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Hassan Benchekroun, Ngo Van Long

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Leader and Follower: A Differential Game Model*

Hassan Benchekroun†, *Ngo Van Long*‡

Résumé / Abstract

On analyse un jeu différentiel entre deux joueurs, dont le premier a l'avantage de prendre sa décision avant son adversaire. On compare le profil de stratégies d'équilibre de ce modèle avec celui d'un modèle d'actions symétriques. On démontre que l'existence d'un leader favorise la conservation dans l'exploitation du stock commun. On analyse les déviations possibles à partir d'un équilibre. On démontre que si le leader peux s'engager à une politique d'exploitation plus modérée, alors le suiveur peux répondre plus ou moins agressivement, selon la durée de la période d'engagement.

We consider a differential game between two players, where one player has the first mover advantage. We compare the equilibrium strategy profile of this model with the one generated by a conventional symmetric model. It is shown that the existence of a first mover results in more conservationist exploitation in the aggregate. We also consider the implication of departures from the equilibrium. We show that if the leader (the first mover) can commit to decrease its effort over a finite interval of time, then the follower (the second mover) may respond by increasing, or decreasing, its effort, depending on the length of the commitment period.

Mots Clés : Leadership, jeux différentiels

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* Corresponding Author: Ngo Van Long, CIRANO, 2020 University Street, 25th floor, Montréal, Qc, Canada H3A 2A5 Tel.: (514) 985-4020 Fax: (514) 985-4039 email: longn@cirano.qc.ca
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† Florida Atlantic University

‡ McGill University and CIRANO

1 Introduction

Migratory fish that travel along the coast line of several nations are subject to sequential catching. This has been the cause of many disputes between nations. For example, the Pacific salmon disputes between Canada and the United States of America are due largely to what is perceived by Canada as Alaska's unfair "interceptions". The Canadian salmon, born in Canadian rivers, have the habit of travelling to the Pacific ocean, staying in the high seas for a few years (3 to 7 years, depending on the species) and then returning to Canada for breeding. On their return trip, they travel along the coast line of Alaska before reaching British Columbia. Alaska's fishers have the chance to catch these fish first. This is called "interception"[1]. If they catch all the Canadian salmon that pass by, then there will be no more fish in the future. So they do have an incentive to conserve the resource. Canadian fishers are the second mover, because they observe the Alaskan interceptions before they decide how much to catch[2].

Several interesting theoretical questions arise: (i) what would be a reasonable definition of equilibrium strategies for this type of game? (ii) in what sense this is a leader-follower game? (iii) does the leader have an incentive to "set an example" by restraining its catch rate relative to a situation where the two players are symmetrical? (iv) if an equilibrium is perturbed by one player, how would the other player react? For example, if one player decides and commits to decrease its fishing effort below its equilibrium value for a specified time interval, would the other player react by also decreasing its fishing effort? These questions have not been raised in the existing theoretical literature on common property resource management (Plourde (1970), Sinn (1984), Clemhout and Wan (1985), Bolle (1986), Thomas (1992), Missios and Plourde (1996,1997)), nor in the literature on fish war, which deals only with symmetric situations (Levhari and Mirman (1980), Chiarella et al. (1984), Kaitala and Pohjola (1988), Plourde and Yeung (1989), Fischer and Mirman (1992), Datta and Mirman (1994), Missios and Plourde (1996)) or

[1] According to Huppert (1995) the interceptions by Alaska accounted for 20 per cent landed value of all salmon species caught by Alaskan fishers over the period 1990-91.

[2] The Pacific salmon dispute also involves another player, namely the fishers from Washington/Oregon. We neglect this aspect for simplicity. For technical and institutional aspects of the disputes, see Munro and Stokes (1987), Munro, McDorman and McKelvey (1997), and Huppert (1995). For theoretical analyses of the Pacific salmon disputes, which abstract from the sequential nature of the problem, see Tian (1998), and Miller (1996).

with situations where one of the parties is myopic (Crabbé and Long (1993).)

This paper is an attempt to model this type of sequential fishing game. We formalize this fishing game as a differential game[3], involving one state variable, the fish stock, and two control variables, the harvest rates, or effort levels, of the players. Using the differential game framework, we shall investigate the answer to the above questions. We consider a continuous time model where at each time t, player one (say Alaska) harvests from the stock before player two (say Canada) can harvest. Player two is the second mover: it has the opportunity to observe the rate of harvest of player one, before deciding on its own rate of harvest. Furthermore, we assume that at any t, the harvest rate of player one (the first mover) has a negative impact on the productivity of player two's fishing efforts. We characterize the Stackelberg equilibrium strategy profile, and compare it with the Naive Nash equilibrium. We show that the first mover (i.e., the Stackelberg leader) in fact has an incentive to be more restrained in its harvest. The combined harvest of the two players in this leader-follower equilibrium is smaller than the total harvest in a Naive Nash equilibrium.

We also consider a deviation from the equilibrium strategies. We suppose player one decreases its fishing effort during a fixed period of time.(This may be due to pressure for more conservation from environmental groups in country one.) We show that player two would respond by modifying its effort. The modification of player two's fishing effort is shown to be an increasing function of time. Furthermore, we show that the best response by player two to player one's decreased fishing effort is ambiguous and is sensitive to the length of the period of time during which player one is decreasing its fishing effort. This period of time can be interpreted as the commitment period to a lower fishing effort of player one. When the period of commitment is short, player two increases its catching effort. However, when the leader's period of commitment is long enough, player two responds by decreasing its catching effort for a certain period of time. This decrease could be even greater than player one's deviation. Our analysis therefore suggests that if environmental groups in country one (the leader) want to achieve an overall moderation in the exploitation rate, but manage only to get their own government to commit to a short term decrease in fishing, they may end up causing more fish to be caught by the other country. Thus a good intention might well

[3]See Clemhout and Wan (1994) for a concise treatment of differential games. For a comprehensive treatment of differential games, see Dockner et al. (2000).

result in a bad outcome. Only a sufficiently long period of commitment by the leader would serve the conservationists's objectives.

In section 2, we set up the model, characterize the Stackelberg equilibrium, and compare it with the Naive Nash equilibrium. Section 3 deals with deviation from the equilibrium strategies. Some concluding remarks are offered in section 4.

2 The Model

We use a continuous time model[4]. Let $x(t)$ be the stock of fish at time t. Player one's (Alaska's) catch rate at t is $h_1(t) = \alpha_1(t)x(t)$, where $\alpha_1(t)$ may be interpreted as its fishing effort. Player one's *net* benefits is $R_1(h_1(t))$, which is independent of player two's fishing effort $\alpha_2(t)$, because player one catches first, before player two. The net benefits to player two, on the contrary, depend on both $h_1(t)$ and $h_2(t)$ as well as the stock of fish and are given by $R_2(h_2(t), h_1(t), x(t))$, where $h_2(t) = \alpha_2(t)x(t)$. We assume R_2 is decreasing in h_1 and increasing in x. This may be justified as follows. The greater the catch rate of player one, the more difficult it is for player two, who must fish after player one has fished, to achieve a given catch rate. This negative impact of player one's catch rate on player two's benefits is stronger the lower the fish stock.

We assume that $R_2(h_2(t), h_1(t), x(t))$ takes the multiplicatively separable form

$$R_2(h_2(t), h_1, x(t)) = R(h_2(t)) \left(1 - \frac{h_1(t)}{x(t)}\right).$$

[4] In an earlier version of the paper, a difference-equation game, in discrete time, was used: we adopted a modification of the Levhari-Mirman (L-M,1980) model of fish wars to allow for the sequentiality of the game under consideration. Using the L-M functional forms, with modification to capture sequential moves, we obtained a puzzling result: Canada's harvest strategy turned out to be independent of Alaska's harvesting rate, i.e., Canada would have a dominant harvesting strategy. This puzzling result seems to be attributable to the L-M specific functional forms for the utility function (log utility) and the reproduction function (a power function, concave in the stock). Unfortunately, to our knowledge, these are the only functional forms for which we can compute analytically equilibrium strategies in a discrete-time sequential game. We have therefore decided to try to capture some aspects of sequentiality using a continuous time model: one group of fishers are disadvantaged by the "prior" harvesting by the other group: the externality goes only one way.

In particular, if we impose the restriction that $R(.) = R_1(.)$, then the above equation indicates that if player one and player two catch the same quantity of fish, the net benefits to player two will be lower, because of the externalities generated by player one's catch. The fact that player two is directly affected by player one's catch is captured by the multiplicative term $\left(1 - \frac{h_1(t)}{x(t)}\right)$. The larger the extraction rate of player one and the smaller the available stock of fish the more costly it is for player two to harvest and the lower will be its *net* benefit for a given harvesting level.

The rate of growth of the fish stock is

$$\dot{x}(t) = G(x(t), \alpha_1(t), \alpha_2(t)). \tag{1}$$

The objective of player one is to maximize J^1, the discounted sum of its instantaneous net benefits

$$J^1 = \int_0^\infty R_1(h_1(t))e^{-rt}dt$$

subject to (1), where r denotes the interest rate which we assume, for simplicity, to be the same for both players.
Similarly, the objective of player two is to maximize J^2:

$$J^2 = \int_0^\infty R_2(h_2(t), h_1, x(t))e^{-rt}dt$$

subject to (1).

We seek a Markov perfect Nash equilibrium where each player uses a time-independent Markovian strategy: $\alpha_1 = \alpha_1(x)$ and $\alpha_2 = \alpha_2(\alpha_1(x), x)$. That is, player one conditions its current effort level only on the current stock, and player two conditions its effort level both on the current stock, and on the observed effort level of its rival, whose catch rate is observed by player two. This feature of the player two's catch strategies allows us to capture the fact that the fishing game is sequential: player one catches first and player two is informed of the catch of player one before taking its action. Since the action of player two depends on the action of player one, which it observes before taking its own action, and since player one knows this, and takes this into account, the solution we are seeking is in fact what economists normally call a Stackelberg equilibrium. But, as pointed out by game theorists, and as will be discussed more fully below, a Stackelberg equilibrium is a Nash equilibrium, where the strategy space of the follower is the set of all feasible reaction functions[5].

The Hamilton-Jacobi-Bellman (HJB) equation for player one is

$$rV_1(x) = \max_{\alpha_1}[R_1(\alpha_1 x) + V_1'(x)G(x, \alpha_1, \alpha_2(\alpha_1(x), x))]$$

and the HJB equation for player two is

$$rV_2(x) = \max_{\alpha_2}[R_2(\alpha_2 x)(1 - \frac{\alpha_1(x)x}{x}) + V_2'(x)G(x, \alpha_1(x), \alpha_2)].$$

To proceed further, it is convenient to specify explicit functional forms. Assume the transition equation is linear in x :

$$G(x(t), \alpha_1(t), \alpha_2(t)) = Ax(t) - \alpha_1(t)x(t) - \alpha_2(t)x(t)$$

The assumed linearity allows us to derive analytical solutions. A possible justification is that even if the true function is, say, quadratic[6] in x, when the two players are facing a low stock level, a linear approximation may be a reasonable price to pay for tractability[7]. Linearity implies, in the absence of human harvesting, an exponential rate of growth of the fish stock: the resource growth is neither limited by space nor by food supply (see Dasgupta and Heal (1979), Clark (1990)). This assumption is not unreasonable, because for most kinds of fish that are commercially exploited, it is human harvesting, rather than natural conditions, that is the relevant limit to the growth of the biomass. Their survival is endangered by mass exploitation rather than by natural forces.

The net benefit function of player 1 is assumed to take the simple form

$$R_1(\alpha_1 x) = (\alpha_1 x)^\beta, \qquad 0 < \beta < 1,$$

It is a concave function in the catch rate, $\alpha_1 x$. For example, if the harvest is sold in an international market at a constant price, then net benefit for

[5] See Dockner et al. (2000), for further discussion in the context of differential games. For a similar point, made in the context of homogenous product duopoly, see Binmore (1992).

[6] A quadratic function is a frequently made assumption in the renewable-resource literature.

[7] It is moreover a realistic assumption in the case of stocks where food and space constraints on growth are negligible.

player one is its total revenue minus its total cost. If total revenue is linear and total cost is convex, then net benefit is concave. In the case the harvest is sold in the home country's market segregated from the world market, the net benefit to the home country is taken to be the sum of consumers' surplus and producers' surplus; in this case, since the consumers' surplus is normally concave and the cost function of catching fish is usually convex, it makes sense to assume that the net benefit is concave[8]. This justifies the assumption $\beta < 1$.

For player two, we specify

$$R_2(\alpha_2 x, \alpha_1 x, x) = (\alpha_2 x)^{\beta}(1 - \alpha_1), \qquad 0 < \beta < 1.$$

Notice that if $1 - \alpha_1 \leq 0$, then player two would set its effort level at $\alpha_2 = 0$. The equilibrium would thus be trivial in that case. Henceforth, we will mainly focus on the cases where parameter values ensure that both players will choose positive effort levels.

Our first task is to show that there exists a pair of Markov perfect Nash equilibrium strategies, as described by the following proposition:

Proposition 1

Assume that $r > \beta A$. There exists a pair of Markov perfect Nash equilibrium strategies, where player one's equilibrium strategy is

$$\alpha_1 = \frac{r - \beta A}{1 - \beta} = \bar{\alpha}_1 > 0 \tag{2}$$

and player two's equilibrium strategy is

$$\alpha_2 = \frac{r - \beta A + \beta \alpha_1}{1 - \beta} = \alpha_2(\alpha_1) \tag{3}$$

if $1 - \alpha_1 > 0$, and $\alpha_2 = 0$ if $1 - \alpha_1 \leq 0$. Given this pair of equilibrium strategies, if $1 - \bar{\alpha}_1 > 0$, the equilibrium effort level of player two is

$$\bar{\alpha}_2 = \frac{(r - \beta A)}{(1 - \beta)^2} = \frac{\bar{\alpha}_1}{(1 - \beta)} > \bar{\alpha}_1 \tag{4}$$

Proof: See Appendix A.

Remarks:

[8] The difference between a positive concave function and a positive convex function is a concave function.

(i) The above pair of strategy is a Nash equilibrium in the following sense. Player two takes the number α_1(to be observed) as independent of its action, and finds, from the set of functions $\alpha_2(\alpha_{1,}x)$, a function that maximizes its payoff. This optimal choice is found to be the function described by (3). Player one, taken as given player two's strategy (3), chooses a function $\alpha_1(x)$ that maximizes its payoff. This function is found to have a constant value which is given by (2). This Nash equilibrium is sometimes called a Stackelberg equilibrium, in the sense that player one harvests first (even though one can say that the strategies are chosen simultaneously).

(ii) Notice that, from (4), player two's harvesting effort is greater than that of player one if $1 - \bar{\alpha}_1 > 0$.

(iii) Player two's strategy, which may be called its reaction function, displays the property of strategic complementarity: an increase in α_1 implies that α_2 will increase too[9]. Moreover, if $\beta > 1/2$, the increase in α_2 is greater than the increase in α_1.

(iv) Had player one not known that α_2 is a function, and had it taken α_2 as a given number[10], like in Cournot games, we would have ended up with a pair of values, denoted by $(\widehat{\alpha}_1, \widehat{\alpha}_2)$, where

$$\widehat{\alpha}_i = \frac{r - \beta A}{1 - 2\beta}, \qquad i = 1, 2 \tag{5}$$

we assume $r - \beta A > 0$ and $\beta < 1/2$. Call this "equilibrium" the Naive E-quilibrium. Comparing (5) with (2), we see that the leader's catch rate $\bar{\alpha}_1$is smaller that the catch rate $\widehat{\alpha}_i$ of each player at the Naive Equilibrium. This makes sense: the leader knows that if it catches more, the follower will also increase its effort. Therefore, the leader has an incentive to exercise restrain-t. Moreover, the overall catching effort under the Stackelberg equilibrium $(\bar{\alpha}_2 + \bar{\alpha}_1)$ is less than the catch effort under the Naive equilibrium $(\widehat{\alpha}_1 + \widehat{\alpha}_2)$.

[9] This strategic complimentarity holds also for more general net benefit functions, provided that the elasticity η of the marginal net benefit with the respect to the catch, i.e., $\eta \equiv \frac{R_2'' \alpha_2 x}{R_2'}$, is such that the following condition is satisfied: $\frac{\eta - (r - A + \alpha_1)\eta'}{\eta^2} > 1$. This condition can be derived from the first order condition of the HJB equation associated with player 2's problem together with the use of the envelope theorem applied to that equation. In general, this condition has no direct relationship with the concavity (or otherwise) of the net benefits function. In the special case where η is a constant, the above inequality is equivalent to requiring $0 < \beta < 1$. (A proof is available upon request.)

[10] Not the number $\bar{\alpha}_2$ above, of course.

This means that, contrary to what might be expected, the fact that one player is given the first mover advantage is not worsening the preservation of the resource.

(vi) We have found a Nash equilibrium (with a leader and a follower) on the assumption that the follower assumes that the leader chooses a constant effort level α_1. The follower then solves for its best reply, which we call the reaction function. Using this reaction function, the leader solves its optimal control problem. It turns out that the solution yields a constant effort level, thus justifying the follower's initial assumption.

(vii) The equilibrium found in Proposition 1 has the property that each player's strategy is individually optimal and is unique, *given* the player's strategy. This can be proven by writing down each player's problem, given the specified strategy of the other player, as an optimal control problem, and applying the standard sufficiency theorem. Note, however, the fact that we have found an equilibrium *strategy pair* does not preclude the possibility that there may exist non-linear strategy pairs that also constitute a Nash equilibrium.

(viii) Note that the equilibrium determined in Proposition 1 is not collectively optimal, that is, the sum of net benefits of the two players is not maximized. In addition to a first source of inefficiency due to the absence of property rights over the resource, there is a second source of inefficiency due to the fact that player one ignores its direct negative impact on player two's net benefits.

In what follows, we shall focus on parameter values yielding positive fishing efforts for both players : $0 < \frac{r-\beta A}{1-\beta} < 1$.

3 Departures from the equilibrium

Given player two's reaction function, player one's choice of α_1 maximizes its objective function. This may be interpreted as follows: Suppose player two has the following best reply function (which is a version of (3)):

$$\alpha_2(t) = \frac{r - \beta A + \beta \alpha_1(t)}{1-\beta} = \alpha_2(\alpha_1(t)) \tag{6}$$

which can be a function of t to the extent that $\alpha_1(t)$ can in principle be a function of time. Player one then has an optimal control problem to solve, where $\alpha_1(t)$ is the control variable. We have shown that this control problem has the solution $\alpha_1(t) =$ constant, given by (2). It follows that no deviation from this path, even for a short time interval, could be optimal for the leader, if player two would **continue** to use the rule (6) at all time. But such continuation is not necessarily optimal for player two. In fact, player two would exercise constant effort only if player one does so. Therefore, it does make sense to ask the following question: if player one deviates from the equilibrium strategy $\bar{\alpha}_1$ for an interval of time, how will player two react? One possible reason for such deviation is that the government of country one may be responding to pressures from conservationist groups within the country, by promising a reduction in the catch rate over a specified number of years. (In a different scenario, the government of country one may promise to subsidize its fishing industry over a fixed period, implying an increase in the catch rate.)

We suppose, in the following analysis, that player one decides to *increase* its effort level to $\bar{\alpha}_1 + \varepsilon$ over a specified time interval $[0, \tau]$. (The case of a decrease can be inferred from the analysis by a simple change of sign.) We should note that the initial equilibrium strategies constitute a Markovian equilibrium and therefore remain equilibrium strategies once the aggressive move of player one ends. Our task will then be to determine the reaction of player two over the interval of time $[0, \tau]$. Let $g(t)$ denote the modification of player two's fishing effort over $[0, \tau]$[11].Before proceeding further we should first note that if player one can commit permanently ($\tau = \infty$) to this increase in its catch effort, then player two's best reply is easily derived from (3) and is

$$g(t) = d\alpha_2 = \frac{\beta}{1-\beta} d\alpha_1 > 0$$

For a finite τ, player two's problem is to choose a function g that maximizes the discounted sum of instantaneous profits:

$$\underset{\{g(t)\}}{Max} \int_0^{\tau} ((\bar{\alpha}_2 + g(t))x)^{\beta}(1 - (\bar{\alpha}_1 + \varepsilon))dt + e^{-r\tau} V_2(x(\tau)) \tag{7}$$

subject to

$$\dot{x} = Ax - (\bar{\alpha}_1 + \varepsilon + \bar{\alpha}_2 + g(t))x \tag{8}$$

[11]Note that we could just as well modelize the modification of player two's fishing effort as a function of the fish stock (i.e. a Markovian strategy). However since player two is the "only decision maker" during $[0, \tau]$ (player one is assumed to follow $\bar{\alpha}_1 + \epsilon$ over $[0, \tau]$) the two formulations are equivalent and lead to the same fishing effort path.

where $V_2(x)$ denotes the discounted sum of profits when each player follows the initial equilibrium strategies and where $\bar{\alpha}_2$ is given by

$$\bar{\alpha}_2 = \frac{r - \beta A + \beta \bar{\alpha}_1}{1 - \beta} \tag{9}$$

Player two's problem is a standard optimal control problem and is thus solved by standard maximum principle techniques (see Leonard and Long (1992)).

Let H denote the Hamiltonian associated to (7) :

$$H = ((\bar{\alpha}_2 + g(t))x)^{\beta}(1 - \bar{\alpha}_1 - \varepsilon)) + \pi (Ax - (\bar{\alpha}_1 + \varepsilon + \bar{\alpha}_2 + g(t))x)$$

where π is the shadow price of the stock of fish. The solution to the problem above is characterized by the following necessary conditions:

$$x\beta \left((\bar{\alpha}_2 + g(t))\, x\right)^{\beta-1}(1 - \bar{\alpha}_1 - \varepsilon) - \pi x = 0 \tag{10}$$

$$\dot{\pi} = r\pi - \beta(\bar{\alpha}_2 + g)\left((\bar{\alpha}_2 + g)\, x\right)^{\beta-1}(1 - \bar{\alpha}_1 - \varepsilon) - \pi(A - \bar{\alpha}_1 - \varepsilon - \bar{\alpha}_2 - g) \tag{11}$$

and the transversality condition

$$\pi(\tau) = V_2'(x(\tau)) \tag{12}$$

By differentiation of (10) with respect to time and substituting $\frac{\dot{\pi}}{\pi}$ into (11) we obtain:

$$\dot{g} - (\bar{\alpha}_2 + \varepsilon \frac{\beta}{\beta - 1})g = \bar{\alpha}_2 \frac{\varepsilon\beta}{\beta - 1} + g^2 \tag{13}$$

where we have made use of (9). Furthermore, substituting π from (10) into (12) and using $V_2(x) = K_2 x^{\beta}$, we obtain

$$(1 - (\bar{\alpha}_1 + \varepsilon))\,(\bar{\alpha}_2 + g(\tau))^{\beta-1} = K_2 \tag{14}$$

Player two's optimal response, $g(t)$, to player one's fishing effort increase is thus determined by the differential equation (13) and (14). This differential equation (13) can be transformed into a Bernouilli equation after a change of variable. For details see Appendix B.

Proposition 2: The optimal modification of player two's fishing effort, $g(t)$, to player one's increased fishing effort is given by

$$g(t) = \frac{1}{\left(\left(\frac{1-(\bar{\alpha}_1+\varepsilon)}{1-\bar{\alpha}_1}\right)^{\frac{1}{\beta-1}} \frac{1}{\bar{\alpha}_2} - \frac{1}{\bar{\alpha}_2 - \varepsilon\frac{\beta}{\beta-1}}\right) e^{(\bar{\alpha}_2 - \varepsilon\frac{\beta}{\beta-1})(t-\tau)} + \frac{1}{\bar{\alpha}_2 - \varepsilon\frac{\beta}{\beta-1}}} - \bar{\alpha}_2 \tag{15}$$

Thus the modification of player two's fishing effort is a decreasing function of time ($g'(t) < 0$), and for $\varepsilon = 0$ we have $g \equiv 0$.

Proposition 3: Player two's reaction to an increase in player one's fishing effort is ambiguous and is sensitive to the period of time during which player one can commit to such an increase. If the period of commitment is small, player two reduces its catching effort. If the period of commitment is large enough, player two's catching effort is initially increased for a certain period of time though it is ultimately reduced. In the limiting case where player one commits permanently to an increase of its harvesting effort, player two unambiguously reacts by increasing its own harvesting effort. This increase could be even greater than player one's deviation.

Proof: see Appendix C.

The intuition behind Proposition 3 is as follows. Player two is facing a trade off: more harvesting increases the instantaneous net benefit at the expense of the stock of fish available when the aggressive move of player one ends, at the instant τ. The main incentive for player two to reduce its catch before τ is to enjoy a larger stock once competition becomes less aggressive (after τ). The larger is τ the less important is the present value of the game after τ for player two and the smaller are the incentives to conserve the resource. For a sufficiently long period of commitment of player one (to more aggressive catching efforts), player two responds by also increasing its catching effort for a certain interval of time.

According to the above propositions, if the commitment period is short, the result is similar to the outcome of a **static** Stackelberg model with two firms, a leader and a follower, both selling in the same market. In the case of strategic substitutes in that static model the best reply of the follower to any increase of output of the leader is to decrease of its own output. However, despite this similarity, our model is different. The two players do not compete in the same market[12]: they either consume their catches, or sell them in distinct markets. Furthermore, we have shown in proposition 1 that the two effort levels are strategic complements. It would be interesting to see how the introduction of rivalry in a common market would alter the behavior of the two players. Since there is no competition in the market place in the case of separate markets, as opposed to the oligopolistic case, it could be expected that the reaction of the follower to an aggressive move of its rival

[12]The conclusions would hold if firms operate in the same foreign market where they have measure zero.

be milder under the case of separate markets. Furthermore one might expect that the longer the period of commitment to the aggressive move, the "shier" should be the response of player two. As can be seen from Propositions 2 and 3, such expectation turns out to be incorrect. The reaction of the follower, in the case of a long period of commitment of the leader to an aggressive move, turns out to be another aggressive move that could be even stronger than the one that provokes it.

As a straightforward implication of propositions 2 and 3, we can conclude that if country one (the leader) is committed to a more conservationist exploitation (i.e., $\varepsilon < 0$) over a specified number of years, the follower will follow suit *only if* the commitment period is long enough. Conservationist groups that manage to get only very short term commitment by country one would find their efforts counter-productive.

4 Concluding Remarks

We have developed a simple model that captures to some extent the sequential aspect of transboundary fishery. We have been able to show that in a Markov perfect equilibrium, the first mover has an incentive to be more conservationist than a normal player in a symmetric situation. This picture must however be seen with guarded optimism, for there is a dark cloud in the horizon for the conservation-minded observer: the leader may deviate from the equilibrium by making an aggressive move, and such a deviation may trigger the follower to react by an even more aggressive move, if player one's period of commitment to the deviation is sufficiently long. Another message from our analysis is that pressures for a unilateral cutback in exploitation would be fruitful only if the commitment horizon is sufficiently long.

Appendix A

Proof of Proposition 1:

Player two takes player one's constant strategy $\alpha_1 = \bar{\alpha}_1$ as given, and chooses its α_2. The HJB equation for player two is

$$rV_2(x) = \max_{\alpha_2}[(\alpha_2 x)^\beta(1-\bar{\alpha}_1) + V_2'(x)(A - \bar{\alpha}_1 - \alpha_2)x] \tag{16}$$

Let us try the following functional form for $V_2(.)$

$$V_2(x) = K_2 x^\beta \tag{17}$$

where

$$K_2 = 0 \text{ if } \ 1 - \bar{\alpha}_1 \leq 0 \tag{18}$$

For $1 - \bar{\alpha}_1 > 0$, equation (16) becomes

$$rK_2 x^\beta = \max_{\alpha_2}[(\alpha_2 x)^\beta(1-\bar{\alpha}_1) + \beta K_2(A - \bar{\alpha}_1 - \alpha_2)x^\beta] \tag{19}$$

The maximization of the right-hand side of (16) yields

$$\alpha_2 = \left[\frac{1-\bar{\alpha}_1}{K_2}\right]^{1/(1-\beta)} \tag{20}$$

Hence

$$K_2 = \frac{1-\bar{\alpha}_1}{\alpha_2^{1-\beta}} \tag{21}$$

Substituting (21) into (19), we get

$$\alpha_2 = \frac{r - \beta A + \beta\bar{\alpha}_1}{1-\beta} = \alpha_2(\bar{\alpha}_1) \text{ if } \ 1 - \bar{\alpha}_1 > 0 \tag{22}$$

Then, from (18),

$$\alpha_2(\bar{\alpha}_1) = 0 \text{ if } 1 - \bar{\alpha}_1 \leq 0 \tag{23}$$

We turn now to player one's problem. We assume that player one knows that player two uses the strategy (22)-(23). Player one's HJB equation is

$$rV_1(x) = \max_{\alpha_1}[(\alpha_1 x)^\beta + V_1'(x)(A - \alpha_2(\alpha_1) - \alpha_1)x] \tag{24}$$

Let us try the functional form $V_1(x) = K_1 x^\beta$. Notice that $\alpha_2(a_1)$ has a kink. We must solve (24) for two separate cases: case (i), where $1 - \bar{\alpha}_1 \leq 0$, and case (ii), where $1 - \bar{\alpha}_1 > 0$.

For case (i), we note that $\alpha_2 = 0$, and hence we have the HJB equation

$$rK_1 x^\beta = \max_{\alpha_1}[(\alpha_1 x)^\beta + \beta K_1 (A - \alpha_1) x^\beta] \tag{25}$$

subject to

$$1 - \bar{\alpha}_1 \leq 0. \tag{26}$$

Solving, we get

$$K_1 = \left[\frac{r - \beta A}{1 - \beta}\right]^{\beta - 1} = K_1^{**}$$

and hence

$$\alpha_1 = \left[\frac{r - \beta A}{1 - \beta}\right] = \alpha_1^{**}$$

which, in view of the constraint (26), is a solution provided that

$$\left[\frac{r - \beta A}{1 - \beta}\right] > 1 \tag{27}$$

If (27) is not satisfied, then (25), subject to (26), has no solution.

For case (ii), we must solve

$$rK_1 x^\beta = \max_{\alpha_1}[(\alpha_1 x)^\beta + \beta K_1 (A - \frac{r - \beta A + \beta \alpha_1}{1 - \beta} - \alpha_1) x^\beta] \tag{28}$$

From this we get

$$\alpha_1^{\beta - 1} = \omega K_1 \tag{29}$$

where

$$\omega = \frac{1}{1 - \beta}$$

Thus, substituting (29) into (28), we get

$$r\omega = \alpha_1 + \beta\omega \left[\frac{A - r}{1 - \beta} - \omega\alpha_1\right] \tag{30}$$

$$\alpha_1 = \frac{(r - \beta A)}{(1 - \beta)} = \alpha_1^{**} \tag{31}$$

This yields

$$K_1 = (1-\beta)\left[\frac{r-\beta A}{1-\beta+\beta^2}\right]^{\beta-1} = K_1^{**}$$

This completes the proof of proposition 1.

Appendix B

In this appendix we determine player two's optimal reply of Canada to player one's catch increase by solving the differential equation

$$\overset{\bullet}{g} - (\bar{\alpha}_2 + \varepsilon\frac{\beta}{\beta-1})g = \bar{\alpha}_2\frac{\varepsilon\beta}{\beta-1} + g^2 \tag{32}$$

with the transversality condition

$$(1-(\bar{\alpha}_1+\varepsilon))\,(\bar{\alpha}_2 + g\,(\tau))^{\beta-1} = K_2 \tag{33}$$

Let us define

$$G\,(t) = g\,(t) + \bar{\alpha}_2$$

The differential equation (13) can be rewritten in terms of $G\,(t)$

$$\overset{\bullet}{G} = G(G - \bar{\alpha}_2 + \varepsilon\frac{\beta}{\beta-1}) \tag{34}$$

This differential equation is a Bernoulli equation. Let $Y = \frac{1}{G}$, (34) gives

$$\overset{\bullet}{Y} = \left(\bar{\alpha}_2 - \varepsilon\frac{\beta}{\beta-1}\right)Y - 1$$

the solution of which is

$$Y\,(t) = Ce^{\left(\bar{\alpha}_2 - \varepsilon\frac{\beta}{\beta-1}\right)t} + \frac{1}{\left(\bar{\alpha}_2 - \varepsilon\frac{\beta}{\beta-1}\right)}$$

where C is constant of integration that can be determined by the transversality condition (14).

After substitution we have

$$g\,(t) = \frac{1}{\left(\left(\frac{K_2}{(1-(\bar{\alpha}_1+\varepsilon))}\right)^{-\frac{1}{\beta-1}} - \frac{1}{\bar{\alpha}_2-\varepsilon\frac{\beta}{\beta-1}}\right)e^{(\bar{\alpha}_2-\varepsilon\frac{\beta}{\beta-1})(t-\tau)} + \frac{1}{\bar{\alpha}_2-\varepsilon\frac{\beta}{\beta-1}}} - \bar{\alpha}_2$$

and thus

$$g(t) = \frac{1}{\left(\left(\frac{1-(\bar{\alpha}_1+\varepsilon)}{1-\bar{\alpha}_1}\right)^{\frac{1}{\beta-1}} \frac{1}{\bar{\alpha}_2} - \frac{1}{\bar{\alpha}_2-\varepsilon\frac{\beta}{\beta-1}}\right) e^{(\bar{\alpha}_2-\varepsilon\frac{\beta}{\beta-1})(t-\tau)} + \frac{1}{\bar{\alpha}_2-\varepsilon\frac{\beta}{\beta-1}}} - \bar{\alpha}_2.$$

Note that

$$\left(\frac{1-\bar{\alpha}_1}{(1-(\bar{\alpha}_1+\varepsilon))}\right)^{-\frac{1}{\beta-1}} \frac{1}{\bar{\alpha}_2} > \frac{1}{\bar{\alpha}_2}$$

Since, for $\varepsilon > 0$ and for $\beta < 1$,

$$0 < \frac{1}{\bar{\alpha}_2 - \varepsilon\frac{\beta}{\beta-1}} < \frac{1}{\bar{\alpha}_2}$$

we have that $g(.)$ is a continuous and a decreasing function of time. Moreover, if $\varepsilon = 0$, $g(.)$ is identically zero.

Appendix C

We now show that if player one increases its effort level by ε over the time interval $[0, \tau]$ then player two will increase its own effort, i.e. $g(t) > 0$ for an interval of time. However player two will ultimately reduce its catching effort, i.e. $g(\tau) < 0$.

We start by noting that $g(\tau) < 0$. Indeed, from (15), it is straightforward that

$$g(\tau) = \bar{\alpha}_2 \left(\left(\frac{1-\bar{\alpha}_1}{(1-(\bar{\alpha}_1+\varepsilon))}\right)^{\frac{1}{\beta-1}} - 1\right) \tag{35}$$

For $\varepsilon > 0$ and for $\beta < 1$, we have $g(\tau) < 0$.

We can also calculate $g(0)$:

$$g(0) = \frac{1}{\left(\left(\frac{1-(\bar{\alpha}_1+\varepsilon)}{1-\bar{\alpha}_1}\right)^{\frac{1}{\beta-1}} \frac{1}{\bar{\alpha}_2} - \frac{1}{\bar{\alpha}_2-\varepsilon\frac{\beta}{\beta-1}}\right) e^{-(\bar{\alpha}_2-\varepsilon\frac{\beta}{\beta-1})\tau} + \frac{1}{\bar{\alpha}_2-\varepsilon\frac{\beta}{\beta-1}}} - \bar{\alpha}_2 \tag{36}$$

Now consider the implication of the length of the commitment period τ. Note that $g(0)$ is a continuous and increasing function of τ, and

$$\lim_{\tau\to+\infty} g(0) = -\varepsilon\frac{\beta}{\beta-1} > 0$$

Thus there a exists a positive $\bar{\tau}$ such that $g(0) > 0$ iff $\tau \geq \bar{\tau}$, and for each given $\tau \geq \bar{\tau}$ there exists $\bar{t} \leq \tau$ such that for all $t \in [0, \bar{t}]$, $g(t) > 0$. In the limiting case where $\tau = \infty$ we have

$$g(t) = -\varepsilon \frac{\beta}{\beta - 1} > 0 \text{ for all } t \in [0, \infty)$$

We now turn to the case of short commitment period. It is clear that

$$\lim_{\tau \to 0} g(0) = \bar{\alpha}_2 \left(\left(\frac{1 - \bar{\alpha}_1}{(1 - (\bar{\alpha}_1 + \varepsilon))} \right)^{\frac{1}{\beta - 1}} - 1 \right) < 0$$

and thus there a exists a positive value τ' such that for each given $\tau \leq \tau'$ we have $g(0) < 0$. Since $g(\tau) < 0$ and g is a continuous function of time we have $g(t) < 0$ for all $t \in [0, \tau]$.

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