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Résumé / Abstract

La rémunération des hauts dirigeants d'entreprise a été grandement documentée. La rémunération des gestionnaires au milieu de la pyramide organisationnelle a été moins documenté toutefois. Le but de ce papier est d'adresser cette lacune. Nous utilisons une approche classique de principal-multiagent où le principal doit distribuer des boni de fin d'année à ses gestionnaires qui possèdent une information privilégiée en ce qui a trait aux coûts de production, une information qui est corrélée d'un gestionnaire à l'autre. Le principal veut structurer le contrat de manière à utiliser optimalement cette information corrélée. Nos résultats montrent qu'un contrat complet peut être trop complexe à implémenter. Ceci peut expliquer pourquoi les corporations se basent très souvent sur de simple mécanismes de rémunération telles les échelles salariales. Notre étude offre également une explication pour l'existence de hiérarchies dans les organisations.

An extensive academic literature exists on the optimal compensation of top executives. A less-developed literature pertains to the optimal compensation of middle management personnel. The goal of this paper is to address that concern. The setup we use is that of a firm's president (the Principal) who must distribute year end bonuses to plant managers (the Agents) who possess private information concerning production costs, information that is correlated across managers. The principal wants to design a contract that uses this correlated information. We find that the fully specified contract may be too complex for corporations to implement. This may explain why firms resort to simple schemes such as pay scales and subjective-based compensation. It gives also a theoretical bias for the existence of hierarchies in an organization.

Keywords : Correlated Information, Auditing, Contract Theory, Complexity.

Mots clés : Information corrélée, Audit, Théorie des contrats, Complexité.

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1 Introduction

An extensive academic literature exists on the optimal compensation of top executives. A less-developed literature pertains to the optimal compensation of middle management personnel. The goal of this paper is to address that concern.

The academic literature addresses the issue of middle management compensation as a contracting problem within a hierarchy. The seminal work of Williamson (1967) addresses the concern that the size of an organization (more precisely the number of layers in the organization) will affect the efficiency with which information is transmitted from the top of the hierarchy to the bottom, and vice versa. Since agent action may not be completely observable, information flow within the organization may lead to opportunism and rent-seeking behavior. This is typically characterized by shirking and/or consuming perks. An organization's top layer accepts such a loss of control over lower layer because, as the organization becomes larger, the top layer' firsthand knowledge of a subordinate's information becomes either too expensive or impossible. Therefore delegation of authority becomes necessary. A possible solution is to use internal audits of the different levels of the hierarchy.

In most studies that involve hierarchies, the recurring assumption is that a single individual (the representative agent) represents a hierarchy's entire layer. Instead, we present a modified principal-multiagent model. We assume a two-layer organization where there is a unique player at the upper layer (the principal), and N players at the lower layer (the agents). Each agent has information that is valuable to the principal and this information is correlated between agents. The principal wants to design a contract that will extract as much information from each agent as possible using the fact that the information is correlated.

The story we have in mind is a company president (the principal) who must handout year-end bonuses to her plant managers (the agents). These bonuses are based on the manager's cost to produce the same exact number of widgets in each plant. What the president ignores is the manager's cost of producing these widgets (for example, economic conditions may change from one plant to the next). To gather that information, the president can choose to engage in a round of costly auditing or she can rely entirely on her managers' reports concerning those costs. Another example involves the allocation of internal investment funding across divisions. As reported by Kole (1997), the compensation contracts of managers, let alone middle managers, are very complex. It is, therefore, essential to develop a model with testable hypotheses first.

A traditional approach to the problem has been to rely on audits of agents to make sure that they tell the truth. The contract is typically of the form: I will audit you with a certain probability, and if the report you made is incorrect, then you will be penalized. Townsend (1979) and Mookherjee and Png (1989) address the problem in this way. They find that it is possible to induce truth-telling on the part of the agent if audits are performed regularly. Since audits are costly, and thus conducting one entails a loss of welfare, it would be welfare-enhancing to be able to extract the correct information without having to conduct those audits. Unfortunately if agents possess information that is independently distributed, *full surplus extraction* (FSE) is not possible (see Crémer and McLean, 1988, and McAfee and Reny, 1992). In fact, the multiagent problem is rarely discussed in this context. When only one agent is responsible for the information at a given layer of the hierarchy, higher layer players must turn to some form of audit to induce truth-telling from the lower layer players because lower layer players do not compete over the information they have to transmit. When more than one agent has the information, there are ways to design contracts that induce truth-telling without resorting to auditing, or at least not as much.

The approach pioneered by Crémer and McLean (1988) and by McAfee and Reny (1992) supposes that the agents' information is correlated, say because of market-wide shocks. In that case, they show that the Principal is able to induce truth-telling without having to sacrifice either efficiency or surplus extraction, even if the correlation is small. This involves making the agent's compensation information-independent, which means that the agent has no incentive to lie. The information an agent willingfully reveals may, however, be used to condition the compensation of the other agents. Because information is correlated across agents, it is logical to use one agent's information, or assumed performance, to condition another agent's compensation.

This type of contract allows the principal extract all the agents' information without resorting to costly audits. Why then is costly monitoring so pervasive (see Abdel-Khalik, 1993) in the economy? The reason is that these compensating schemes quickly become extremely risky for the agents: for instance, albeit with some small probability, an agent that has achieved a very good performance could be submitted to a hefty penalty (or the opposite) depending on the other agents' reports. Crémer and McLean (1988) and McAfee and Reny (1992) rely heavily on the assumption that agents can bear risk at no cost; i.e., agents are risk neutral. Risk neutrality allows the design of arbitrarily high transfers (or penalties) to implement the mechanism.

If agents are risk averse or have limited liability, however, such penalties turn out to be too large to induce participation. Robert (1991) has shown that the FSE is no longer possible when agents are risk averse. Demougin and Garvie (1991) did the same exercise using a model where liability constraints are imposed. As a consequence private information becomes valuable so that one may consider spending resources to gather it through costly audits. Risk aversion and/or liability constraint may then be the reason why audits are so frequently used.

For any given amount of correlation in the information structure, the efficiency of a compensation scheme is seen to depend ultimately on the capacity of agents to bear risk. The need for auditing should be present when such capacity is lower and when the information across agents is less correlated. When information is independent from one agent to the next, auditing an agent provides no information on another. On the other hand the effectiveness of auditing increases as the correlation increases.[1] We assume throughout that the Principal is able to commit firmly to an auditing procedure so that there are no renegotiation issues to deal with (see Boyer, 2000, and Khalil, 1997 for issues dealing with renegotiation).

We show that compensation schemes become extremely complex very quickly when agents' types are partially correlated. These contracts may become too complex to be implemented realistically so that firms resort to simple compensation schemes instead of spending huge resources in writing a complete contract. These simple schemes allow workers to extract a rent from the firm (such as shirking on the job or consuming perquisites), rents that are smaller than the cost of writing the complete contract. Another implication of our results is that it gives a rationale for the existence of hierarchies in an organization. If individuals have a limited ability to manage complex contract, then the use of a hierarchy may allow to have lower management manage an optimal number of workers, and have upper management manage an optimal number of lower managers. The number of levels in a hierarchy then depends on the ability of individuals to keep track of provisions in the optimal complete contract.

The first case we study involves the case where audits are costless. It is clear that in this context the first best will be achieved. We then move to the cases where types are independent, and where types are perfectly correlated. The most interesting case is presented in section 3 where we assume that types are partially correlated. In that case, the contract becomes very complex very rapidly

[1] This is obvious in the limit case of perfect correlation. In these cases, the principal only needs to audit a unique agent to acquire all information. See section 2.3.

when information that is partially correlated. We discuss the implications for corporations of such complex contracts in section 4. Section 5 concludes.

2 Modeling compensation

As mentioned in the introduction, the context we have in mind is a company president (the Principal) who must handout year-end bonuses to her plant managers (the agents). These bonuses are based on the manager's production cost, which is not known to the company president. To gather that information, the president can choose to engage in a round of costly auditing or she can rely entirely on her managers' reports concerning those costs. Another example is the use of *cost-plus* clauses to the defense contractor who must deliver equipment to the military. The military agrees to compensate the contractor for the cost incurred in producing the delivered equipment, plus some amount that we could call profit.

Consider first for simplicity an organization composed of only 3 players. There is a president (player 0) facing two managers (players 1 and 2). Each manager i produces widgets for the president for which he entails a private random cost (a type) of $\theta^i \in \{\theta^L, \theta^H\}$. There are thus four possible combination of types with two managers ((θ^H, θ^H), (θ^H, θ^L), (θ^L, θ^H) and (θ^L, θ^L)). To compensate the manager for his cost, each shall be given a wage w^i that may be contingent on the announced production cost (messages) sent by all managers and on the result of performed costly audits.

Auditing one manager costs c to the president and reveals the manager's type. We assume that there is a constant return to scale technology in audits. An audit policy specifies how audits are to be carried out as a function of the managers' reports. All audits are simultaneous.

Let $q_{\{n\}}$ represent the probability that some subset of managers is audited. With two managers, $q_{\{1\}}$ (resp. $q_{\{2\}}$) is the probability that only manager 1 (resp. manager 2) is audited; $q_{\{1,2\}}$ is the probability that both managers are audited. Wages and the audit policy may be contingent on the managers' announced types.

The objective of the principal is to maximize profits by minimizing the expected cost of compensating the managers and the cost of the auditing policy. Given wages w^1 and w^2, the cost of auditing c and the audit policy $\{q_{\{1\}}, q_{\{2\}}, q_{\{1,2\}}\}$ the expected cost of any contract is

$$w^1 + w^2 + c\left[q_{\{1\}} + q_{\{2\}} + 2q_{\{1,2\}}\right]$$

The principal's choice variables in this problem are the different wages and the audit policy.

The contract between the president and the managers specifies wages to be paid and the audit policy to be performed. We model the contracting process using a standard approach:

1. The company president hires the managers by offering them a contract. The contract is such that all managers accept it.

2. Each manager i accomplishes a task for which a wage is paid.

3. Each manager learns the private cost θ^i incurred by executing the task for the president.

4. Each manager sends simultaneously a message to the president regarding the production cost. The profile of messages is denoted by

$$m \in \left\{ \left(\theta^H, \theta^H\right), \left(\theta^H, \theta^L\right), \left(\theta^L, \theta^H\right), \left(\theta^L, \theta^L\right) \right\}$$

5. According to the audit policy induced by the messages, random audits are performed which reveal information θ_n. Manager i's wage (including the penalty), noted $w^i_{\theta_n}(m)$, is contingent on the profile of messages and on the information produced by the audits.

We can restrict our attention to truthful messages using the Revelation Principle. Manager i's ex post payoff is simply $U_i\left(w^i - \theta^i\right)$, where w^i is the wage paid to the worker, and θ^i is the cost incurred by the worker. We assume that all wages must be positive or zero (managers have a limited liability). Hence, the lowest payoff any manager may receive is $U_i\left(-\theta^i\right)$. Wages are paid at the end of the game, similarly to year-end bonuses.

2.1 Contracting with costless information

When auditing is costless ($c = 0$) we are in effect in a case of complete information: we can infer that the president always audits both managers. It is straightforward to see that the president can extract all the information and the surplus of the agents. This is done by setting each manager's wage to his private cost ($w^i = \theta^i$).

2.2 Contracting with independent types

When types are independent, the information the president receives from one manager provides no information on any other. Since types are independent, we can concentrate our analysis on a representative manager. Consider what happens to the manager who incurred cost $\theta^L < \theta^H$. A manager of type θ^L is guaranteed to receive utility $U\left(w^L - \theta^L\right)$ by telling the truth. By lying (announcing type θ^H), he may receive utility $U\left(w^H - \theta^L\right)$ if he is not audited, or utility $U\left(-\theta^L\right)$ if he is audited. The auditing strategy such that the manager is indifferent between announcing θ^H and θ^L is such that he must audit with probability q_H^* that is given by

$$U\left(w^L - \theta^L\right) \geq q_H^* U\left(-\theta^L\right) + \left(1 - q_H^*\right) U\left(w^H - \theta^L\right)$$

The auditing probability that makes the manager as well off telling the truth is

$$q_H^* = \frac{U\left(w^H - \theta^L\right) - U\left(w^L - \theta^L\right)}{U\left(w^H - \theta^L\right) - U\left(-\theta^L\right)}$$

By auditing with probability $q_H = Max\left\{q_H^*, 0\right\}$,[2] the president knows that the type θ^L manager will always tell the truth since he has nothing to gain by lying.

There is unfortunately a waste of resources in this economy in the sense that costly audits must be conducted. In other words, truth-telling always is achieved only at the expense of costly audits. These audits have expected cost $c\left(\pi q_H + (1 - \pi) q_L\right)$, where π is the probability that the manager is of type θ^H.

In this setup where types are independent, the president is not able to use one player's message to determine her strategy concerning the other player, since such message has no informational value. The next logical step would be to examine what happens when types are correlated.

[2] Consider now the case of an agent who faces cost $\theta^H > \theta^L$. Telling the truth yields utility $U\left(w^H - \theta^H\right)$. Lying (announcing type θ^L) yields utility $U\left(-\theta^H\right)$ if the agent is audited, and utility $U\left(w^L - \theta^H\right)$ if not. For a manager to be as well off telling the truth then lying, we need q_L^*, the probability with which the principal audits a manager who announced cost θ^L, must be such that

$$U\left(w^H - \theta^H\right) \geq q_L^* U\left(-\theta^H\right) + \left(1 - q_L^*\right) U\left(w^L - \theta^H\right)$$

The auditing probability that makes the manager indifferent between telling the truth and lying is

$$q_L^* = \frac{U\left(w^L - \theta^H\right) - U\left(w^H - \theta^H\right)}{U\left(w^L - \theta^H\right) - U\left(-\theta^H\right)}$$

By auditing with probability $q_L = Max\left\{q_L^*, 0\right\}$, the president knows that the manager will always tell the truth since he has nothing to gain by lying.

If $w^H < w^L$ (resp. $w^H > w^L$) we have $q_H^* < 0$ (resp. $q_L^* < 0$) which is not possible, which means that q_H and q_L must be bounded by zero. Of course, only one of these conditions will hold, unless $w^H = w^L$ in which case the labor contract offers no insurance provision.

2.3 Contracting with perfectly correlated types

Consider the extreme case where types are perfectly positively correlated. It is then easy to build a contract that yields an efficient outcome and full surplus extraction to the principal. The contract exploits the fact that it is common knowledge that the cost faced by all managers are identical, and thus that any manager knows what is the other manager's cost. This allows the president to put the managers in a so-called prisoner's dilemma where one-sided defection is rewarded. The president is then able to extract all information as in the complete information case, without having to incur audits in equilibrium. The way to proceed is for the president to: 1-audit any one manager whenever a non-valid messages profile (i.e., not the same message) was sent; and 2-reward the managers that have told the truth while giving the ones that lied a wage of zero. A high reward for saying the truth is then sufficient to put the managers in a prisoner's dilemma.

Since manager types are perfectly correlated, the only possible sets of type are $\theta_i = \theta^j$ for all $i \in \{1, ..., N\}$ and $j \in \{1, ..., T\}$ (all agents have the same cost). There are then 2^N possible message profiles that can be made for a given type, but only $N + 1$ distinct message profiles if we assume that agents are symmetric ex ante. After the managers have filed their reports, the president must decide to audit or not. She must incur cost c for each audited manager.

The president ignores the managers' types. She only knows that the managers all have the same type, which means that she knows for sure that some managers lied whenever reports are not the same. The only case when the president does not know that a manager lied is when all managers agree to lie. If they don't agree, each lying manager knows he will be found out, which is an undesirable outcome since a manager's payoff after a lie and an audit is less than his payoff when he tells the truth, whatever his true type (i.e., $U\left(-\theta^j\right) < U\left(w^j - \theta^j\right)$).

It is clear that agents always tell the truth if they see no gain in telling a lie. A simple way to prevent such collusion is to force managers into a prisoner's dilemma. Consider the case where all managers announce θ^k when their true type is θ^j. If the president never audits, each manager receives payoff $U\left(w^k - \theta^j\right) > U\left(w^j - \theta^j\right)$. We see, however, that a manager may have an incentive to deviate from this strategy by reporting the truth. To see why, let b^j be the wage paid to a θ^j type manager who reported the truth when the other managers reported type θ^k and were audited. By setting $b^j > w^k$, the manager who told the truth when the others lied receives payoff $U\left(b^j - \theta^j\right) > U\left(w^k - \theta^j\right)$, while the lying manager receives payoff $U\left(-\theta^j\right)$. We see that every manager has an incentive to deviate from the collusion strategy. It is therefore individually rational for every manager to tell the truth.

A similar argument applies if types are perfectly correlated, but not necessarily perfectly positively correlated. The president then audits the managers whenever they did not send a proper message profile. Rearranging the game slightly yields the same three important results as in the case of perfectly positively correlated information. It follows that the principal is able to achieve FSE since all the information is extracted and no agent is ever audited in equilibrium.

3 Contracting with imperfectly correlated types

3.1 General case

The contract devised in the preceding section is not optimal when agent types are only partially correlated. When types are partially correlated, the principal can use that information to condition her auditing strategy and the wages she offers the agents. In fact, when types are partially correlated, the principal should audit more than when type are perfectly correlated, but less than when type are independent. Boyer and González (1999) show in a two-agents-two-types environment that the more correlated the agents' types (in absolute terms), the lower the probability of auditing and the lower the wages paid. It then becomes imperative for the principal to use the partially correlated information in devising the optimal contract. What is left to find is the number of variables that must be specified in the complete contract.

Theorem 1 *In the general case where N managers can be of any T possible types (i.e., each agent has the same number T of possible types), each contract must specify $\left[\left(2^N-1\right)+N\left(T+1\right)^N\right]T^N$ variables. There are $\left(2^N-1\right)T^N$ audit policies that need to be specified, as well as $NT^N\left(T+1\right)^N$ wages.*

Proof. Part 1. *Each agent i performs a task for the Principal from which he entails a private random cost (a type) of $\theta^i=\theta^i_k\in\Theta^i$ where the type value strictly increases with k. We let $\Theta=\times_{i\in I}\Theta^i$ be the set of $|\Theta|=T^N$ types profiles where $|\cdot|$ is the cardinal operator*

The Principal introduces audits by committing to an audit policy *that specifies how audits are to be carried conditionally on the agents' reports. Let m denote a profile of messages. By the Revelation Principle, a message from agent i to the Principal can be restricted to an element $m^i\in\Theta^i$. With simultaneous audits, an audit policy is then a mapping from Θ into some distribution*

q over $\mathcal{P}(I) = 2^N$, the power set of agents; that is, with some probability q_n, only the subset n of agents, $0 \leq |n| \leq N$, will be audited, with $\sum_{n \varepsilon \mathcal{P}(I)} q_n = 1$. For a given type profile, the set of audit policies is the unit simplex $\mathbb{S}^K$ in $\mathbb{R}_+^K$ where $K = 2^N - 1$.[3] Given there are $J = T^N$ type profiles, we will have $KJ = T^N \left(2^N - 1\right)$ possible audit policies.

<u>Part 2</u>. Auditing the subset n of agents reveals surely their type profile θ_n. Given their messages m_n, we define an audit result *to be a $|n|$-tuple a where each ordered element a_j of a is an integer from 0 to $T-1$ that specifies by how many indexes the agent in n with the j^{th} name overstated his cost. A $|n|$-tuple of zeroes is then equivalent to say that all audited agents told the truth.[4] The set of possible audit results for any subset $n \neq \emptyset$ of agent is $A_{|n|}$ and has $|A_{|n|}| = T^n$ elements. If $n = \emptyset$, $|A_0|$ is defined to be 1 in the sense that the only new piece of information brought by performing the random audit policy was that no audits were actually performed.*

The principal compensates each agent using a wage w^i that is contingent on the messages sent to the Principal about the agents' types and on the result of performed audits. A wage to agent i contingent on the profile of messages m and on the information a produced by the audits is noted $w^i_{n,a}(m)$.

Let us count the number of wages that must be specified following the announce of a message profile m by the agents. For each subset n of audited agents, there are $|A_{|n|}|$ possible results. The number of subsets in $\mathcal{P}(I)$ that have $k = |n|$ elements is $\binom{N}{k}$. It follows that, given any m, there are $1 + \sum_{k=1}^{N} \binom{N}{k} T^k = (T+1)^N = L$ possible contingencies we must take into account.[5] Given there are $J = T^N$ type profiles for which N wages must be specified, the number of wages that need to be specified in the complete contract is $NJL = NT^N (T+1)^N$.

It follows that a complete contract is composed of an audit function that maps Θ into $\mathbb{S}^K$ (which gives us $T^N \left(2^N - 1\right)$ audit policies) and of N non negative wages functions that each maps $\Theta \times \mathbb{E}$ into the wage set, $\mathbb{R}_+$ (which gives us $NT^N (T+1)^N$ wages). •

[3] There are only $2^N - 1$ audit probabilities for a given distribution of types because the audit probabilities must sum to one (which removes one degree of freedom).

[4] For example, suppose there are eight possible types ($T = 8$). Suppose agents 2, 4 and 5 sent message profile $m_{\{2,4,5\}} = [\theta_3, \theta_5, \theta_2]$. We have audit result $a = (0, 2, 7)$, if agent 2 told the truth, agent 4 lied by over-reporting his true cost by 2 and agent 5 lied by over-reporting by 7 (that is, underreporting his true cost by 1). This means that the three agents' true types is θ_3.

[5] Another way to get that result is to augment the type set of each agent by a "null" type which represents the ex post type of an agent that has not been audited. Then, either the agent is audited, with T possible outcomes, or he is not audited and we say that he has the null type. There are then $(T+1)^N$ possible outcomes to the audits.

A complete contract must then specify $J(K+NL)$ numbers. In the 2X2-CASE, that of two agents ($N=2$) with two types ($T=2$), the complete contract must specify $4(3+2\cdot 9)=84$ variables. With six agents each having six types, the complete contract must specify $6^6\left[\left(2^6-1\right)+6\left(6+1\right)^6\right]=32,937,129,792$ variables.

3.2 Symmetry

3.2.1 Agents and Types

Suppose that there are N agents who may take any of T possible types. By De Moivre's theorem, the number of T-tuples of non negative integers that sum to N is $S(N,T)=\binom{T+N-1}{N}$. The state space is then built as follows. Let $\mu:\Theta\to\mathbb{N}^T$ be the function that maps any type profile into the $1\times T$ vector that specifies the number of agents that have type 1, 2, etc. Clearly, the image of μ has $S(N,T)$ elements. We specify a distribution f over the image $\mu_i(\Theta)$. Then we independently draw an element in the set of permutations of I which gives us a ranking r of the N agents and an element μ of $\mu_i(\Theta)$ according to the distribution f. From μ we build an urn (a set) $B(\mu)$ where we put μ_1 balls labeled θ_1, μ_2 balls labeled θ_2, etc. By construction, there are N balls in $B(\mu)$. Finally, we draw successively the N balls out of $B(\mu)$ without replacement and we assign r_n the type labeled on the n^{th} ball drawn. That is, if $r_3=7$ and the third ball is labeled θ_4, then agent 7 has type θ_4 ($\theta^7=\theta_4$).

The probability of observing a given type profile θ is then

$$p(\theta)=\frac{f(\mu_i(\theta))}{\binom{N}{\mu_i(\theta)}},$$

where the denominator is the multinomial coefficient

$$\binom{N}{\mu_i(\theta)}=\frac{N!}{\Pi_{t=1}^{T}\mu_t!}.$$

That probability distribution function depends on θ through μ; it follows that if $\mu_i(\theta)=\mu_i(\theta')$, then $p(\theta)=p(\theta')$. This implies that all agents are completely symmetrical: like for the type for profile, the probability distribution of any vector $\theta_{n'}$ conditional on the realization of some vector θ_n depends only on $\mu_i(\theta_n)$. It is then straightforward to show that all agents have the same marginal type distribution. Under that construction, we say that all agents are *symmetric* ex ante.

Not only do we get a simplified state space with symmetry but a contract is likely to be simplified as well; that is, an optimal contract will pool many agents that are of an indistinguishable nature to the Principal. There are now only $S(N,T) < J$ distinct profiles of messages[6] that can be sent to the Principal since each message profile m information content is resumed by the reduced message profile $\mu_i(m)$. Beside, all agents that share the same type ex post are still identical with respect to their information set. We argue that there exists an optimal contract that treats all agents equally ex ante and all (announced) types equally at the interim stage, that is, before audits are performed.

Let I_{θ_k} be the subset of agents that declare being of type θ_k at the interim stage. We say that a contract is *symmetric* if the following conditions are satisfied: $\forall i,j \in I_{\theta_k}, \forall \theta_k \in \Theta^1, \forall n, n' \in \mathcal{P}(I)$ and $m, m' \in \Theta$ such that $\mu_i(m) = \mu_i(m')$ and $\mu_i(m_n) = \mu_i(m'_{n'})$; and $\forall a \in A(n)$,

$$q_n(m) = q_{n'}(m'),$$
$$w^i_{n,a}(m) = w^j_{n',a}(m').$$

Note that this implies that two agents i and j that have declared the same type have the same marginal probability of being audited. If $i,j \in I_{\theta_k}$, then $\mu_i(m_{\{i\}\cup n}) = \mu_i(m_{\{j\}\cup n}), \forall n \in \mathcal{P}(I \setminus i,j)$. Hence, $q_{\{i\}\cup n}(m) = q_{\{j\}\cup n}(m)$ for these n. Consider any n such that $j \in n$ but $i \notin n$; permuting j by i in n yields n' such that $\mu_i(m_{\{i\}\cup n}) = \mu_i(m_{\{j\}\cup n'})$ and $q_{\{i\}\cup n}(m) = q_{\{j\}\cup n'}(m)$. These subsets come by pairs and exhaust the set of subsets to which i and j might respectively be joined to.

We can then restrict our search of an optimal contract to the class of symmetric contracts since all agents are symmetric ex ante.

Let μ be a reduced message profile; two agents that have sent the same message should have the same probability of being audited. For two reduced profile μ^1 and μ^2 that are permutations to one another, we have to specify the same value to each profile. We can associate all these permuted profiles to a single partition η of N into T or less integers $\eta_1 + \eta_2 + \ldots + \eta_{|\eta|} = N$ which we represent as a $|\eta|$-tuple $\eta = (\eta_1, \eta_2, \ldots, \eta_{|\eta|})$ such that $1 \leq |\eta| \leq T$. Let $\mathcal{N}(N,T)$ be the set of these partitions; there are $|\mathcal{N}(N,T)|$ of them.[7] A partition η is an event that says that there were $|\eta|$ *kinds* of types announced: η_1 agents announced a type of the first kind, η_2 agents announced a type of the second kind; etc. If all agents send the same message then $\eta = (N)$, whatever that message was.

[6]That account can be also be obtained as a special case of Polya's theorem: there are N identical agents that form a symmetric group S_N and each is to be painted of one of T colors. We distinguish colors (types) but not names. The total number of combinations is given by $|S_N|^{-1}(\sum_{g\in S_N} T^{\mathrm{cyc}(g)})$ where $\mathrm{cyc}(g)$ is the number of cycles in permutation g. That total amounts to $S(N,T)$.

[7]That number is the coefficient of x^N in the series expansion of $\Pi_{k=1}^{T}(1-x)^{-k}$. If T is large, $|\mathcal{N}(N,T)|$ does not increase as T increases

For each partition η, we must compute the number of reduced profiles of messages that are associated to it. This amounts to compute the number of distinct ways we can assign the elements of η to T types. Suppose first that $\eta_k \neq \eta_{k'}, \forall 1 \leq k, k' \leq |\eta|$. Then there are T ways to assign a type to η_1, $T-1$ ways to assign a type to η_2, ..., $T+1-|\eta|$ ways to assign a type to $\eta_{|\eta|}$; for a total of $\Pi_{k=1}^{d(\eta)}(T-k)$ ways to assign types to η.

But that formula will lead to double counting if some elements of η are repeated. For instance, if $\eta = (1,1)$ and $T = 3$ then there are only three ways to having two agents spread into 2 equal formations of 1. We have T types that we must assign to the elements of η. Clearly, if there are repetition in the elements of η, e.g. $\eta_1 = \eta_2$ like in the preceding example, we should not count assignment of θ_1 to η_1 and θ_2 to η_2 as a distinct assignment than that of θ_1 to η_2 and θ_2 to η_1. For each repeated number of η, we need to divide by the number of indistinguishable permutations it generates. For instance, if 8 appears three times in η, than we must divide the permutations associated to 8 by 3!. Furthermore, $T - |\eta|$ are left out of θ; the same reasoning implies that the total should be pondered by $(T - |\eta|)!$. Suppose there are r_1 1's into η, r_2 2's, etc.; and let $r(\eta)$ be the vector of the r_k's. There are thus $T - d(\delta)$ types that are left out which we count as a last element of r. Then, the total number of reduced message profiles associated to η is the multinomial coefficient $\binom{T}{r(\eta)}$.

Let Π denote the product of the elements of a t-tuple and let $\eta + 1$ be the t-tuple such that one was added to each element of η. For each η we must specify $\Pi_{n+1} - 1$ numbers for an audit policy; that is, we can audit from 0 to $\eta_k \geq 0$ of the agents that have announced the k^{th} kind of type of η, times those of the k'^{th} kind of type, etc. For instance, if $N = 8$ and $\mu_1 = [0,4,1,3]$ and $\mu_2 = [1,3,4,0]$; then both are permutation of the partition $8 = 4+3+1$ that we note $\eta = (4,3,1)$ and $\Pi_{\eta+1} = (4+1)(3+1)(1+1)(0+1) = 40$. So there are 40 configurations of types we must consider to audit and that requires an audit policy composed of $40 - 1 = 39$ numbers.

Hence, to compute the total number of numbers that must be specified by a symmetric audit policy, we first list all the partitions η of N and, for each of them, we count the number $\binom{T}{r(\eta)}(\Pi_{\eta+1} - 1)$. Summing these products gives us the true complexity of the symmetric audit policy:

$$\sum_{\eta \in \mathcal{N}(N,T)} \binom{T}{r(\eta)} (\Pi_{\eta+1} - 1) . \tag{1}$$

For instance, if $N = 2$ and $T = 2$, the possible partitions are $\eta^1 = (2)$ and $\eta^2 = (1,1)$. We then compute

$$\binom{T}{r(\eta^1)}\left(\Pi_{\eta^1+1}-1\right)+\binom{T}{r(\eta^2)}\left(\Pi_{\eta^2+1}-1\right) = 2\left[(2+1)(0+1)-1\right]+1\left[(1+1)(1+1)-1\right] = 2(3-1)+1(4-1)=7$$

When $N=5$ and $T=3$, there are five possible partitions: $\mathcal{N}(N,T)=\{(5),(4,1),(3,2),(3,1,1),(2,2,1)\}$. There are only three ways to have the first partition: 5,0,0; 0,5,0; 0,0,5. There are six ways to have the second partition: 4,1,0; 4,0,1; 0,1,4; 0,4,1; 1,4,0; 1,0,4. The number of audit policies that need to be specified is then

$$\begin{aligned}\sum_{\eta\in\mathcal{N}(N,T)}\binom{T}{r(\eta)}\left(\Pi_{\eta+1}-1\right) &= \left[\begin{array}{c}3\left((5+1)(0+1)(0+1)-1\right)+6\left((4+1)(1+1)(0+1)-1\right)\\ +6\left((3+1)(2+1)(0+1)-1\right)+3\left((3+1)(1+1)(1+1)-1\right)\\ +3\left((2+1)(2+1)(1+1)-1\right)\end{array}\right]\\ &= 3(6-1)+6(10-1)+6(12-1)+3(16-1)+3(18-1)=231\end{aligned}$$

There are seven possible partitions when there are six agents ($N=6$), each having three possible types ($T=3$). These partitions are $\mathcal{N}(N,T)=\{(6),(5,1),(4,2),(3,2,1),(3,3),(4,1,1),(2,2,2)\}$. We then have

$$\begin{aligned}\sum_{\eta\in\mathcal{N}(N,T)}\binom{T}{r(\eta)}\left(\Pi_{\eta+1}-1\right) &= \left[\begin{array}{c}3\left((6+1)(0+1)(0+1)-1\right)+6\left((5+1)(1+1)(0+1)-1\right)\\ +6\left((4+1)(2+1)(0+1)-1\right)+6\left((3+1)(2+1)(1+1)-1\right)\\ +3\left((3+1)(3+1)(0+1)-1\right)+3\left((4+1)(1+1)(1+1)-1\right)\\ +1\left((2+1)(2+1)(2+1)-1\right)\end{array}\right]\\ &= \left[\begin{array}{c}3(7-1)+6(12-1)+6(15-1)+6(24-1)\\ +3(16-1)+3(20-1)+1(27-1)\end{array}\right]=434\end{aligned}$$

audit policies to specify.

3.2.2 Wages

We now attempt to reduce the number of wages to be explicitly specified in a contract. Again, two permuted profiles μ^1 and μ^2 must specify the same number of wages so that we can work from η and sum over $\mathcal{N}(N,T)$. We will resume the contingency n,a with a single $(T+1)\times|\eta|$ matrix α. The first row is the number of agents of each type that where not audited. The T subsequent rows $k=2,\ldots,T+1$ are the number of audited agents of each announced kind of type that have overstated their cost by k indexes. A matrix that has only zeros in the T lowest rows implies that no agent was audited. If only the second of the lowest T rows has some positive integers, then all audited agents told the truth. If the first row is zero, then all agents were audited. The sum of all elements of that matrix is N.

For example, if $N = 18$ and $T = 4$, we could have $\eta = (10, 5, 3, 0)$ and

$$\alpha = \begin{pmatrix} 4 & 5 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \text{Not audited} \\ \text{Audited and told the truth} \\ \text{Audited and overstated by 1} \\ \text{Audited and overstated by 2} \\ \text{Audited and overstated by 3} \end{pmatrix}$$

This would read that six of the ten agents in the first kind of type of η were audited: 2 were telling the truth, one was overstating his cost by 1, three by 2 and none by 3. The five agents in the second kind of type in η were not audited. All agents in the third kind of type of η were audited, two were telling the truth and one was overstating his cost by 1. No one reported being of the fourth kind. Given any partition η, there is a set $\mathcal{A}(\eta)$ of such matrices.

We can divide the agents into classes according to:

1. Those that declared being the k^{th} kind of type in η and were not audited; there are $\alpha_{1,k}$ of them per column k.

2. Those that declared being of the k^{th} kind of type in η, were audited and had their message confirmed; there are $\alpha_{2,k}$ of them per kind k.

3. Those that were audited and that lied; there are $\sum_{t=3}^{T+1} \alpha_{t,k}$ of them per column k.

If the contract is incentive-compatible[8] then out-of-equilibrium payments have no bearing on the Principal's program nor on the truth-telling constraints of any agents. Moreover, by setting all off-equilibrium payments to zero only relaxes the truth-telling constraints. Hence, without loss of generality, we can assume that agents in the third class always get zero.

It follows that we only need to specify a number of payments equal to the number of non zeros entries there are in the first two rows of α.[9] Let $\kappa(\alpha)$ be that number. For all possible α given η, we count $\kappa(\alpha)$ non-zero entries. There are $\binom{T}{r(\eta)}$ distinct reduced message profiles that yield η. Summing over $\mathcal{N}(N, T)$, we need to specify

$$\sum_{\eta \in \mathcal{N}(N,T)} \binom{T}{r(\eta)} \left(\sum_{j=1}^{|\mathcal{A}(\eta)|} \kappa(\alpha^j) \right),$$

[8] To illustrate the notation, this example does not use the fact that the contract is incentive compatible. If it were incentive compatible, we know that we would see no entries in the $T-1$ lower lines since agents always tell the truth.

[9] We need to specify wages for the case where an agent is audited and found to have told the truth because even if the contract is IC (and thus all agents tell the truth in equilibrium) the auditing scheme is precisely used to induce truth-telling. And since the principal can commit to an auditing strategy, the players do not need to enter a renegotiation phase.

payments.[10] The difficult part is to compute the second term.

Given η, the number of configurations of column k of α that can be made by partitioning η_k within $T+1$ rows is $S(\eta_k, T+1)$. Let $\sigma(\eta)$ be the vector of these numbers. The number of distinct matrices α that can be made from η is $\pi(\sigma(\eta))$. A first approximation of $\kappa(\eta)$ would then be $2|\eta|\pi(\sigma(\eta))$ but many of these α have zeros in their first two rows for which no payment needs to be specified. Now, if we knew how many zeros appears in all these matrices α, then we would know that a proportion $\frac{2}{T+1}$ of them appear in the first two rows and we could subtract these zeros. Let's first count how many times zero might appear if we rearrange column k in all possible fashions:

1. We may have from $\max(0, T+1-\eta_k)$ to T zeros in column k; pick z of them.
2. These z zeros may be disposed in $\binom{T+1}{z}$ ways in column k.
3. Once the z zeros have been disposed, there are $\binom{\eta_k-1}{T-z}$ ways of disposing the η_k unlabeled units into the remaining $T+1-z$ labeled locations.
4. It follows that the number of zeros that will appear in column k is
$$\sum_{z=\max(0,T+1-\eta_k)}^{T} z\binom{T+1}{z}\binom{\eta_k-1}{T-z}.$$

Each of these configurations of column k is to be matched with many different configurations of the other columns. If we do the exercise for all columns at once, then we find that there will be

$$Z(\eta) = \sum_{z_1=\max(0,T+1-\eta_1)}^{T} \cdots\cdot\cdot \sum_{z_{|\eta|}=\max(0,T+1-\eta_{|\eta|})}^{T} \left(\sum\nolimits_{k=1}^{|\eta|} z_k\right) \Pi_{k=1}^{|\eta|}\tbinom{T+1}{z_k}\tbinom{\eta_k-1}{T-z_k}$$

zeros in all $\alpha \in A(\eta)$.

The number of payments that must be specified for a symmetric contracts is thus

$$\frac{2}{T+1}\sum_{\eta\in\mathcal{N}(N,T)}\binom{T}{r(\eta)}\left(|\eta|(T+1)\pi(\sigma(\eta)) - Z(\eta)\right). \tag{2}$$

In table 1 we tabulated some of these totals for values of N and T up to 6. The numerator in each cell presents the number of optimal audit policies and the number of wages needed in an

[10] Note that two distinct matrices $\alpha, \alpha' \in \mathcal{A}(\eta)$ that have the same first two rows must nevertheless specify different wages since they represent different events (the same number of agents of each kind of type lied but in a different fashion). Below, we argue that we can disregard these distinct events when we consider Bayesian-Nash implementation as all agents expect nothing but the other agents to tell the truth in these equilibria.

optimal contract. The denominator gives the same sum in the general case where the symmetric assumption is relaxed.

The first term in the denominator (the number of audit policies) is always a factor of the number of types since the formula to find the number of audit policies is $\left(2^N - 1\right) T^N$. The second term in the denominator (the number of wages) is always a factor of the number of agents and the number of types since the formula to find the number of wages is $NT^N \left(T+1\right)^N$.

Table 1.
Number of Variables to Be Specified
For the Symmetric Contract (Numerator) and for the Full Contract (Denominator)

N \ T	2	3	4	5	6
2	$\frac{7+24}{12+72}$	$\frac{15+72}{27+288}$	$\frac{26+160}{48+800}$	$\frac{40+312}{75+1{,}800}$	$\frac{57+504}{108+3\,528}$
3	$\frac{16+84}{56+648}$	$\frac{46+468}{189+5{,}184}$	$\frac{100+1{,}680}{448+24{,}000}$	$\frac{185+4{,}650}{875+81{,}000}$	$\frac{308+10{,}836}{1{,}512+222{,}264}$
4	$\frac{30+224}{240+5{,}184}$	$\frac{111+2{,}184}{1{,}215+82{,}944}$	$\frac{295+12{,}320}{3{,}840+64{,}000}$	$\frac{645+49{,}606}{9{,}375+3{,}240{,}000}$	$\frac{1{,}239+158{,}928}{19{,}440+12{,}446{,}784}$
5	$\frac{50+504}{992+38{,}880}$	$\frac{231+8{,}190}{7{,}533+1{,}244{,}160}$	$\frac{736+70{,}840}{31{,}744+16{,}000{,}000}$	$\frac{1{,}876+409{,}202}{96{,}875+121{,}500{,}000}$	$\frac{4{,}116+1{,}787{,}940}{241{,}056+653{,}456{,}160}$
6	$\frac{77+1{,}008}{4{,}032+279{,}936}$	$\frac{434+26{,}208}{45{,}927+17{,}915{,}904}$	$\frac{1{,}632+340{,}032}{258{,}048+384\times 10^6}$	$\frac{4{,}795+2{,}782{,}560}{984{,}375+4{,}374\times 10^6}$	$\frac{11{,}914+16{,}449{,}048}{2{,}939{,}328+32{,}934\times 10^6}$

The first entry at the numerator (Symmetric contract) and at the denominator (Full contract) represents the number of audit policies that need to be specified; the second entry represents the number of wages that need to be specified.

In our case, saying that all agents are symmetric ex ante means that an agent's type is drawn from the same probability distribution. By assuming this ex ante symmetry reduces complexity because the principal does not need to consider more than one distribution function of types. Instead of assuming that agents are symmetric, one could also assume that the principal knows the distribution function of each agent without knowing each agent's type. We would have the same reduction in complexity. The symmetry assumption is less restrictive, however, since we could say that although agents are not symmetric at the time when their type is drawn (i.e., the distribution function of types is not the same for all agents), the distribution of the distribution function is symmetric. In other words, distribution functions over types are not the same for all agents, but the distribution function of the distribution function is the same for all agents.

3.3 Nash implementation

We can further tackle the curse of dimensionality using the Bayesian-Nash implementation approach. Since every agent expect all others to tell the truth, we do not need to specify payments for all contingencies following an audit: We only need to specify payments made in equilibrium or off-equilibrium given a one-sided defection only. Within a symmetric contract, this implies that for each reduced message profile μ (with associated partition η) and given that any audited agent that lied gets zero, we only need to specify a different set of wages for each configuration of audits. Given that all agents expect all other agents to tell the truth, we only need α matrices composed of two rows: Agents who were audited, and agents who were not. There are η_k+1 ways of auditing agents that have declared being of the k^{th} kind of type. If this was the only reported type, we would only need to specify $2\left(\eta_k-1\right)$ payments. If there is a second kind of type k' that can be audited in $\eta_{k'}+1$ ways, we need $2\left(\eta_k-1\right)\left(\eta_{k'}+1\right)$ payments for kind k and $2\left(\eta_{k'}-1\right)\left(\eta_k+1\right)$ payments for kind k'. Hence, the number of payments to be specified for η is

$$2\pi(\eta+1)\sum_{k=1}^{|\eta|}\frac{\eta_k-1}{\eta_k+1}.$$

This reduces the total number of numbers to be specified for a symmetric contract in Bayesian-Nash implementation to

$$\sum_{\eta\in\mathcal{N}(N,T)}\binom{T}{r(\eta)}\left((\pi(\eta+1)\left(1+2\sum_{k=1}^{|\eta|}\frac{\eta_k-1}{\eta_k+1}\right)-1\right). \tag{3}$$

Some of these numbers are tabulated in table 2 below.

Table 2.
Number of Variables to Be Specified
Bayesian Nash implementation of a Symmetric Contract

N \ T	2	3	4	5	6
2	23	51	90	140	201
3	56	172	388	735	1,244
4	110	447	1,255	2,845	5,607
5	190	987	3,376	9,026	20,496
6	301	1,946	7,968	24,815	64,330

The number represents the sum of the number of audit policies and of the number of wages that need to be specified under a symmetric contract when out-off-equilibrium payoffs are not considered.

Table 3 shows by how much the complexity of contracts is reduced when we assume symmetry and we ignore contingencies that never occur in equilibrium. As before, the number of types appears on the horizontal axis, while the number of managers appears on the vertical axis. The entry represents the percentage reduction in contract complexity once Bayesian-Nash implementation is included in the symmetric contract compared to the general contract.

Table 3.
Reduction in the Complexity of the Compensation Contract.
Bayesian Nash Implementation of a Symmetric Contract versus Full Contract

N \ T	2	3	4	5	6
2	76.619%	83.810%	89.387%	92.533%	94.472%
3	92.045%	96.799%	98.413%	99.102%	99.444%
4	97.972%	99.469%	99.805%	99.912%	99.955%
5	99.523%	99.921%	99.979%	99.993%	99.997%
6	99.894%	99.989%	99.998%	99.9994%	99.9998%

This percentage is found by using the following formula: $1 - \frac{\text{Table 3 entry}}{\text{Denominator of Table 2 entry}}$.
In other words, by what percentage is the complexity of the contract reduced once consider Bayesian-Nash implementation of a symmetric contract versus the full contract.

The analysis of the optimal auditing strategy in that context becomes quickly very complex. The president will commit to audit whenever some "suspect" profile of types is announced and to reward the whistle-blower if some fraud is revealed afterward with a bonus at least as high as his payoff had he participated in the conspiracy. Such a contract induces truth telling by all types of managers. Truth-telling is achieved, however, at the expense of some costly audits since all profiles of types have a positive probability of occurrence so that there will be a positive probability of auditing. At the margin, the principal will want to economize on the cost of auditing by reducing the probability of auditing.

Reducing the probability of performing an audit has a double effect on the managers' expected payoffs: i) it increases the probability that a misreport could go undetected; ii) it decreases the probability that a whistle-blower will be identified and rewarded.

3.4 Special case: Two agents and two types

Formally, when there are $N = 2$ managers each having $T = 2$ possible types, a complete contract should specify 84 elements. That number can be reduced to 23, a fourfold reduction, by assuming that 1- the two managers are basically identical before learning their private information, 2- all agents that have the same observable characteristic must be treated the same and 3- agents are playing a Nash strategy.

The $\left[\left(2^N - 1\right) + N\left(T+1\right)^N\right] T^N$ formula allows one to compute these numbers for arbitrarily N and T. Our reduction approach has some merit since, for instance, when $N = T = 6$ we obtain a half-million fold reduction. Although the reduction from 33 billion contract elements to $64,330$ is sizable, having to juggle over $64,000$ variables is still a difficult task even for the best operational research algorythm.

When there are $N = 2$ managers each having $T = 2$ possible types, the 23 elements the contract needs to specify are

The probability of auditing both managers when both report low cost (θ^L)	The wage paid when both managers report low cost and none are audited	The wage paid to the high cost manager when reports are different and both are audited
The probability of auditing a single manager when both report low cost	The wage paid to the audited manager when both managers report low cost and only one is audited	The wage paid to the low cost manager when reports are different and only the low cost manager is audited
The probability of auditing both managers when both report high cost (θ^H)	The wage paid to the non-audited manager when both managers report low cost and only one is audited	The wage paid to the high cost manager when reports are different and only the low cost manager is audited
The probability of auditing a single manager when both report high cost	The wage paid when both managers report high cost and both are audited	The wage paid to the low cost manager when reports are different and only the high cost manager is audited
The probability of auditing both managers when they report different costs	The wage paid when both managers report high cost and none are audited	The wage paid to the high cost manager when reports are different and only the high cost manager is audited
The probability of auditing only the low cost manager when they report different costs	The wage paid to the audited manager when both managers report high cost and only one is audited	The wage paid to the low cost manager when reports are different and none are audited
The probability of auditing only the high cost manager when they report different costs	The wage paid to the non-audited manager when both managers report high cost and only one is audited	The wage paid to the high cost manager when reports are different and none are audited
The wage paid when both managers report low cost and both are audited	The wage paid to the low cost manager when reports are different and both are audited	

4 Implications

The complexity of contracts where information is correlated across managers implies, amongst other things, that it may be too costly in the real world to implement. The way corporations approach this problem is to design labor contracts that approximate the fully specified contract. A corporation that is only willing to specify 10 different salaries would end up bunching together many elements of the fully specified contract. A corporation then decides implicitly that some workers will extract some rent for the corporation in the sense that they may lie about their true type and receive a compensation that is not commensurate with their cost of production. Another way to simplify contract is to resort to qualitative assessments of a worker's job.

Most if not all contracts in reality do not specify all possible contingencies, mainly because all contingencies are too numerous to count. This is why we often time see clauses in contracts such as "hard working", "team player", "shows potential", "conscientious", etc.... These contract provisions are arguably subjective; one's idea of hard working may not be someone else's. The efficiency of these contract provisions rests on the trust that workers put in management in acknowledging who fits in the subjective category and who does not.

Another implication of our results is that it gives a rationale for hierarchies. Suppose that any individuals has the intellectual ability to manage at most 200 variables. If there are only two types of employees in the organization, then, from the previous table, we know that the optimal number of workers under his command is five. This means that if the organization has ten employees, two individuals will need to be hired to manage these ten employees. If the organization has 500 employees, then the organization needs to hire 100 lower managers to manage the 500 employees, 20 middle managers to manage the 100 lower managers, and 4 upper managers to manage the 20 middle managers. This means that an organization that has 500 employees will have 124 managers distributed over three levels.

Suppose some technological advance increases the number of variables any individual can keep track of to 301. This means that any individual can manage six employees, if employees can take only two types. The optimal number of managers needed in an organization of 500 employees is then 101 distributed over three levels (84 lower managers, 14 middle managers, 3 upper managers). This technical advance allows the organization to reduce its managerial staff by about 20 percent.

In Boyer and González (1999) it is shown that the more correlated the information, the lower the ability of agents to extract a rent from the principal. This means that the more correlated the information, the less likely is the principal to audit and the lower the wages he needs to pay to induce truth-telling. Let us apply this result to our hierarchy. Suppose each time information reaches a new echelon of our hierarchy that a white noise is added to the signal. This means that information is less and less correlated as it moves up the hierarchy. Given the Boyer and González (1999) results, it should then be expected to have upper managers audited with greater probability and be more highly rewarded for their work.

5 Conclusion

The goal of this paper was to examine what happened to wage contracts when managers must file a report based on information that is correlated. In the literature we encounter basically only two cases. First, when managers are risk neutral, it has been shown that it is possible to extract all the information from the managers using arbitrarily large transfer schemes. When managers are risk averse or have limited liability, this is no longer the case. The second case is where the information is independent from one manager to the next. In this context it is not possible to design a contract that extracts all the information from the managers. In fact, the optimal contract is such that some rent must be paid to the manager who is best qualified. The case of correlated information with auditing and risk averse managers was never addressed in the scientific literature.

The complexity associated with designing contracts where information is correlated across managers is astonishing. In simple settings such as two managers having two possible types, a contract must specify at least 84 variables (or 23 when making simplifying assumptions). When the number of managers and the number of types increase, the number of variables that need to be specified grows exponentially. In an organization with six managers having three possible types, almost 18 million variables ($1,946$ when making simplifying assumptions) must be specified in the contract.

The implications of our results are two-fold. First, the complexity issue raised implies that fully specified contracts may be too expensive to implement in reality. We infer that this may be why corporation resort to pay scales and subjective wording in their labor contracts. It may be less precise and it may allow some workers to shirk, but it may also be easier and less costly to implement. The second important implication of our results is that it gives a rationale for hierarchies. Suppose individuals have a limited ability to handle complex contracts in the sense that individuals can only keep track of a limited number of variables. It may then be best for an organization to give itself a pyramidal structure where individuals manage an optimal number of workers based on their ability to keep track of the different parameters of the optimal complete contract.

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