisfies (7). (See Long and Soubeyran (2000) for sufficient conditions for uniqueness). Thus we write

$$\widehat{Q} = \widehat{Q}(\boldsymbol{\omega}, \mathbf{u}) \tag{8}$$

where $\omega \equiv (\omega_1, \omega_2, ..., \omega_n)$ and $\mathbf{u} \equiv (u_1, u_2, ...u_n)$.

We now express the equilibrium profit of firm i as follows

$$\widehat{\pi}_i = \left(\widehat{P} - \widehat{\rho}_i\right) \widehat{q}_i - u_i h_i = \left[\left(\widehat{P} - \widehat{\theta}_i\right) + \left(\widehat{\theta}_i - \widehat{\rho}_i\right) \right] \widehat{q}_i - u_i h_i$$

$$= [-\widehat{P'}]\widehat{q}_i^2 + \widehat{r}_i(\widehat{z}_i)u_i\widehat{z}_i - u_i h_i$$

[4] For assumptions ensuring existence and uniqueness of equilibrium, see Long and Soubeyran (2000).

$$= [-\widehat{P'}]\widehat{q}_i^2 + u_i\left[\widehat{z}_i c_i'(\widehat{z}_i) - c_i(\widehat{z}_i)\right] - u_i h_i \tag{9}$$

where we have made use of the Cournot equilibrium condition

$$\widehat{P} - \widehat{\theta}_i = [-\widehat{P'}]\widehat{q}_i \tag{10}$$

Expression (9) deserves some comments. Since the profit expression in (9) incorporates the Cournot equilibrium condition (10), it indicates that, in the first stage of the game, while the government can manipulate the $\widehat{q}_i$ and $\widehat{Q}$ via the choice of the policy parameters ω_i, it cannot violate the Cournot equilibrium condition. (Technically, this is very much like the incentive compatibility constraint in principal-agent problems: the principal cannot ignore economic agents' equilibrium conditions.) We now turn to a complete analysis of the first stage of the game.

2.3 The first stage: optimization by the government

The objective of the government is to maximize a weighted sum of profits, consumers' surplus, and tax revenue, minus the damage cost caused by pollution

$$W = \sum_{i \in I} \pi_i + \beta S + \gamma \sum_{i \in I} t_i e_i - D(E)$$

where $E = \sum_{i \in I} u_i e_i$, $D(E)$ is the damage cost, and S is the consumers' surplus

$$S = \int_0^Q P(\widetilde{Q})d\widetilde{Q} - P(Q)Q$$

The weight given to consumers' surplus is $\beta > 0$. The weight $\gamma > 0$ is a measure of the marginal cost of public fund (see Ballard et al. (1985)).

In what follows, we assume that the damage cost function is linear, $D(E) = \sigma E > 0$. We define

$$\delta_i \equiv \sigma \varepsilon_i / \gamma \tag{11}$$

Then the social welfare at a Cournot equilibrium may be written as

$$\widehat{W} = \sum_{i \in I} \widehat{\pi}_i + \beta S(\widehat{Q}) + \gamma \sum_{i \in I} (\tau_i - \delta_i)\widehat{q}_i(\widehat{Q}, \omega_i, u_i) \tag{12}$$

where $\widehat{Q} = \widehat{Q}(\boldsymbol{\omega}, \mathbf{u})$, and $\widehat{\pi}_i$ is given by (9). Note that the right-hand side of (12) contains tax payments by firm in the expression $\widehat{\pi}_i$, and the social value of tax revenue $\gamma\tau_i\widehat{q}_i$. These two terms do not cancel out when $\gamma \neq 1$.

From expression (12), we see that welfare can be maximized by an appropriate choice of the firm-specic tax rates τ_i. However, as we demonstrate below, it is analytically much more convenient to solve the same welfare maximization problem using the equilibrium outputs $\widehat{q}_i$ as choice variables, and afterward infer the optimal taxes. The two methods yield the same solution. We now transform variables so that the τ_i's are no longer explicitly present in the objective function. We replace the τ_i's in (12) by equilibrium quantities. From the equilibrium condition (4),

$$\widehat{P'}\widehat{q}_i + \widehat{P} = d_i + \tau_i + c'(\widehat{q}_i/u_i)$$

we get

$$\tau_i - \delta_i = \widehat{q}_i\widehat{P'} + (\widehat{P} - d_i - \delta_i) - c'(\widehat{q}_i/u_i) \tag{13}$$

Substituting (13) into (12), we get

$$\widehat{W} = F(\widehat{Q}) - \sum_{i \in I} f_i(\widehat{q}_i, \widehat{Q}) \tag{14}$$

where

$$F(\widehat{Q}) \equiv \beta S(\widehat{Q}) + \gamma \widehat{P}\widehat{Q} - H \tag{15}$$

$$H \equiv \sum_{i \in I} u_i h_i$$

and

$$f_i(\widehat{q}_i, \widehat{Q}) \equiv (d_i + \delta_i)\gamma\widehat{q}_i + (\gamma - 1)[-\widehat{P'}]\widehat{q}_i^2 + u_i\phi_i \tag{16}$$

$$\phi_i \equiv (\gamma - 1)\widehat{z}_i c_i'(\widehat{z}_i) + c_i(\widehat{z}_i)$$

Note that for given $\widehat{Q}$, f_i is strictly convex in $\widehat{q}_i$ if $\gamma > 1$, $c_i'' \geq 0$ and $c_i''' \geq 0$. Expression (14) shows that welfare is directly dependent on the $\widehat{q}_i$'s. *The tax rates do not (explicitly) appear in this expression.* Thus $\widehat{W}$ can be maximized by the direct choice of the equilibium outputs. Afterwards, the taxes can be inferred from (13).

We have thus obtained a very useful lemma:

Lemma 1: In the welfare maximization problem, there is a one-to-one correspondence between determining firm-specific emission tax rates to maximize welfare, expression (12), and determining Cournot equilibrium outputs to maximize welfare, expression (14).

We note that taxing firms is a way of manipulating their marginal costs. Thus, our Pigouvian taxation problem is a special case of the “cost manipulation approach.” (See Long and Soubeyran, 1997a, 1997b, 2001a).

2.4 A benchmark case: perfect competition

Before solving for optimal emission taxes in an asymmetric oligopoly, it is useful to consider a benchmark case, with perfect competition, using our "cost manipulation approach." We assume in this subsection that the marginal cost of public fund is unity, $\gamma = 1$, and the weight given to consumers' surplus is also unity, $\beta = 1$.

Perfect competition means that each firm thinks that its output has no effect on the price, i.e., the term $P'(Q)$ does not appear (i.e., is assigned the value zero) in the firm's first order condition. Therefore (13) reduces to

$$\tau_i = \widehat{P} - d_i - c'(\widehat{q}_i/u_i) \tag{17}$$

With $\gamma = \beta = 1$ social welfare (14) becomes

$$\widehat{W} = S(\widehat{Q}) + \widehat{P}\widehat{Q} - H - \sum_{i \in I}[(d_i + \delta_i)\widehat{q}_i + u_i c_i(\widehat{z}_i)] \tag{18}$$

Writing $\widehat{Q} = \sum_{i \in I} \widehat{q}_i$ and maximizing (18) with respect to the $\widehat{q}_i$'s, we obtain

$$\widehat{P} - d_i - \delta_i - c_i'(\widehat{z}_i) = 0 \tag{19}$$

which says that marginal social cost of firm i's output must be equated to price, a standard result. From (19) and (17), we get $\tau_i = \delta_i$, and hence, using (1) and (11),

$$t_i = \frac{\sigma}{\gamma} \equiv \delta \text{ for all } i \in I \tag{20}$$

(where $\gamma = 1$) that is, the tax per unit of pollutant discharged by firm i is equal to the marginal damage cost. Thus, under perfect competition, all firms are treated equally. (We will later show that this "equal treatment" result does not apply to the oligopoly case.) Even though this result is well known, it is useful for future reference to state it as a proposition:

Proposition 1: (Benchmark Pigouvian tax) Under *perfect competition,* the optimal Pigouvian tax t_i (per unit of pollutant discharge) is the same for all firms and equal to the marginal damage cost δ. Thus all firms are treated equally.

Proposition 1 implies that, under perfect competition, the optimal *tax per unit of output* is $\tau_i^B = \frac{\sigma}{\gamma}\varepsilon_i \equiv \delta_i$ where the superscript B indicates the optimal value for the benchmark case, and δ_i is the (adjusted) marginal damage caused by a unit of output of firm i.

2.5 An oligopoly with constant marginal cost

Now we turn to the case of an oligopoly with constant marginal cost: $c_i(z) = \alpha_i z$. (The increasing marginal cost case is treated in Appendix 1.) In this case, (16) becomes

$$f_i(\widehat{q}_i, \widehat{Q}) \equiv (d_i + \delta_i + \alpha_i)\gamma \widehat{q}_i + (\gamma - 1)[-\widehat{P'}]\widehat{q}_i^2 \tag{21}$$

We define the *marginal social cost* of firm i's output as

$$s_i \equiv d_i + \delta_i + \alpha_i \tag{22}$$

We consider two sub-cases: (a) $\gamma = 1$ and (b) $\gamma > 1$.

2.5.1 Sub-case (a): $\gamma = 1$

In this sub-case, the marginal cost of public fund is unity. It is easy to show that the optimal policy is to design taxes so that only the firm with the lowest marginal social cost will produce. Without loss of generality, assume $s_1 < s_2 < s_3 < ... < s_n$. Then, if $\partial\widehat{W}/\partial\widehat{q}_1 = F'(\widehat{Q}) - s_1 = 0$ it must be true that, for all $j > 1$, $\partial\widehat{W}/\partial\widehat{q}_j = F'(\widehat{Q}) - s_j = s_1 - s_j < 0$, implying that $\widehat{q}_j = 0$. It follows that at the social optimum, only firm 1 produces.

An intuitive explanation of this result is as follows. Suppose that at an equilibrium both firms 1 and 2 produce positive outputs, and they satisfy the Cournot equilibrium conditions

$$P(\widehat{Q}) + P'(\widehat{Q})\widehat{q}_1 = d_1 + \alpha_1 + \tau_1$$

and

$$P(\widehat{Q}) + P'(\widehat{Q})\widehat{q}_2 = d_2 + \alpha_2 + \tau_2$$

Then, social welfare can be increased by raising $\tau_2/[-P'(\widehat{Q})]$ by $\Delta > 0$ and reducing $\tau_1/[-P'(\widehat{Q})]$ by Δ, so that firm 1's output will increase by Δ and firm 2's output will fall by Δ, leaving industry output and price unchanged. Social welfare increases because the total cost of producing the given output $\widehat{Q}$ is now lower. Tax revenue will change, but industry profit, defined as sales revenue, minus production cost, minus tax payment) will change by the same amount, therefore, given that $\gamma = 1$, the tax revenue change does not matter.

2.5.2 Sub-case (b):

Let us turn to sub-case (b), where $\gamma > 1$. In this case, (16) becomes

$$f_i(\widehat{q}_i, \widehat{Q}) \equiv (d_i + \delta_i + \alpha_i)\gamma \widehat{q}_i + (\gamma - 1)[-\widehat{P'}]\widehat{q}_i^2 \equiv a_i \widehat{q}_i + b(\widehat{Q})\widehat{q}_i^2 \tag{23}$$

where $a_i \equiv \gamma s_i$ is the weighted marginal social cost of firm i's output, and $b(\widehat{Q}) \equiv (\gamma - 1)[-\widehat{P'}]$. For given $\widehat{Q}$, the function $f_i(\widehat{q}_i, \widehat{Q})$ is quadratic and strictly convex in $\widehat{q}_i$. A very effective way to characterize the optimal outputs is to use the following *two-step procedure.*

The two-step procedure:

In step 1, we fix an arbitrary level of industry output, $\widehat{Q}$, and maximize welfare by choosing the $\widehat{q}_i$'s subject to the constraint that $\sum_{i \in I} \widehat{q}_i = \widehat{Q}$. This gives the optimal value of $\widehat{q}_i$, conditional on the given $\widehat{Q}$. In the second step, we determine the optimal industry output.

Step 1:

Given $\widehat{Q}$, we write the Lagrangian as

$$L = F(\widehat{Q}) - \sum_{i \in I} f_i(\widehat{q}_i, \widehat{Q}) + \lambda \left[\sum_{i \in I} \widehat{q}_i - \widehat{Q} \right] \tag{24}$$

From this we obtain the conditions

$$-a_i - 2b(\widehat{Q})\widetilde{q}_i + \lambda = 0, \text{ for all } i \in I \tag{25}$$

where $\widetilde{q}_i$ denotes the optimal value of $\widehat{q}_i$, conditional on the given $\widehat{Q}$. From (25),

$$\widetilde{q}_i = \frac{\lambda - a_i}{2b(\widehat{Q})} \text{ for all } i \in I \tag{26}$$

Summing (26) over all i, we obtain an expression showing that λ is uniquely determined by $\widehat{Q}$

$$\lambda = \widetilde{\lambda}(\widehat{Q}) = a_I + \frac{2}{n} b(\widehat{Q})\widehat{Q} \tag{27}$$

where $a_I \equiv (1/n) \sum_{i \in I} a_i$. Substituting (27) into (26), and letting $\widehat{q}_I \equiv \widehat{Q}/n$, we get

$$\widetilde{q}_i(\widehat{Q}) = \widehat{q}_I - \frac{1}{2b(\widehat{Q})} [a_i - a_I] \quad \text{for all } i \in I \tag{28}$$

This equation gives us:

Lemma 2: The optimal deviation of the output of firm i from average industry output is a linear function of the deviation of its marginal social cost from the industry average.

Remark: To illustrate, consider a pair of firms $(1, 2)$ with marginal social costs $s_1 < s_2$. Then (28) gives

$$\widetilde{q}_1(\widehat{Q}) - \widetilde{q}_2(\widehat{Q}) = \frac{\gamma}{2(\gamma - 1)[-\widehat{P'}]}[s_2 - s_1] \tag{29}$$

That is, the solution of the optimization problem has the property that the firm with higher marginal social cost produces less than the firm with lower marginal social cost. Note that, for a given $\widehat{Q}$, a greater γ implies a smaller gap between $\widetilde{q}_1$ and $\widetilde{q}_2$, but this gap is always positive and greater than $[s_2 - s_1]/2[-\widehat{P'}]$. This may be explained as follows: a greater γ implies that a greater weight is given to tax revenue. Thus, for any given $\widehat{Q}$, a marginal increase in γ would increase the government's desire to increase tax revenue at the cost of reduced productive efficiency (here, productive efficiency includes not only private cost considerations, but also environmental cost). The government would therefore raise τ_1 by some small amount $\epsilon > 0$ and at the same time reduce τ_2 by ϵ, thus leaving total output $\widehat{Q}$ constant. The increase in tax revenue is approximately $(\widetilde{q}_1 - \widetilde{q}_2)\epsilon$ and this must be balanced against the marginal loss in productive efficiency associated with the increase in the output of the high-cost firm and the reduced output of the low-cost firm.

Step 2:

Using the results in step 1, we are now ready to find the optimal industry output. We make use of the fact that the optimal value of the Lagrangian, given $\widehat{Q}$, is equal to the maximized W, given $\widehat{Q}$. Thus

$$\widetilde{L}(\widehat{Q}) = F(\widehat{Q}) - \widetilde{\lambda}(\widehat{Q})\widehat{Q} + \sum_{i \in I} f_i^* \left[\widetilde{\lambda}(\widehat{Q}), \widehat{Q}\right]$$

where

$$f_i^*(\lambda, \widehat{Q}) \equiv \sup_{q_i} \lambda \widehat{q}_i - f_i(\widehat{q}_i, \widehat{Q})$$

($f_i^*(\lambda, \widehat{Q})$ is called the conjugate function of $f_i(\widehat{q}_i, \widehat{Q})$, see Rockafellar, 1970, section 12.) In the present case,

$$f_i^*(\lambda, \widehat{Q}) = [a_I + 2b\widehat{q}_I]\,\widetilde{q}_i - \left[a_i\widetilde{q}_i + b\widetilde{q}_i^2\right]$$

$$= -(a_i - a_I)\widetilde{q}_i - b\left[\widetilde{q}_i - \widehat{q}_I\right]^2 + b\widehat{q}_I^2$$

Thus, using (28)

$$f_i^*(\lambda, \widehat{Q}) = -(a_i - a_I)\widehat{q}_I + b\widehat{q}_I^2 + \frac{1}{4b(\widehat{Q})}(a_i - a_I)^2$$

and, using (27), the maximized welfare, for given $\widehat{Q}$, is

$$\widetilde{W}(\widehat{Q}) = F(\widehat{Q}) - \left[a_I + 2b(\widehat{Q})\frac{\widehat{Q}}{n}\right]\widehat{Q}$$

$$+nb(\widehat{Q})\left(\frac{\widehat{Q}}{n}\right)^2 + \frac{1}{4b(\widehat{Q})}\sum_{i\in I}(a_i - a_I)^2$$

Hence

$$\widetilde{W}(\widehat{Q}) = F(\widehat{Q}) - a_I\widehat{Q} - \frac{1}{n}b(\widehat{Q})\widehat{Q}^2 + \frac{1}{4b(\widehat{Q})}\sum_{i\in I}(a_i - a_I)^2 \tag{30}$$

Maximizing (30) with respect to $\widehat{Q}$, we get the necessary condition

$$F'(\widehat{Q}) - a_I - \frac{1}{n}b'(\widehat{Q})\widehat{Q}^2 - \frac{2}{n}b(\widehat{Q})\widehat{Q} - \frac{b'(\widehat{Q})}{4b^2(\widehat{Q})}\sum_{i\in I}(a_i - a_I)^2 = 0$$

This equation determines the optimal value of $\widehat{Q}$, which we denote by $\widetilde{Q}$. The optimal output for firm i is

$$\widetilde{q}_i^* = \frac{\lambda\left(\widetilde{Q}\right) - a_i}{2b(\widetilde{Q})} \text{ for all } i \in I$$

From this, we derive the optimal firm-specific tax $\widetilde{\tau}_i$, using (13):

$$\widetilde{\tau}_i = \widetilde{q}_i^* P'(\widetilde{Q}) + (P(\widetilde{Q}) - d_i) - c'(\widetilde{q}_i^*/u_i). \tag{31}$$

Let

$$\Delta_i \equiv \widetilde{\tau}_i - \delta_i \tag{32}$$

denote the gap between the optimal firm-specific tax τ_i and the benchmark tax δ_i (as described in Proposition 1).

$$\Delta_i = \widetilde{q}_i^* P'(\widetilde{Q}) + P(\widetilde{Q}) - s_i = \frac{(2-\gamma)s_i}{2(\gamma-1)} + P(\widetilde{Q}) - \frac{\lambda\left(\widetilde{Q}\right)}{2(\gamma-1)} \tag{33}$$

In particular, for any pair of firm (i, j), we have

$$\Delta_i - \Delta_j = (\widetilde{\tau}_i - \delta_i) - (\widetilde{\tau}_j - \delta_j) = \frac{2-\gamma}{2(\gamma-1)}\left(s_i - s_j\right). \tag{34}$$

Thus, if $\delta_i = \delta_j$ and $s_i < s_j$ then $\widetilde{\tau}_i < \widetilde{\tau}_j$, provided $\gamma < 2$. That is, the more efficient firm pays a lower tax rate. However, note that $(2-\gamma)/2(\gamma-1)$ is a decreasing function of γ; therefore the gap $\widetilde{\tau}_i - \widetilde{\tau}_j$ becomes narrower as γ increases (because with a greater γ, revenue considerations become more important, and the government increases the tax rate on the bigger firms.) The next section is devoted to further economic interpretations of these results.

3 Properties of Optimal Firm-specific Pigouvian Taxes

In an oligopoly with firms having different production costs, it is in general not optimal to tax firms equally for their pollution. This is because the Pigouvian taxes now serve two purposes: correction for pollution externalities, and correction for market power and for production inefficiency (because oligopolists do not equalize marginal production costs among themselves.) We now seek to characterize the optimal departure from the benchmark Pigouvian taxes δ_i (i.e., the one given in Proposition 1).

3.1 A theorem on optimal Pigouvian distortions

From (29), at the optimal solution, the taxes are such that the more efficient firms (those firms with low s_i) always produce more than the less efficient ones. The quantity Δ_i measures the deviation of optimal firm-specific tax *under oligopoly* from the firm-specific marginal damage cost caused by a unit of output of firm i. In what follows, we assume $2 > \gamma > 1$ (to be in line with the empirical estimation of the marginal cost of public fund by Ballard et al.,1985) and $c_i(z) = \alpha_i z$. We will call Δ_i the *optimal Pigouvian distortion* for firm i. Let us first compute the gap between Δ_i and the industry average Δ_I. From (34),

$$\Delta_i - \Delta_I = \frac{2-\gamma}{2(\gamma-1)} (s_i - s_I) \tag{35}$$

where s_i is the marginal social cost of firm i's output.

From this, we can compute the variance of the statistical distribution of the Pigouvian distortions:

$$Var\Delta = \left[\frac{2-\gamma}{2(\gamma-1)}\right]^2 Var\left[s\right] \tag{36}$$

Proposition 3.1: **(Optimal distortions theorem).**The variance of the statistical distribution of the Pigouvian distortions is given by (36), and

(i) An increase in γ will lead to a decrease in this variance.

(ii) In the empirically relevant range of γ, i.e., $1 < \gamma < 2$, if the marginal social cost s_i of firm i is *greater* than the industry average, the Pigouvian distortion for firm i will be *greater* than average Pigouvian distortion.

(iii) The variance of the Pigouvian distortions is greater (respectively, smaller) than the variance of the marginal social costs if $1 < \gamma < 4/3$ (respectively, $4/3 < \gamma < 2$).

Remark: In the rather extreme case where $\gamma > 2$ (which is unlikely from empirical data) we obtain the reversal of (ii) above: When $\gamma > 2$, if the marginal social cost s_i of firm i is *greater* than the industry average, the Pigouvian distortion for firm i will be *smaller* than average Pigouvian distortion. It remains true that the optimal solution implies that the more efficient firms have greater outputs, see (29).

The Optimal Distortion Theorem provides a link between the ex-ante heterogeneity of the oligopoly's cost structure and the ex-post dispersion of the firm-specific Pigouvian tax rates.

3.2 A geometric interpretation: the Projection Theorem

We now provide a geometric interpretation of the optimal choice of outputs. Consider the *first step* in the two-step procedure explained in section 2.5.2.

That step is equivalent to the program of choosing the $\widehat{q}_i$ $(i \in I)$ to

$$\min \sum_{i \in I} f_i(\widehat{q}_i, \widehat{Q})$$

subject to $\sum_{i \in I} \widehat{q}_i = \widehat{Q}$ (given) and $\widehat{q}_i$ positive. This step can be described by the following Projection Theorem.

Proposition 3.2 (Projection Theorem) The determination of the optimal composition of industry output is equivalent to choosing a vector $\widehat{\mathbf{q}} \equiv (\widehat{q}_1, ..., \widehat{q}_n)$ from an $n-1$ dimensional simplex $\mathcal{S}$ so as to minimize the distance between the vector $\widehat{\mathbf{q}}$ and a reference vector $\mathbf{q}^{\square} \equiv (q_1^{\square}, ..., q_n^{\square})$ where

$$q_i^{\square} \equiv -\frac{a_i}{2b(\widehat{Q})} \tag{37}$$

and where

$$\mathcal{S} \equiv \{\widehat{q} \geq 0 : \sum_{i \in I} \widehat{q}_i = \widehat{Q}\}$$

Proof:

$$f_i(\widehat{q}_i, \widehat{Q}) = a_i \widehat{q}_i + b(\widehat{Q})\widehat{q}_i^2 = b(\widehat{Q}) \left[\left(\widehat{q}_i + \frac{a_i}{2b} \right)^2 - \frac{a_i^2}{4b^2} \right]$$

$$= b \left[\left(\widehat{q}_i - q_i^{\square} \right)^2 - \left(q_i^{\square} \right)^2 \right]$$

Thus

$$\sum_{i \in I} f_i(\widehat{q}_i, \widehat{Q}) = b \| \widehat{\mathbf{q}} - \mathbf{q}^{\square} \|^2 - b \sum_{i \in I} \left(q_i^{\square} \right)^2$$

where the second term on the right-hand side depends only on $\widehat{Q}$, which is fixed, and the first term on the right-hand side is b times the square of the distance of the point $\widehat{\mathbf{q}}$ in the set S (which is an $n-$simplex) to the given point $\mathbf{q}^{\square}$. Given $\widehat{Q}$, both b and $\mathbf{q}^{\square}$ are fixed. It follows that the first step (24) of the program is equivalent to finding the minimal distance between $\widehat{\mathbf{q}}$ and the given point $\mathbf{q}^{\square}$.

The optimal $\widetilde{\mathbf{q}}$ which achieves the minimal distance $\|\widehat{\mathbf{q}} - \mathbf{q}^{\square}\|$ is the projection of $\mathbf{q}^{\square}$ on the $n-$simplex $\mathcal{S}$. Its components are given by

$$\widetilde{q}_i = \widetilde{q}_I - \frac{1}{2b(\widehat{Q})}(a_i - a_I) \tag{38}$$

which is (28). Figure 1 illustrates the case $n = 2$. The projection $\widetilde{\mathbf{q}}$ satisfies

$$\widetilde{\mathbf{q}} = \mathbf{q}^{\square} + (\widetilde{q}_I - q_I^{\square})\mathbf{u}$$

where $\mathbf{u} = (1, 1, ..., 1)$ where $q_I^{\square} = -\frac{a_I}{2b(\widehat{Q})}$. ■

3.3 The Concentration Motive

Our result shows that firm-specific Pigouvian taxes in a polluting oligopoly serve two functions: the usual function of correcting for externalities, and the function of correcting for production efficiency. For this second function, the optimal tax vector depends on two elements (i) the degree of unit-cost asymmetry in the oligopoly, and (ii) the cost of public fund. The first element is measured by the variance of the statistical distribution of unit costs before and after taxation (this variance is related to the Herfindahl index.) The second element is measured by γ and reflects the trade-off between profits and tax revenue.

Does the optimal Pigouvian tax structure increase or decrease the concentration of the industry? Before answering this question, it is necessary to examine the relationships among the variance of the distribution of the unit costs, the Herfindahl index of concentration, industry profit, and welfare. We now state a number of lemmas concerning these relationships. First, recall that the Herfindahl index of concentration is

$$\mathcal{H} = \sum_{i \in I} \left[\frac{q_i}{Q}\right]^2$$

Given that there are n firms, this index attains its maximum value ($\mathcal{H} = 1$) when one firm produces the whole industry's output and the remaning $n - 1$ firms produce zero output, and it attains its *mimimum* value ($\mathcal{H} = 1/n$) when each of the n firms produces $q_i = Q/n$. Now all firms will produce the same amount of output if they have the same tax-inclusive marginal costs.

Lemma 3.1: For a given output level $\widehat{Q}$, the Cournot equilibrium industry profit is an increasing function of the Herfindahl index of concentration.

Proof: Recall that at a Cournot equilibrium, firm i's profit is $\widetilde{\pi}_i = [-\widehat{P'}]\widehat{q}_i^2 + u_i \left[\widehat{z}_i c_i'(\widehat{z}_i) - c_i(\widehat{z}_i)\right] - u_i h_i$. With $c_i(z_i) = \alpha_i z_i$, the industry profit is

$$\widehat{\Pi} = \sum_{i \in I} \widetilde{\pi}_i = \left[-\widehat{P}'\right] \widehat{Q}^2 \widehat{\mathcal{H}} + \text{constant}$$

where

$$\widehat{\mathcal{H}} = \sum_{i \in I} \left[\frac{\widehat{q}_i}{\widehat{Q}} \right]^2 \tag{39}$$

Lemma 3.2: Given the output level $\widehat{Q}$, the Herfindahl index of concentration is an increasing function of the variance $Var(\widehat{\theta})$ of the distribution of the tax-inclusive marginal costs in a Cournot equilibrium.

$$\widehat{\mathcal{H}} = \frac{1}{n} \left[1 + \frac{Var(\widehat{\theta})}{\left[(-\widehat{P}')\widehat{q}_I \right]^2} \right] \geq \frac{1}{n} \tag{40}$$

Thus any policy that maximizes [respectively, minimizes] the variance of the distribution of tax-inclusive marginal costs will maximize [respectively, minimizes] the concentration of the industry, and, for a given $\widehat{Q}$, maximizes the profit of the industry.

Proof:

From (4),

$$\widehat{q}_i = \frac{\widehat{P} - \widehat{\theta}_i}{(-\widehat{P}')} \tag{41}$$

we obtain

$$\begin{aligned} \sum_{i \in I} \widehat{q}_i^2 &= \sum_{i \in I} \frac{1}{(-\widehat{P}')^2} \left[(\widehat{P} - \widehat{\theta}_I) - (\widehat{\theta}_i - \widehat{\theta}_I) \right]^2 \\ &= \frac{1}{(-\widehat{P}')^2} \left[n(\widehat{P} - \widehat{\theta}_I)^2 + \sum_{i \in I} (\widehat{\theta}_i - \widehat{\theta}_I)^2 \right] \\ &= \frac{1}{(-\widehat{P}')^2} \left[n \left((-\widehat{P}')\widehat{q}_I \right)^2 + nVar(\widehat{\theta}) \right] \end{aligned} \tag{42}$$

The result (40) follows from (42) and (39). ■

Proposition 3.2 (A pro-concentration motive theorem)

Assume that all firms have the same emission coefficients: $\epsilon_i = \epsilon$ for all i. Given $2 > \gamma > 1$, the *optimal* firm-specific Pigouvian tax structure *increases* the variance of the statistical distribution of tax-inclusive marginal costs within the oligopoly relative to the variance of the statistical distribution of pre-tax marginal costs[5]. The relationship between the two variances is given

[5]In the empirically unlileky case where $\gamma > 2$, replace "*increases*" by "*decreases*".

by:

$$Var(\widetilde{\theta}) = \Omega(\gamma)Var(\theta^0) \tag{43}$$

where $\theta_i^0 = d_i + \alpha_i$ is the equilibrium marginal cost of firm i in a Cournot equilibrium where all the taxes are zero, and where

$$\Omega(\gamma) \equiv \left[\frac{\gamma}{2(\gamma-1)}\right]^2 > 1.$$

Proof: First, note that if all the taxes are zero, then

$$\theta_i^0 - \theta_I^0 = (d_i - d_I) - (d_I + \alpha_I) \tag{44}$$

Recall that $\widehat{q}_i$ denote the equilibrium Cournot output of firm i given an arbitrary vector of firm-specific taxes, and $\widetilde{q}_i$ is the equilibrium Cournot output of firm i when the taxes are optimized. From (28), and (41), which is true also when the tilda replaces the hat,

$$\widetilde{q}_i - \widetilde{q}_I = -\frac{(\widetilde{\theta}_i - \widetilde{\theta}_I)}{\left[-\widetilde{P}\prime\right]} = -\frac{(a_i - a_I)}{2(\gamma-1)\left[-\widetilde{P}\prime\right]}$$

Hence

$$\widetilde{\theta}_i - \widetilde{\theta}_I = \frac{\gamma}{2(\gamma-1)}\left[(d_i+\alpha_i) - (d_I+\alpha_I) + (\delta_i - \delta_I)\right] \tag{45}$$

Therefore

$$\widetilde{\theta}_i - \widetilde{\theta}_I = \frac{\gamma}{2(\gamma-1)}\left[(\theta_i^0 - \theta_I^0) + (\delta_i - \delta_I)\right]$$

$$Var(\widetilde{\theta}) = \left[\frac{\gamma}{2(\gamma-1)}\right]^2\left[Var(\theta^0) + Var\delta + 2cov(\theta^0, \delta)\right] \tag{46}$$

If $\epsilon_i = \epsilon$ for all i, then, in view of (1) and (11), (46) reduces to (43). Note that $\Omega(\gamma) > 1$ if $1 < \gamma < 2$.

Remark: The intuition behind the pro-concentration motive theorem is as follows. If $\delta_i = \delta$ for all i, the marginal cost of public fund is within the empirically likely range ($1 < \gamma < 2$), then, for any given industry output level, the optimal firm-specific tax structure increases the variance of marginal costs (from $Var(\theta^0)$ to $\Omega(\gamma)Var(\theta^0)$) by taxing more efficient firms at a lower rate, see (34), because this helps the lower cost firms to expand output relative to the higher cost firms, and as a result improves productive efficiency. However, if γ is great, the tax revenue becomes a very important consideration, and it becomes optimal to tax more efficient firms at a higher rate, so as to generate more revenue. Take for example the case of a duopoly, where firm 2 has higher production cost. For a given level of industry output $\widehat{Q}$, we must maintain $t_1+t_2 =$ constant, say $2\bar{t}$. From an initial assignment $(t_1, t_2) = (\bar{t}, \bar{t})$, consider deviation of t_2 from $\bar{t}$, say $t_2 = \bar{t} + \kappa$, and hence $t_1 = \bar{t} - \kappa$. An increase in κ yields marginal gain in production efficiency, because the same level of industry output $\widehat{Q}$ is produced, but the lower cost firm increases its output and the higher cost firm reduces its output. However, an increase in κ by $\Delta\kappa$ implies reduced tax revenue, by approximately $(\widehat{q}_1 - \widehat{q}_2)\Delta\kappa$,(plus the effect of induced changes in composition of industry output) and this implies increased distortion cost, approximately $(\gamma - 1)\Delta\kappa(\widehat{q}_1 - \widehat{q}_2)$. For a given $\gamma > 1$, the optimal deviation κ^* is at the point where the marginal gain in productive efficiency is equated to the marginal increase in distortion cost. Clearly, a higher γ shifts the marginal distortion cost upwards, implying a smaller κ^*.

4 Pollution Standards and Choice of Technology with Set-up Costs

The model of sections 2 and 3 is appropriate if there is a fixed relationship between output level and emission. Such a specification is quite realistic when the time horizon is sufficiently short. However, if we are dealing with a longer run optimization problem, we must take into account the fact that firms will be able to choose among various techniques of production, some of which are less polluting then others. The cleaner the technique, the higher is the set-up cost. We now turn to this type of long run considerations while retaining the oligopoly framework. Unlike the preceding section, we will focus on a different instrument of regulation: the use of firm-specific pollution standards. By this, we mean that the regulator sets an upper limit, denoted by $\bar{e}_i$, on pollution emission per period for firm i.

We assume that the actual level of emission generated by firm i is denoted by e_i. The amount e_i is a function of the output q_i and a technology parameter μ_i chosen by firm i. A greater μ_i indicates a more polluting technology.

Thus, we assume

$$e_i = e_i(q_i, \mu_i), \qquad \frac{\partial e_i}{\partial \mu_i} > 0, \qquad \frac{\partial e_i}{\partial q_i} > 0 \tag{47}$$

In choosing q_i and μ_i, the firm i must make sure that

$$e_i(q_i, \mu_i) \leq \overline{e}_i. \tag{48}$$

If the firm i chooses the technology μ_i, it incurs a **set-up cost** $K_i(\mu_i)$. We assume $K_i'(\mu_i) < 0$, i.e., cleaner techniques involve higher set-up costs. Inverting (47), we get

$$\mu_i = \mu_i(q_i, e_i)$$

It follows that

$$K_i = K_i\left[\mu_i(q_i, e_i)\right] = K_i(q_i, e_i)$$

Firm i's profit function is

$$\pi_i = P(Q)q_i - d_i q_i - u_i c_i\left(\frac{q_i}{u_i}\right) - K_i(q_i, e_i) - u_i h_i \tag{49}$$

We are dealing with optimal regulation by means of choosing firm-specific standards. Clearly, the problem would be trivial if at the social optimum, none of the n constraints (48) bind. In what follows, we assume that all these constraints hold with equality. Thus we will use e_i and $\bar{e}_i$ interchangeably. We suppose that firm i takes $\bar{e}_i$ as given, and chooses q_i to maximize its profit, given the outputs of other firms. Then the first order condition for the output choice of firm i is

$$\widehat{q}_i P'(\widehat{Q}) + P(\widehat{Q}) = \theta_i \equiv d_i + c'(\widehat{z}) + \frac{\partial K_i(\widehat{q}_i, e_i)}{\partial q_i} \tag{50}$$

For concreteness, we focus on the case where the production cost is linear in output $c'(\widehat{z}) = \alpha_i$ and the emission function is linear in the technology choice parameter μ,

$$e_i = e_i(q_i, \mu_i) = \mu_i q_i \tag{51}$$

Thus μ_i may be called the *pollution content* per unit of output. We specifies that

$$K_i(\mu_i) = \bar{K} - \mu_i \tag{52}$$

where μ_i can be chosen from the interval $[0, \bar{K}]$. If $\mu_i = \bar{K}$ (a very polluting technology) then the set-up cost is zero. If the firm chooses a very clean technology, i.e., $\mu_i = 0$, then the set-up cost is $\bar{K}$.

The specification (51) implies that, *given the maximum permissible emission* $\overline{e}_i$, if the firm wants to produce the amount q_i, it must choose the technology

$$\bar{\mu}_i = \frac{\overline{e}_i}{q_i}$$

or a cleaner technology, $\mu_i \leq \bar{\mu}_i$. But cleaner technologies cost more. So the firm has no incentive to choose any technology other than $\bar{\mu}_i$. The cost K_i can therefore be expressed as a function of $\overline{e}_i$ and q_i:

$$K_i(q_i, \overline{e}_i) = \bar{K}_i - \frac{\overline{e}_i}{q_i} \tag{53}$$

For simplicity, from this point we will omit the bar over e_i. Then using (53), the average variable cost is

$$\widehat{\rho}_i = d_i + \alpha_i + \frac{\bar{K}_i}{\widehat{q}_i} - \frac{e_i}{\widehat{q}_i^2} \tag{54}$$

and (50) becomes

$$\widehat{q}_i P'(\widehat{Q}) + P(\widehat{Q}) = d_i + \alpha_i + \frac{e_i}{\widehat{q}_i^2} \equiv \widehat{\theta}_i \qquad i \in I \tag{55}$$

From (55), given the vector $(e_1, e_2, ..., e_n)$ we can obtain the vector of equilibrium Cournot outputs $(\widehat{q}_1, \widehat{q}_2, ..., \widehat{q}_n)$. Conversely, if we know the the equilibrium pair $(\widehat{q}_i, \widehat{Q})$ then we can infer from (55) (a) the equilibrium emission by firm i:

$$e_i = e_i(\widehat{q}_i, \widehat{Q}) = \left[\widehat{q}_i P'(\widehat{Q}) + P(\widehat{Q}) - d_i - \alpha_i\right] \widehat{q}_i^2 \tag{56}$$

and (b) the technology choice

$$\mu_i = \frac{e_i}{q_i} = \left[\widehat{q}_i P'(\widehat{Q}) + P(\widehat{Q}) - d_i - \alpha_i\right] \widehat{q}_i \tag{57}$$

Using (54),

$$\widehat{\pi}_i = \left(\widehat{P} - \widehat{\rho}_i\right) \widehat{q}_i - u_i h_i = \left[\left(\widehat{P} - \widehat{\theta}_i\right) + \left(\widehat{\theta}_i - \widehat{\rho}_i\right)\right] \widehat{q}_i - u_i h_i$$

$$= \left[-P'(\widehat{Q})\right] \widehat{q}_i^2 + \left(\widehat{\theta}_i - \widehat{\rho}_i\right) \widehat{q}_i - u_i h_i \tag{58}$$

Therefore the equilibrium profit of firm i is a function of e_i

$$\widehat{\pi}_i = \left[-P'(\widehat{Q})\right] \widehat{q}_i^2 + \left[\frac{2e_i}{\widehat{q}_i}\right] - \bar{K}_i - u_i h_i \tag{59}$$

In this section, by assumption, there are no taxes, because we want to focus on firm-specific pollution standards. Social welfare at a Cournot equilibrium is a weighted sum of of consumers' surplus (net of pollution damage cost) and profits:

$$\widehat{W} = \beta S(\widehat{Q}) - \sigma \widehat{E} + \sum_{i \in I} \widehat{\pi}_i \tag{60}$$

To solve the welfare maximization problem, we may proceed in two different (but equivalent) ways. The first way is to express the $\widehat{q}_i$ as functions of the vector $(e_1, e_2, ..., e_n)$, so that $\widehat{W} = \widehat{W}(e_1, e_2, ..., e_n)$, and we maximize $\widehat{W}$ by choosing the vector $(e_1, e_2, ..., e_n)$, which can be interpreted as pollution quotas (or standards) that are assigned to firms. The alternative way is to make use of the fact that there is an inverse relationship, given by (56), so that we can formulate the problem as one of choosing the $\widehat{q}_i$. In what follows, we use this approach, because it is mathematically simpler, and its methodology is essentially identical to the one we used in the Pigouvian tax case. This vindicates our claim in the introduction section that we have found a unified approach for both types of regulations.

Substituting (56) into (59), we get

$$\widehat{\pi}_i = 2\widehat{P}\widehat{q}_i - \bar{K}_i - u_i h_i - \left[2(d_i + \alpha_i)\widehat{q}_i + \left(-\widehat{P}'\right) \widehat{q}_i^2\right] \tag{61}$$

and industry profit is

$$\widehat{\Pi} = 2\widehat{P}\widehat{Q} - n\bar{K}_I - H - \sum_{i \in N} \left[2(d_i + \alpha_i)\widehat{q}_i + \left(-\widehat{P}'\right) \widehat{q}_i^2\right] \tag{62}$$

Social welfare, expression (60) can be written as

$$\widehat{W} = F(\widehat{Q}) - \sum_{i \in I} f_i(\widehat{q}_i, \widehat{Q}) \tag{63}$$

where

$$f_i(\widehat{q}_i, \widehat{Q}) = 2(d_i + \alpha_i)\widehat{q}_i + \left(-\widehat{P}'\right) \widehat{q}_i^2 + \sigma e_i(\widehat{q}_i, \widehat{Q})$$

where, as can be seen from (56), $e_i(\widehat{q}_i, \widehat{Q})$ is a cubic function of $\widehat{q}_i$, for given $\widehat{Q}$. To maximize (63), we use the familiar two-step procedure, just as in the Pigouvian tax case. In the *first step,* we fix $\widehat{Q}$, and choose the $\widehat{q}_i$, $i \in I$, to solve

$$\max_{\widehat{q}_i} F(\widehat{Q}) - \sum_{i \in I} f_i(\widehat{q}_i, \widehat{Q}) \tag{64}$$

subject to

$$\sum_{i \in I} \widehat{q}_i = \widehat{Q} \tag{65}$$

The objective function (64) is *strictly concave* in the $\widehat{q}_i$, $i \in I$, *if* $\partial^2 f_i / \partial \widehat{q}_i^2 > 0$. Now

$$\frac{\partial^2 f_i}{\partial \widehat{q}_i^2} = 2\left(-\widehat{P}'\right)[1 - 3\sigma \widehat{q}_i] + 2\sigma\left[\widehat{P} - d_i - \alpha_i\right] > 0$$

Since $\widehat{P} - d_i - \alpha_i > 0$ from (55), a sufficient condition for $\partial^2 f_i / \partial \widehat{q}_i^2 > 0$ is $1 - 3\sigma \widehat{q}_i > 0$. Let $\bar{Q}$ be defined by $P(\bar{Q}) = 0$. Then, given that $\widehat{q}_i < \bar{Q}$, $1 - 3\sigma \widehat{q}_i > 0$ if $\sigma > 1/(3\bar{Q})$. We assume that this inequality holds. Then (64) is *strictly concave* in the $\widehat{q}_i$. Let λ be the Lagrange multiplier associated with (65). Then in the first step, we obtain the necessary condition

$$\frac{\partial f_i}{\partial \widehat{q}_i} = \lambda$$

from which we can express $\widehat{q}_i$ as a function of λ and $\widehat{Q}$:

$$\widehat{q}_i(\lambda, \widehat{Q}) = \frac{1}{6\sigma \widehat{P}'}\left\{2\widehat{P}' - 2\sigma(\widehat{P} - d_i - \alpha_i) + \sqrt{Y_i}\right\}$$

where

$$Y_i \equiv 4\left[\left(-\widehat{P}'\right) + \sigma\left(\widehat{P} - d_i - \alpha_i\right)\right]^2 + 24\sigma\left(-\widehat{P}'\right)[2(d_i + \alpha_i) - \lambda]$$

The equation $\sum_{i \in I} \widehat{q}_i(\lambda, \widehat{Q}) = \widehat{Q}$ determine $\lambda(\widehat{Q})$.

In step 2, we find the optimal value for $\widehat{Q}$, which we denote as $\widetilde{Q}$. (This is similar to the method used in Section 2.5.2, so we will not give details here.) We then obtain $\widetilde{\lambda} = \lambda(\widetilde{Q})$ and $\widetilde{q}_i = \widehat{q}_i(\widetilde{\lambda}, \widetilde{Q})$. The optimal pollution quota (standard) for firm i is then, from (56)

$$\widetilde{e}_i = \left[\widetilde{q}_i P'(\widetilde{Q}) + P(\widetilde{Q}) - d_i - \alpha_i\right] \widetilde{q}_i^2$$

Proposition 4.1: (Optimal standards in the presence of set-up costs: equal treatment of equals) Assume that firms can choose their *pollution content parameter* μ_i at the cost given by (52), and $\sigma > 1/(3\bar{Q})$. Then if all firms are ex ante identical, the social planner will give them equal treatment.

Remark: The "equal treatment of equals" result follows from the concavity of the social welfare function. In the next section, we will show that, for certain abatement cost functions, the social welfare function is convex in the equilibrium outputs, and it will then be optimal to have unequal treatment of equals.

5 Pollution Standards and Abatement Costs

We now turn to a model in which firms can reduce emission at any given output level, by incurring abatement costs (which is a function of both the output level and the emission level. Unlike the preceding section, there is no set-up cost. As an illustration, consider the following example.

Example 1: A firm produces an output q by using raw material M and a composite input Y which itself is produced using three inputs, labor, L, capital, K, and emission, e. Assume the production function is

$$q = \min\{M, Y\}$$

where

$$Y = F(L, K, e) = \sqrt{(1 + \min\{e, z\})KL} \tag{66}$$

where z is a positive constant. In this example, for any given level of output, the emission level e can be reduced by increasing K or L. Thus if the regulating agency reduces the maximum permissible $\overline{e}$, the firm can comply by increasing K without reducing q. In this sense, investment in K is a pollution-abating activity.

Let c be the unit cost of raw material.The wage rate is w and the rental rate is r, while emission costs nothing. In the absence of regulation, the firm will choose $e = z$, to save on capital and labour inputs. Suppose, however, that the firm is not allowed to emit more than $\overline{e}$. We assume $\overline{e} < z$. Then the firm's cost of producing q units of output can be derived from the following cost minimization problem with respect to K, M, L and e :

$$\min[rK + wL + cM]$$

subject to

$$\sqrt{(1+\min\{e,z\})KL} \geq q$$

$$M \geq q$$

and

$$e \leq \overline{e}$$

Since by assumption, $\overline{e} < z$, the above minimization problem yields the cost function

$$C(c,w,r;q;\overline{e}) = cq + [2(rw)^{1/2}]\frac{q}{[1+\overline{e}]^{1/2}} \equiv cq + A(\overline{e},q)$$

where the dependence of A on (r,w) has been suppressed for notational simplicity. The function $A(\overline{e},q)$ may be called the "abatement cost"function: for any given q, to achieve a smaller $\bar{e}$, the firm must incur a greater cost, that is, $\partial A/\partial \bar{e} < 0$.**(End of example.)**

In what follows, we adopt a more general abatement cost function, and write

$$A_i(\overline{e}_i, q_i) = a_i(\overline{e}_i)v(q_i) \tag{67}$$

with $a_i(\overline{e}_i) > 0$ for all $\overline{e}_i \geq 0$, $a_i'(\overline{e}_i) < 0$, $a_i''(\overline{e}_i) \geq 0$, $v'(q_i) > 0$, $v''(q_i) \geq 0$, and $v(0) = 0$. Thus A_i is convex in both arguments, and $A_i(\overline{e}_i, 0) = 0$.Then firm i's profit is

$$\pi_i = q_i P(Q) - c_i q_i - A_i(q_i, e_i) \tag{68}$$

(Note that there is a formal similarity between (68) and (49), but in the specification (53), K_i is concave in q_i while, from (67), A_i is convex in q_i, with $A_i(\overline{e}_i, 0) = 0$.)

We assume that the regulating agency specifies an amount $\overline{e}_i$ (i.e., maximum pollution per period) that firm i must not exceed. We take it that the fines for violation are sufficiently high to ensure perfect compliance. It follows that if the firm wants to produce quantity q_i then it must spend the amount $A_i(\overline{e}_i, q_i)$. We call $\overline{e}_i$ the "firm-specific emission standard" set by the regulatory agency.

We wish to determine the optimal configuration of firm-specific standards that maximizes social welfare, given the constraints that firms are oligopolists. As usual, social welfare consists of consumers' surplus, profits, minus the environmental damage caused by emissions:

$$W = S(Q) + \sum_{i \in I} \pi_i - D(E) \tag{69}$$

where $E = \sum_{i \in N} e_i$ and $D(E)$ is the damage function. For simplicity, assume that the damage function is linear: $D(E) = \sigma E$. We assume that the *only* policy instrument that the agency has is the setting of standards.

Before determining the optimal firm-specific standards, we must investigate how these standards affect the pattern of production and profits within the oligopoly. It is instructive to begin our analysis with an example.

Example 2: (Asymmetric emission standards, applied to identical firms, can improve profits and welfare)

Consider a duopoly with *identical* firms facing a linear inverse demand function $P = 1 - Q$. Assume the function $A(\overline{e}, q)$ takes the form

$$A(\overline{e}, q) = 3a(\overline{e})q^2$$

where $a' < 0$ and $a'' > 0$, and in particular, $a(0.5) = 1/2$, $a(0.4) = 3/2$, and $a(0.6) = 1/4$. Suppose that the emission standards are fixed at $\overline{e}_1$ and $\overline{e}_2$. Then, the equilibrium outputs satisfy the first order conditions (assuming an interior solution):

$$-\hat{q}_1 + (1 - \hat{Q}) = 6a(\overline{e}_1)\hat{q}_1 \equiv \theta_1$$

$$-\hat{q}_2 + (1 - \hat{Q}) = 6a(\overline{e}_2)\hat{q}_1 \equiv \theta_2$$

Adding these two equations yields

$$\hat{Q} + 2(1 - \hat{Q}) = \theta_1 + \theta_2 \equiv 2\theta_N$$

It follows from this equation that the regulator can change the firm-specific pollution standards in such a way that θ_N remains unchanged (implying that $\hat{Q}$ is unchanged) with the possibility of improving welfare. To see this, assume that initially $\overline{e}_1 = \overline{e}_2 = 0.5$. Then equilibrium outputs are $\hat{q}_1 = \hat{q}_2 = 1/6$ and $\hat{Q} = 1/3$. Equilibrium price is $2/3$, and equilibrium marginal costs are $\theta_1 = \theta_2 = 1/2$. Now suppose the regulator wants to keep θ_N at $1/2$, while increasing θ_1 to $6/10$ and decreasing θ_2 to $4/10$. Then $\hat{Q}$ remains unchanged, and

$$\hat{q}_1 = (1 - \hat{Q}) - \theta_1 = 2/30$$

$$\hat{q}_2 = (1 - \hat{Q}) - \theta_2 = 8/30$$

This implies that the new standards, denoted by $\overline{e}_1^*$ and $\overline{e}_2^*$ must satisfy

$$a(\overline{e}_1^*) = \frac{\theta_1}{6\hat{q}_1} = 3/2$$

$$a(\overline{e}_2^*) = \frac{\theta_2}{6\hat{q}_2} = 1/4$$

Our assumptions on $a(.)$ then imply that $\overline{e}_1^* = 0.4$ and $\overline{e}_2^* = 0.6$. It follows, in this example, that aggregate emission, industry output and price are unchanged. The sum of profits increases, however. Before the changes in standard, both firms earn a profit of 1/60 each. After the changes, firm 1's profit is 22/900 and firm 2's profit is 112/900. Industry profit rises from 1/30 to 134/900. **(End of example 2)**

The above example was constructed in such a way that when the regulatory agency changes the emission standards, the total emission does not change, while industry profit increases, because the low-cost firm expands its output, and the high-cost firm reduces its output. In general, however, one would expect that there are cases where the regulator must make a tradeoff between pollution and profit. We now show that even in these cases, welfare can be increased by setting non-identical firm-specific standards.

Given e_i, firm i's marginal cost of production is $c_i + a_i(e_i)v'(q_i)$ Then, if $\hat{Q}$ is the Cournot equilibrium industry output, firm i's *equilibrium* output satisfies

$$\hat{q}_i P'(\hat{Q}) + P(\hat{Q}) = c_i + a_i(e_i)v'(\hat{q}_i) \equiv \theta_i \tag{70}$$

where θ_i is firm i's marginal cost at a Cournot equilibrium. We will exploit the following *equilibrium* relationship between e_i and $\hat{q}_i$, for a given $\hat{Q}$:

$$a_i(e_i) = \frac{P'(\hat{Q})\hat{q}_i + P(\hat{Q}) - c_i}{v'(\hat{q}_i)} \tag{71}$$

That is,

$$e_i(\hat{q}_i, \hat{Q}) = a_i^{-1}\left[\frac{\hat{q}_i\hat{P}' + \hat{P} - c_i}{v'(\hat{q}_i)}\right] \tag{72}$$

Thus in equibrium, firm i's abatement cost is

$$a_i(e_i)v(\hat{q}_i) = \frac{[P'(\hat{Q})\hat{q}_i + P(\hat{Q}) - c_i]\hat{q}_i}{\eta(\hat{q}_i)} \tag{73}$$

where $\eta(\hat{q}_i)$ is defined as the elasticity of $v(q_i)$: $\eta(q_i) = q_i v'(q_i)/v(q_i)$.

Equilibrium profit of firm i is, from (68) and (73),

$$\hat{\pi}_i = \hat{q}_i\{\hat{P} - c_i\} + \{[-\hat{P}']\hat{q}_i - (\hat{P} - c_i)\}[\hat{q}_i/\eta(\hat{q}_i)]$$
$$= (\hat{P} - c_i)\left[1 - \frac{1}{\eta(\hat{q}_i)}\right]\hat{q}_i + [-\hat{P}']\frac{\hat{q}_i^2}{\eta(\hat{q}_i)} \tag{74}$$

Industry profit in equilibrium is

$$\hat{\Pi} = \sum_i \hat{\pi}_i = \hat{Q}^2[-\hat{P}']\widehat{\mathcal{H}} + \sum_i (\hat{P} - c_i)\left[1 - \frac{1}{\eta(\hat{q}_i)}\right]\hat{q}_i$$

where $\widehat{\mathcal{H}}$ is a "modified Herfindahl index" of concentration:

$$\widehat{\mathcal{H}} = \sum_{i \in I} \frac{\hat{q}_i^2}{\eta(\hat{q}_i)\hat{Q}^2}$$

Using (69), (74), and (72), we can express social welfare as

$$\widehat{W} = S(\hat{Q}) - \sum_{i \in N} f_i(\hat{q}_i, \hat{Q}) \tag{75}$$

where

$$f(\hat{q}_i, \hat{Q}) \equiv -[P(\hat{Q}) - c_i]\left[1 - \frac{1}{\eta(\hat{q}_i)}\right]\hat{q}_i$$
$$+\hat{P}'\frac{\hat{q}_i^2}{\eta(\hat{q}_i)} + \sigma e_i(\hat{q}_i, \hat{Q}) \tag{76}$$

For any **given** $\hat{Q}$, the regulator can choose the $\hat{q}_i$'s to maximize social welfare subject to $\sum_{i \in I} \hat{q}_i = \hat{Q}$. An interesting property of the social welfare function (75) is that, under certain reasonable assumptions, it is *convex* in the $\hat{q}_i$'s, for a given $\hat{Q}$. For example, we obtain this convexity property if $v(q) = q$, $a(e) = B - e$ where $B > 0$, and $P(Q) = 1 - Q$. We can now the following proposition.

Proposition 5.1: When pollution abatement cost is of the form given by (67), optimal standards satisfy the following properties:

(i) For a given level of industry output, $\hat{Q}$, the optimal firm-specific pollution standards are

$$\widetilde{e}_i = a_i^{-1}\left[\frac{\widetilde{q}_i\widetilde{P}'(\hat{Q}) + \widetilde{P}(\hat{Q}) - c_i}{v'[\widetilde{q}_i]}\right]$$

(ii) **(Unequal treatment of equals)**: Assume the social welfare function (75) is convex in the $\hat{q}_i$'s. Then at any given $\hat{Q}$, the optimum choice of the $\hat{q}_i$'s is achieved by giving non-identical treatments to identical firms.

6 Concluding remarks

We have developed a general method of solving for optimal firm-specific measures for regulating a polluting oligopoly. The method applies to two different methods of regulating polluting firms: firm-specific Pigouvian taxes, and firm-specific standards. This method can also be applied to problems such as optimal allocation of tradeable pollution permits, and delocation. (See Long and Soubeyran, 2001b,c.)

Our method makes use of the properties of the inf-convolution approach in mathematical programming, using duality. It has an elegant geometric interpretation. Several important insights emerge. It is shown that optimal firm-specific regulations are partly driven by the motive to increase the industry concentration, because increased concentration can enhance productive efficiency. However, tax revenue can be an important consideration, and any increase in the marginal cost of public funds would lead to an increased tax rate on the more efficient firms. Whether the degree of concentration is increased or decreased by regulation depends on the trade-off between productive efficiency and the cost of public funds. Another interesting results is that it may be optimal to give unequal treatments to ex ante identical firms.

Our analysis can be extended to study the role of strategic trade policy in the presence of a polluting international oligopoly. There are a number of insightful papers that deal with this topics (Conrad (1993), Barrett (1994), Kennedy (1994), Ulph and Ulph (1996), Ulph (1996a,b), Neary (1999)). However, the possibility of unequal treatments of equals was not explored in these papers, because the models did not allow for asymmetry within the domestic industry, and firm-specific taxes or standards were ruled out.

APPENDIX 1

Specific Pigouvian Taxes with Non-linear Costs

We now examine the the case where $c_i(z_i)$ is strictly convex. To simplify the exposition, we assume that the marginal cost of public fund is unity:

$\gamma = 1$. In this case, the functions $f_i(\widehat{q}_i, \widehat{Q})$ become

$$f_i(\widehat{q}_i, \widehat{Q}) = (d_i + \delta_i)\,\widehat{q}_i + u_i c_i \left(\frac{\widehat{q}_i}{u_i}\right) = g_i(\widehat{q}_i)$$

The first stage of the game can be solved in two steps: In step (i), we solve

$$\max_{\widehat{q}_i} \widehat{W} = F\left(\widehat{Q}\right) - \sum_{i \in I} f_i(\widehat{q}_i, \widehat{Q})$$

where

$$F\left(\widehat{Q}\right) \equiv \beta S(\widehat{Q}) + \widehat{Q}P(\widehat{Q}) - H \tag{77}$$

subject to

$$\sum_{i \in I} \widehat{q}_i = \widehat{Q}$$

where $\widehat{Q}$ is given, and $q_i \geq 0$. In step (ii), we determine the optimal $\widehat{Q}$.

To solve step (i), we form the Lagrangian

$$\widehat{L} = F\left(\widehat{Q}\right) - \sum_{i \in I} f_i(\widehat{q}_i, \widehat{Q}) + \lambda \left[\sum_{i \in I} \widehat{q}_i - \widehat{Q}\right]$$

or

$$\widehat{L} = F\left(\widehat{Q}\right) - \lambda \widehat{Q} + \sum_{i \in I} \left[\lambda \widehat{q}_i - f_i(\widehat{q}_i, \widehat{Q})\right]$$

We obtain the first order conditions

$$\frac{\partial f_i}{\partial \widehat{q}_i} = d_i + \delta_i + c_i'(\widetilde{z}_i) = \lambda$$

hence

$$\widetilde{z}_i = c_i^x(\lambda - d_i - \delta_i)$$

where $c_i^x(.)$ is the inverse function of $c_i'(.)$. Then $\widetilde{q}_i(\lambda) = u_i c_i^x(\lambda - d_i - \delta_i)$. The equation $\sum_{i \in I} \widetilde{q}_i(\lambda) = \widehat{Q}$ determines a unique $\lambda(\widehat{Q})$.

The second step: We now determine the optimal $\widehat{Q}$. We follow the duality method used in Rockafellar (1970). Following Rockafellar, we define the conjugate function

$$f_i^*(\lambda, \widehat{Q}) = \sup_{\widehat{q}_i} \left[\lambda \widehat{q}_i - f_i(\widehat{q}_i, \widehat{Q})\right]$$

then

$$f_i^*(\lambda, \widehat{Q}) = \lambda \widetilde{q}_i(\lambda) - f_i(\widetilde{q}_i(\lambda), \widehat{Q})$$

It follows that the optimal value of the Lagrangian (optimized with respect to the $\widehat{q}_i$) is

$$\widetilde{L}(\widehat{Q}) = \left[F(\widehat{Q}) - \lambda(\widehat{Q})\widehat{Q}\right] + \sum_{i \in I} f_i^*(\lambda(\widehat{Q}), \widehat{Q})$$

Differentiating $\widetilde{L}(\widehat{Q})$ with respect to $\widehat{Q}$ and equating it to zero yields

$$F'(\widehat{Q}) - \widehat{Q}\frac{d\lambda}{d\widehat{Q}} - \lambda(\widehat{Q}) + \sum \frac{\partial f_i^*}{\partial \lambda}\frac{d\lambda}{d\widehat{Q}} + \sum \frac{\partial f_i^*}{\partial \widehat{Q}} = 0 \tag{78}$$

Since

$$\frac{\partial f_i^*}{\partial \lambda} = \widetilde{q}_i(\lambda) + \left[\lambda - \frac{\partial f_i}{\partial \widehat{q}_i}\right]\frac{\partial \widetilde{q}_i}{\partial \lambda} = \widetilde{q}_i(\lambda)$$

(78) reduces to

$$F'(\widehat{Q}) - \widehat{Q}\frac{d\lambda}{d\widehat{Q}} - \lambda(\widehat{Q}) + \sum_{i \in I} \widetilde{q}_i(\lambda(\widehat{Q}))\frac{d\lambda}{d\widehat{Q}} = 0$$

or

$$F'(\widehat{Q}) - \lambda(\widehat{Q}) = 0 \tag{79}$$

This equation determines the optimal $\widetilde{Q}$.

Now, from (77)

$$F'(\widehat{Q}) = \widehat{P} - (1-\beta)\left[-\widehat{P}'\right]\widehat{Q} \tag{80}$$

From (79) and (80),

$$\widehat{P} - \lambda = (1-\beta)\left[-\widehat{P}'\right]\widehat{Q} \tag{81}$$

.

The difference between the optimal per unit tax and the marginal damage is given by

$$\tau_i - \delta_i = \widetilde{P} + \widetilde{P}'\widetilde{q}_i - \left[(d_i + \delta_i) + c_i'(\widetilde{z}_i)\right]$$

But, recall that

$$(d_i + \delta_i) + c_i'(\widetilde{z}_i) = \lambda \tag{82}$$

Therefore

$$\tau_i - \delta_i = \left[\widetilde{P} - \lambda\right] + \widetilde{P}'\widetilde{q}_i \tag{83}$$

From (81) and (83),

$$\tau_i - \delta_i = \left[-\widetilde{P}'\right] \widetilde{Q} \left[(1-\beta) - \frac{\widetilde{q}_i}{\widetilde{Q}}\right]$$

Thus the specific Pigouvian tax t_i on pollution by firm i is

$$t_i \equiv \frac{\tau_i}{\epsilon_i} = \delta + \frac{1}{\varepsilon_i} S'(\widetilde{Q}) \left[(1-\beta) - \frac{\widetilde{q}_i}{\widetilde{Q}}\right] \tag{84}$$

We conclude that (i) t_i is greater, the greater is the marginal damage cost, (ii) t_i is negatively related to the weight attached to consumers' surplus, and (iii) in equilibrium, among all firms that have the same emission coefficient ε_i, smaller firms are taxed at a higher rate. This is because smaller firms are less efficient, and optimal policy seeks to reduce their outputs.

Optimal policy also favors firms with more plants. To see this, consider two firms, say firm i and firm j with $d_i = d_j$, $\delta_i = \delta_j$, and the same cost function *at the plant level*, i.e., $c_i(.) = c_j(.)$. Then, from (82), $\widetilde{z}_i = \widetilde{z}_j$. It follows that $u_i > u_j$ then $\widetilde{q}_i > \widetilde{q}_j$, and therefore, from (84), firm i will pay less tax per unit of output than firm j. Intuitively, this is because, *at the firm level,* firm i has a lower marginal cost curve. It is in this sense a more efficient firm, and accordingly it is better treated. (This happens only under oligopoly; under perfect competition, both firms would be taxed at the same rate.)

Remark: (79) is a first order condition. The second order condition is

$$F'' - \frac{d\lambda}{d\widetilde{Q}} < 0$$

where $d\lambda/d\widetilde{Q}$ is obtained from differentiating the constraint $\sum_{i\in I} \widetilde{q}_i(\lambda(\widetilde{Q})) = \widetilde{Q}$

$$\sum_{i\in I} \frac{d\widetilde{q}_i}{d\lambda}\frac{d\lambda}{d\widetilde{Q}} = 1$$

and where $d\widetilde{q}_i/d\lambda$ is obtained from (82)

$$\frac{d\widetilde{q}_i}{d\lambda} = \frac{u_i}{c_i''(\widetilde{z}_i)}$$

Thus

$$\frac{d\lambda}{d\widetilde{Q}} = \sum_{i \in I} \frac{u_i}{c_i''(\widetilde{z}_i)} > 0$$

Now $F'' = (2-\beta)\widetilde{P}' + (1-\beta)\widetilde{P}''\widetilde{Q}$. Recall that $2\widetilde{P}' + \widetilde{P}''\widetilde{Q} < 0$ is the stability condition for Cournot oligopoly. So if β is small enough, then $F'' < 0$.

APPENDIX 2
The Duality Approach

The following is the outline of the duality approach contained in Rockafellar (1970). Consider the problem

$$\max_{x_i} J = F(X) - \sum_{i \in I} f_i(x_i, X)$$

subject to

$$\sum_{i \in I} x_i = X, \qquad x_i \geq 0, i \in I$$

where X is given, $f_i(x_i, X)$ convex with respect to x_i and differentiable with respect to (x_i, X), and proper ($f_i(x_i, X)$ is never $-\infty$, and is not identically $+\infty$).

To solve this problem, define the extended functions $g_i(x_i, X) = f_i(x_i, X)$ if $x_i \geq 0$ and $g_i(x_i, X) = +\infty$ if $x_i < 0$. Then we have the program

$$\max_{x_i} J = F(X) - \sum_{i \in I} g_i(x_i, X)$$

subject to

$$\sum_{i \in I} x_i = X, \qquad i \in I$$

For agiven X, the Lagrangian of this problem is

$$L(x, \lambda, X) = [F(X) - \lambda X] + \sum_{i \in I} [\lambda x_i - g_i(x_i, X)]$$

where $x = (x_1, ..., x_n)$.

The saddlepoint duality theorem (see Rockafellar, 1970, pp 284-5) states that $\widetilde{x} = (\widetilde{x}_1, ..., \widetilde{x}_n)$ is an optimal solution of the program if and only if (i) given $\widetilde{\lambda}$, $\widetilde{x}$ maximixes the function $L(x, \widetilde{\lambda}, X)$, and (ii) $\widetilde{\lambda}$ minimizes

$L(\widetilde{x}(\lambda, X), \lambda, X)$ with respect to λ, where $\widetilde{x}(\lambda, X)$ achieves the minimum of $L(x, \lambda, X)$ for each given λ.

The determination of $\widetilde{x}_i(\lambda, X)$ is given by the first order condition of the program

$$\sup_{x_i} [\lambda x_i - g_i(x_i, X)] = g_i^*(\lambda, X)$$

$g_i^*(\lambda, X)$ is called the conjugate function of $g_i(x_i, X)$. We have

$$\frac{\partial g_i^*}{\partial \lambda} = \widetilde{x}_i(\lambda, X) + \left[\lambda - \frac{\partial g_i}{\partial x_i}\right] \frac{\partial \widetilde{x}_i}{\partial \lambda} = \widetilde{x}_i(\lambda, X)$$

We also have

$$\widetilde{L} = L[\widetilde{x}(\lambda, X), \lambda, X] = [F(X) - \lambda X] + \sum_{i \in I} g_i^*(\lambda, X)$$

and $\widetilde{\lambda}(X) = \widetilde{\lambda}$ achieves the minimum of $\widetilde{L}$ with respect to λ. The first order condition for that is

$$\sum_{i \in I} \frac{\partial g_i^*}{\partial \lambda}(\widetilde{\lambda}, X) = X$$

that is,

$$\sum_{i \in I} \widetilde{x}_i(\lambda, X) = X$$

This equation gives $\widetilde{\lambda}(X)$. The optimal $\widetilde{x}_i$ follows: $\widetilde{x}_i(X) = \widetilde{x}_i\left[\widetilde{\lambda}(X), X\right]$, $i \in I$.

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