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Favoritism In Vertical Relationship: Input Prices And Access Quality

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Résumé / *Abstract*

On étudie le favoritisme qui existe dans la relation verticale entre une firme à l'amont et plusieurs firmes à l'aval. On démontre que le favoritisme est le résultat de la maximisation de profit. On considère les questions suivantes. Premièrement, si la firme à l'amont peut fixer des prix différents pour le même produit qu'elle vend aux firmes à l'aval, est-ce qu'elle traite mieux les firmes qui sont moins efficaces? Deuxièmement, si la firme à l'amont peut offrir aux firmes à l'aval des niveaux de qualité d'accès à son réseau, est-ce que la qualité sera uniforme? La réponse à la première question dépend de l'aptitude de l'auto-provision des firmes à l'aval. Quant à la deuxième question, on montre que certaines firmes sont favorisées.

Mots clés : Relation verticale, le prix des inputs, la qualité d'accès, oligopole.

Favoritism in vertical relationship is a situation in which an upstream firm sets favorable exchange conditions to some agents at the expense of others. This paper explores the reason for, and direction of, favoritism in the vertical relationship between an upstream firm and a number of downstream firms that are Cournot rivals relying on the inputs provided by the upstream firm. We show that favoritism may arise from profit maximization. We address the following questions: (i) if the upstream firm can charge different prices to different downstream firms, will it treat the less efficient firms more favorably? (ii) if the upstream firm can provide different levels of quality of access to several ex ante identical downstream firms, will it provide a uniform quality of access? We show that the answer to (i) depends on whether downstream firms can self-supply, and we characterize the structure of favors. As for (ii), we show that among ex-ante equal firms, some firms will be selected for favorable treatment.

Keywords: *Vertical Relationship, Input Pricing, Access Quality, Oligopoly.*

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1. Introduction:Favoritism in Vertical Relationship

Vertical relationship between upstream …rms and downstream …rms is one of the most important topics in industrial organization, both at the theoretical level and in regulatory practices. The following examples illustrate the prevalence of vertical relationships. In the petroleum industry, the upstream …rm is the supplier of crude oil, and the downstream …rms are oil re…neries. In telecommunications, the downstream …rms serve the market for long distance calls, while the upstream …rm is the owner of the local network, without which the long-distance telephone companies cannot sell their products to the consumers. In the market for electricity, it is often the case that electricity transmission and distribution is controlled by one …rm, but electricity generation is not. Downstream …rms generate electricity and sell it to consumers, using an essential input which is the transmission network provided by the owner of the network, considered as an "upstream" …rm. In some situations, an upstream …rm can also be integrated with a downstream …rm (e.g., the case of the owner of a local telephone network who also provides long distance services, in direct competition with several other "downstream" long-distance service …rms).

In regulatary practices, the pricing of intermediate input supplied by an upstream monopolist has been a matter of concern. For example, according to the Economist (Feb. 5, 2000, p. 60), America's Federal Trade Commission (FTC) decided to challenge the proposed merger between BP Amoco, and Atlantic Rich…eld, known as Arco, because the merged …rm would control 70% of Alaska's oil reserves, and this control would allow it to unfairly raise the price of crude oil that it sells to re…neries on the west coast of America. In particular, the FTC argued that BP Amoco has discriminated among customers, unfairly charging higher prices to re…ners that cannot easily switch to imported oil. According to the FTC, this …rm "should therefore not be trusted with an even bigger market share" (p.60).

If some agents are unfairly treated compared with others, a complaint about favoritism can be made. If agents are identical, but receive unequal treatments, favoritism is easy to establish. But favoritism is more di¢cult to prove when agents are heterogenous. In this model, we consider an asymmetric downstream oligopoly: …rms are ex-ante di¤erent with respect to production cost. In this setting, we examine the incentive for an upstream …rm to pratice favoritism. Some of the major questions concerning favoritism in vertical relationship are: (i) what is the direction of favoritism when an upstream monopolist can practice input price discrimination among downstream …rms that are not identical? (i.e., what is the impact of downstream heterogeneity?) (ii) is the answer to (i) sensitive to the curvature properties of the demand function, (iii) how do the answers to the above questions change if (a) the downstream …rms can also produce, perhaps at higher costs, the input themselves, or (b) the upstream …rm is integrated with a downstream …rm to supply the …nal good to consumers, in direct competition with other downstream …rms? (iv) if the upstream …rm can provide input at di¤erent quality levels to di¤erent downstream …rms, will it put at a disadvantage a subset of ex ante identical downstream …rms?

Partial answers to some of the above questions have been provided by DeGraba (1990) and Katz (1987). Assuming that downstream …rms cannot produce the input, DeGraba shows[1] that, under linear demand and constant marginal costs, "when the supplier is allowed to price-discriminate, he charges the …rms with lower marginal cost a higher price than he charges the …rm with the higher marginal cost"(p.1248). This kind of favoritism has also been termed "discount reversal" because it predicts the exact opposite of the "quantity discount" phenomenon, i.e., the empirical observation that larger buyers tend to be charged less per unit than smaller ones. DeGraba explains that "the apparent contradiction stems from the fact that quantity discounts are used as a self-selection mechanism when the seller does not know the demand curves of the buyers"(p.1248). In DeGraba's model, because

[1] DeGraba pointed out (p.1248) that this result was presented in Katz (1987) for Cournot players. DeGraba's main interest is in how price discrimination a¤ects downstream producers' long-run choice of technology.

of the assumption of perfect information, such quantity discounts do not arise[2]. DeGraba's explanation seems to suggest that under perfect information, one would not observe quantity discount.

However, Katz (1987) has shown that quantity discounts may arise even under perfect information, if the input supplied by the upstream …rm can also be produced by the downstream …rms, under a special form of increasing returns: constant marginal cost, and falling average cost, owing to a strictly positive …xed cost in the production of the intermediate input. Thus, according to Katz, a monopolist that sells an input would o¤er to a large buyer, such as a chain store, a better deal than the ones it o¤ers to local stores, because the chain store can make the credible threat of producing the input itself as it has the potential advantage of economies of scale. Katz's model seems to suggest that increasing return in self-supply is a crucial factor for quantity discount under perfect information.

In addition to the above "positive" issues, the "normative" issues of regulation have received a great deal of attention in the industrial organization literature. If there exists a regulator that seeks to maximize social welfare, what are the appropriate regulations on input prices (or access prices) and input quality? Recent works by Vickers (1995), Armstrong, Doyles, and Vickers (or ADV, 1996), La¤ont, Rey, and Tirole (1996a, 1996b) have shed much light on these topics. Vickers (1995) considers the case where the downstream …rms are symmetric Cournot rivals under free entry (implying zero pro…ts), while ADV (1996) considers a downstream competitive fringe, that takes as given the price announced by a dominant integrated …rm. ADV provides an ECPR (e¢cient component pricing rule) formula that relates the input price (or access price) to the direct cost and to the opportunity cost of providing access.

This paper explores the economic rationale for favoritism in vertical relationship, and explores the direction of favoritism. First we

[2] In our Appendix A1, we show that DeGraba's discount reversal result holds also in a more general model with non-linear demand and non-constant marginal costs.

consider the case where downstream firms can self-supply, and show that favoritism against the weak firms, i.e., quantity discount (in the sense of lower input price for larger firms) can occur even under decreasing returns, in contrast to Katz's assumption of increasing returns[3].We also derive an access pricing formula for the case in which downstream firms are asymmetric Cournot rivals. Since we postulate that the objective is to maximize the profit of the upstream firm (or, in some cases, the vertically integrated firm) rather than to maximize social welfare, our access pricing formula is not directly comparable to those of ADV. However, broadly speaking, there is a certain similarity in interpretation.

Another important form of favoritism that we address in this paper is input quality discrimination. As pointed out by Vickers (1995, p.14), input price is only one of several possible ways that an integrated firm could use to restrict access. Another dimension of restriction is the quality of access. Quality discrimination gives an integrated firm an alternative way of raising rivals' costs. An example is the interconnection of telecommunication networks. According to Vickers, "though the pricing terms on which British Telecom was to give access to its rival Mercury were set in 1985, there has been continuing dispute about the quality of that access in terms of delay, the quality of the lines of exchanges, etc., and the impact on Mercury's competitive position."(p. 14). Our paper complements Vickers' informal discussion on quality discrimination by providing a formal analysis of a model of input quality favoritism, where an integrated firm can provide access at different quality levels to several downstream rivals. We show that it can be optimal for the integrated firm to treat ex-ante identical rivals in non-identical ways. Our result, that it may be optimal to practice "favoritism among equals", indicates that models in which identical firms are assumed to be treated equally, can be misleading[4].

[3]We also show, in Appendix A1, that if downstream firms cannot self-supply the input, then discount reversal occurs, even when the demand curve is not linear and marginal cost is not constant.

[4]For other instances of "unequal treatment of equals", see Salant and Shaffer

This paper, by characterising the direction of favoritism, goes beyond the grounds covered by Long and Soubeyran (1997a,1997b, 2001). They examined a general class of games called "Cost Manipulation Games with Costs of Manipulating." This class includes two-stage games, where, in the second stage, firms compete as Cournot, or Bertrand, rivals, and in the first stage, some outside agent manipulates some variables, to maximise its own objective. These manipulations generate costs of manipulating. In general terms, this class of games involvs a number of agents $i \in I$, who, in the second stage, play a non-cooperative game G, and the pay-off to agent i is $\pi_i = \pi_i(a_i; a_{-i}; m_i; m_0)$, where a_i denotes agent i's action, $a_i \in A_i$. In the first stage, a principal[5], who may be an outsider, or a sub-group of these agents, manipulates the variables $m_i \in M_i$; $i \in I$, and $m_0 \in M_0$ to influence the nature of the second stage game. In this paper the manipulating agent is the upstream supplier and the variables for manipulation are prices or quality levels of the intermediate goods. The costs of manipulating are upstream production costs. If the second stage game has a unique equilibrium, we show that, by an appropriate change of variable, the first cooperative stage reduces to a decomposable program[6]. This program is a sup-convolution program and can be solved globally using the mathematical duality theory (Rockafellar 1970). It has an elegant geometric interpretation.

Our global method of resolution helps us to determine the direction (not just the existence) of favoritism.(For the existence results, see Salant-Shaffer (1996,1999) for local conditions, and Long and Soubeyran (1997a,1997b, 2001), for a global geometric approach, well adapted to capture non-linearities). Our paper shows that input-price favoritism puts higher cost firms at a disadvantage, but among

(1996,1999), Long and Soubeyran (1997a,b, 2001).

[5] The extension to the case of two rival principals presents no conceptual difficulties.

[6] This decomposable program is of the form: $\max \sum_{i \in N} f_i(z_i; z_N)$ with respect to the z_i, under the constraint $\sum_{i \in N} z_i = n z_N$.

identical downstream …rms there is no favoritism in input price. By contrast, input quality favoritism can occur among identical …rms[7]. (In the input quality case, we do not seek to characterize the direction of favoritism when …rms are heterogenous, because the convexity of the objective function makes it di¢cult to determine the bias.)

2. Input price favoritism in the presence of self-supply by downstream …rms

In this section we focus on the case where all downstream …rms have constant[8] marginal downstream costs of production of the …-nal good, using an intermediate good that they can either produce themselves, or buy from the upstream supplier (or both). We wish to determine whether the upstream …rm would …nd it pro…table to practice favoritism against the large …rms, i.e. to practice "discount reversal" (charging higher prices to larger downstream …rms.)

2.1. The timing of the game

We assume that there are n downstream …rms, and a single up-stream …rm. Let $N = \{1, 2, ..., n\}$ denote the set of downstream …rms. The downstream …rms are oligopolists that produce a homogenous …-nal good, and compete as Cournot rivals in the …nal good market. The inverse demand function is $P = P(Q)$. In order to produce q_i units of the …nal good, the downstream …rm i needs $D_i(q_i)$ units of the intermediate input. It can satisfy this need by purchasing y_i units of the intermediate input from the upstream …rm S, and producing x_i units of the intermediate input itself, such that $y_i + x_i = D_i(q_i)$. Let t_i be the …rm-speci…c price of the intermediate good charged by the upstream …rm S to the downstream …rm i. Let $U_i(x_i)$ be the cost to

[7] The reason is that the quality variable a¤ects, in a non-linear way, the down-stream …rms' unit cost of producing the …nal good.

[8] The case of non-constant marginal costs is analyzed in Appendix A1.

firm i of producing x_i. We assume that $U_i(x_i)$ is strictly convex.The profit function of firm i is

$$\pi_i = P(Q)q_i - t_i[D_i(q_i) - x_i] - U_i(x_i)$$

Following De Graba and Katz, we focus mainly on the case where the supplier S cannot charge a fixed fee. It is important to note that since the downstream firms can produce the intermediate input, the upstream firm can never charge a fixed fee that would reduce firm i's profit to zero.

The timing of the game is as follows. In the first stage, the upstream firm S sets discriminatory input prices t_i, $i = 1, \ldots, n$. Its profit is

$$\Pi_S = \sum_{i \in N} t_i[D_i(q_i) - x_i(t_i)] - c_S \left[\sum_{i \in N} \{D_i(q_i) - x_i(t_i)\} \right] + \sum_{i \in N} T_i$$

where $T_i = 0$ if two-part tariff is not allowed, and where c_S is firm S's constant marginal cost.

In the second stage, each downstream firm i makes both its procurement decision and its final output decision at the same time[9]: it chooses the quantity q_i and also decides how much of the required input $D_i(q_i)$ is to be self-supplied, $x_i \geq 0$, and how much to be purchased from the upstream firm S, $y_i = D_i(q_i) - x_i \geq 0$.

2.2. The equilibrium in stage two

As usual, the game is solved backwards. We consider first the choice made in the second stage. Given t_i, each firm i solves the program:

$$\max_{q_i, x_i} \pi_i = P(q_i + Q_{-i})q_i - t_i[D_i(q_i) - x_i] - U_i(x_i)$$

[9] One could consider two other alternative formulations. In one formulation, the downstream firms choose q_i in stage 2, and make procurement decision in stage 3. Another alternative formulation would be to reverse the order: to make procurement decision in stage 2 and final output decision q_i in stage 3. It can be shown that our results are unchanged, because of the separability of x_i and q_i in the profit function of firm i.

subject to

$$D_i(q_i) - x_i \geq 0 \qquad (1)$$

and non-negativity constraints.

We will restrict attention to the case where the cost $U_i(x_i)$ is sufficiently convex so that the constraint $D_i(q_i) \geq x_i$ is not binding. Then the first order conditions are:

$$P'(\hat{Q})\hat{q}_i + P(\hat{Q}) = t_i D_i'(\hat{q}_i); \quad i \in N \qquad (2)$$

and

$$t_i - U_i'(\hat{x}_i) = 0; \quad i \in N \qquad (3)$$

where the hat denotes equilibrium values. From (3), we obtain $\hat{x}_i = \hat{x}_i(t_i)$ with

$$\hat{x}_i'(t_i) = 1/U_i'' > 0$$

In what follows, we will focuss on the case where the input requirement function $D_i(.)$ is linear, i.e.,

$$D_i' = d_i > 0$$

and the function $U_i(.)$ is quadratic

$$U_i(x_i) = \frac{u_i x_i^2}{2}$$

Then

$$\hat{x}_i(t_i) = \frac{t_i}{u_i}$$

Let us define firm i's marginal cost of producing the final good as

$$\mu_i \equiv t_i d_i$$

Then (2) becomes

$$\hat{q}_i P'(\hat{Q}) + P(\hat{Q}) = \theta_i; \quad i \in N \tag{4}$$

Summing (4) over all $i \in N$; we get

$$\hat{Q} P'(\hat{Q}) + nP(\hat{Q}) = n\theta_N \tag{5}$$

where

$$\theta_N \equiv \frac{1}{n} \sum_{i \in N} \theta_i$$

It follows from (5) that the equilibrium output $\hat{Q}$ depends only on θ_N, and we may write

$$\hat{Q} = \hat{Q}(\theta_N) \tag{6}$$

We obtain from (4) and (6) the following relationship between equilibrium output of firm i and $(\theta_N; \theta_i)$:

$$\hat{q}_i = \frac{\left[P(\hat{Q}(\theta_N)) - \theta_i \right]}{-P'(\hat{Q}(\theta_N))} \equiv \hat{q}_i(\theta_N; \theta_i) \tag{7}$$

The quantity of input that firm i purchases from the upstream monopoly is:

$$y_i = d_i \hat{q}_i - b_i(t_i) = d_i \hat{q}_i(\theta_N; \theta_i) - \theta_i/(u_i d_i) \equiv y_i(\theta_N; \theta_i) \tag{8}$$

2.3. Stage one: input price favoritism

Now consider stage 1. The upstream monopolist sets the t_i's (and hence $\theta_i = d_i t_i$ and θ_N, to maximize its profit:

$$\max_{\theta_i} \Pi_S = \sum_{i \in N} \left(\frac{\theta_i}{d_i} - c_S \right) y_i(\theta_N; \theta_i) \tag{9}$$

or, using (8),

$$\max_{\theta_i} \pi_S = \sum_{i \in N} \left(\frac{\theta_i}{d_i} - c_S \right) [d_i \hat{q}_i(\theta_N; \theta_i) - \theta_i/(u_i d_i)] \tag{10}$$

Thus, the profit of the upstream firm depends on the parameters d_i and u_i. The profit function can be written in a more compact form

$$\max_{\theta_i} \pi_S = \sum_{i \in N} f_i(\theta_N; \theta_i) \tag{11}$$

where $f_i(\theta_N; \theta_i) \equiv \left[\frac{\theta_i}{d_i} - c_S \right] [d_i \hat{q}_i(\theta_N; \theta_i) - \theta_i/(u_i d_i)] \equiv -A_i(\theta_N)\theta_i^2 + B_i(\theta_N)\theta_i - E_i(\theta_N)$, with

$$A_i(\theta_N) \equiv \frac{1}{\left[-P'(\hat{Q}(\theta_N)) \right]} + \frac{1}{u_i d_i^2}$$

$$B_i(\theta_N) \equiv \frac{P(\hat{Q}(\theta_N))}{\left[-P'(\hat{Q}(\theta_N)) \right]} + c_S d_i A_i(\theta_N)$$

$$E_i(\theta_i) \equiv \frac{c_S d_i P(\hat{Q}(\theta_N))}{\left[-P'(\hat{Q}(\theta_N)) \right]}$$

We wish to determine conditions under which the monopolist finds it profitable to practice discount reversal. To do this, it is convenient to solve the problem (11) in two steps. In the first step, θ_N is fixed, so that $\hat{Q} = \hat{Q}(\theta_N)$ is fixed, and the optimal θ_i's are determined subject to $\sum_{i \in N} \theta_i = n\theta_N$. In the second step we determine θ_N. This decomposition is useful, because the first step amounts to fixing the price of the final good, which allows us to focus on the input price discrimination aspect of firm S's optimization problem. This aspect is quite separate from the firm 's exploitation of consumers by setting the price of the final good.

2.3.1. Solving the first step

The Lagrangian for the first step is

$$L = \sum_{i\in N} f_i(\theta_N;\theta_i) + \lambda\left[\sum_{i\in N}\theta_i - n\theta_N\right] \tag{12}$$

The program is strictly concave, because $\partial^2 f_i(\theta_N;\theta_i)/\partial\theta_i^2 = -A_i(\theta_N) < 0$. Assuming an interior maximum, we get the first order conditions

$$\frac{y_i}{d_i} + \frac{\theta_i}{d_i}\frac{\partial y_i}{\partial\theta_i} - c_S\frac{\partial y_i}{\partial\theta_i} + \lambda = 0 \tag{13}$$

This equation yields $\theta_i^* = \theta_i^*(\lambda;\theta_N)$. Therefore, substituting into the constraint, we get

$$\sum_{i\in N}\theta_i^*(\lambda;\theta_N) - n\theta_N = 0 \tag{14}$$

which yields $\lambda = \lambda(\theta_N)$. Given θ_N, the monopolist's optimal input price t_i is implicitly given by

$$t_i^* = \frac{\theta_i^*}{d_i} = c_S - \frac{1}{\frac{\partial y_i}{\partial\theta_i}}\left[\frac{y_i(\theta_N;\theta_i^*)}{d_i} + \lambda(\theta_N)\right] \tag{15}$$

where

$$\frac{\partial y_i(\theta_N;\theta_i)}{\partial\theta_i} = \frac{d_i}{P'(\hat{Q})} - \frac{1}{d_i u_i} < 0$$

Then (15) gives, explicitly

$$t_i^* = \frac{1}{2}\left[c_S + [u_i d_i]\frac{P(\hat{Q}) + \lambda(\theta_N)[-P'(\hat{Q})]}{d_i^2 u_i + [-P'(\hat{Q})]}\right] \tag{16}$$

From (16), we obtains the following result:

Proposition 1 (Direction of favoritism)

(a) If any pair (i, j) of downstream firms that have high and identical costs of self-supply, the monopolist will practise discount reversal, i.e., firms with lower downstream costs d_i are charged a higher t_i :

$$\text{sign}\{t_i^* - t_j^*\} = -\text{sign}\{d_i - d_j\}$$

(b) If the costs of downstream self-supply are very low then the monopolist will not practise discount reversal.

(c) For any pair of downstream firms (i, j) with the same input-requirement functions (i.e., $d_i = d_j$), the firm with a lower cost of self-supply (i.e., a lower u) will be charged a lower input price.

Proof: (a) and (b): from (16)

$$\text{sign}\frac{\partial t_i}{\partial d_i} = \text{sign}\{[-P'] - d_i^2 u_i\}$$

The right-hand side is negative if u_i is sufficiently great.

(c) from (16), $\partial t_i / \partial u_i > 0$. $\square$

Part (b) of Proposition 1 is broadly in agreement with the result obtained by Katz (1987), who showed that if downstream firms can threaten to self-supply then the the upstream monopolist will give discounts to larger firms. However, Katz (1987) relied on the assumptions that self-supply involves a positive fixed cost and a constant marginal cost. On the contrary, we assume that self-supply involves no fixed cost, and the marginal cost of self-supply is increasing. Also, part (a) indicates that discount reversal (i.e., lower t_i for smaller firms) can occur even if firms can self-supply, provided the marginal cost curve of self-supply is steep enough.

Proposition 2: (Absence of favoritism among equals) If all downstream firms are ex-ante identical, the upstream supplier will treat them equally, by charging all of them an identical input price $t_i^* = t^*$, for all $i \in N$:

$$t_i^* = t^* = \frac{1}{2}\left[c_S + [ud]\frac{P(\hat{Q}) + \lambda(\theta_N)[-P'(\hat{Q})]}{d^2u + [-P'(\hat{Q})]}\right] \tag{17}$$

Proof: Use the fact that the Lagrangian is strictly concave.

2.3.2. The second step

For the determination of θ_N, we use a procedure similar to that given in the Appendix A1. Substituting $\theta_i^* = \theta_i(\lambda^*(\theta_N); \theta_N)$ into the objective function (11), we obtain

$$\Pi_S^* = \sum_{i \in N} f_i(\theta_N; \theta_i(\lambda^*(\theta_N); \theta_N)) \tag{18}$$

Differentiating Π_S^* with respect to θ_N and equating the derivative to zero, making use of the envelope theorem, we get

$$\frac{d\Pi_S^*}{d\theta_N} = \sum_{i \in N} (\partial f_i/\partial \theta_i) d\theta_i^*/d\theta_N + \sum_{i \in N} \partial f_i/\partial \theta_N = \sum_{i \in N} \partial f_i/\partial \theta_N = 0$$

This condition determines the optimal θ_N.

3. Vertically Integrated Input Supplier and Favoritism in Access Pricing

In the preceding section, the input supplier does not compete in the downstream market. We now consider the case where the input supplier is vertically integrated with a downstream firm and therefore treats other downstream firms as rivals. For instance, in telecommunications, the downstream sector serves the market for long-distance calls, and the upstream firm is the owner of the local telephone network, which may be vertically integrated with a long-distance service provider. Similarly, in the market for electricity, electricity transmission and distribution may be controlled by one firm, that also owns an electricity generation plant, in competition with other plants that rely on the transmission network provided by the integrated firm.

Using the model introduced in this section, we seek answers to the following questions: (i) does the vertically integrated firm has an incentive to practice discount reversal? (ii) how strong is the incentive to raise rivals' cost? (iii) what form does the "Efficient Component Pricing Rule" (ECPR) take when the downstream firms are non-identical Cournot oligopolists?

Vickers (1995) addresses the question of access pricing under the assumptions that the downstream firms are identical Cournot rivals, and that downstream profits are zero due to free entry. Armstrong et al. (1996) assume that the downstream firms constitute a competitive fringe (i.e., they take the price of their output as given). We consider the case of asymmetric downstream firms that are Cournot rivals, and their number is fixed.

Let $N = \{1, 2, ..., n\}$ be the set of downstream firms. Partition this set into two subsets, denoted by $I = \{1, 2, .., n_I\}$ and $J = \{n_I + 1, ..., n_I + n_J\}$ where $n_I + n_J = n$. All members of I are integrated with the upstream firm S while all members of J are independent rivals. If $n_I \geq 2$, we assume that these downstream firms also compete with each others, i.e., the integrated firm behaves as if it has a multi-divisional structure that discourages collusion between the downstream divisions.

If the output of downstream firm h is q_h, its input need is $D_h(q_h)$. This need is satisfied partly by purchasing y_h from the upstream division of the integrated firm, and partly by self-supplying the quantity $x_h = D_h(q_h) - y_h$. The cost of self-supply is $U_h(x_h)$. The profits of the downstream firms are

$$\pi_h = Pq_h - t_h y_h - U_h(x_h); \quad h \in I \cup J \equiv N$$

where $y_h + x_h = D_h(q_h)$. For simplicity, we assume that $D_h(q_h) = d_h q_h$; $h \in N$. The profit of the upstream division (i.e., the input supplier S) of the integrated firm is

$$\Pi_S = \sum_{h \in N} (t_h - c_S) y_h$$

The total profit of the integrated firm is

$$\Pi_{IS} = \Pi_S + \sum_{h\in I} \pi_h = \sum_{j\in J} (t_j - c_S)y_j + \sum_{i\in I} (P - d_i c_S)q_i + \sum_{i\in I} [c_S x_i - U_i(x_i)]$$

The timing of the game is as followed. In the last stage, all the n downstream entities (the n_I divisions of the integrated firms and the n_J independent downstream firms) compete as Cournot rivals. Each downstream entity h chooses simultaneously its final output level q_h and its own production of intermediate input $x_h \le d_h q_h$, while taking as given all the pairs $(q_k; x_k)$ for $k \neq h$. They also take as predetermined the input prices t_h dictated by the upstream entity S. Thus entity h seeks to maximize

$$\pi_h = P(Q_{-h} + q_h)q_h - t_h d_h q_h + [t_h x_h - U_h(x_h)] \tag{19}$$

subject to $d_h q_h \ge x_h \ge 0$.

Assuming an interior solution, we have $2n$ first-order conditions:

$$P'(Q)q_h + P(Q) = t_h d_h$$

$$t_h - U_h'(x_h) = 0$$

These conditions give

$$\hat{q}_h = \hat{q}_h(\theta_N; \theta_h) = \frac{\left[P(\hat{Q}(\theta_N)) - \theta_i\right]}{-P'(\hat{Q}(\theta_N))} \tag{20}$$

and $\hat{x}_h = \hat{x}_h(t_h)$, where $\theta_h = t_h d_h$ and $\theta_N = (1/n)\sum_{h\in N} \theta_h$. This stage gives the equilibrium profit of the downstream entities (using (20) and (19))

$$\hat{\pi}_h = \left[P(\hat{Q}(\theta_N)) - \theta_i\right]\hat{q}_h + V_h^*(t_h) = [-P'(\hat{Q}(\theta_N))]\hat{q}_h^2 + V_h^*(t_h)$$

where $V_h^*(t_h) \equiv \max_{x_h}\{t_h x_h - U_h(x_h)\}$ s.t. $x_h \ge 0$.

We now turn to the first stage of the game, when the integrated firm chooses the t_h's to maximize its profit

$$\pi_{IS} = \sum_{j \in J} (t_j - c_S)\hat{y}_j + \sum_{i \in I} (\hat{P} - d_i c_S)\hat{q}_i + \sum_{i \in I} [c_S \hat{x}_i - U_i(\hat{x}_i)]$$

where $\hat{P} \equiv P(\hat{Q}(\theta_N))$, $\hat{q}_h = [\hat{P} - d_h t_h]/[-\hat{P}']$ for all $h \in N$, $\hat{y}_j = d_j \hat{q}_j$ for all $j \in J$, and $\hat{x}_i = \hat{x}_i(t_i)$ for all $i \in I$. Recalling that $t_h = \theta_h/d_h$, we can formulate the optimization problem of the integrated firm as that of choosing the t_h's to maximize π_{IS}:

As in the preceding section, we solve this problem in two steps. In step 1, we fix θ_N (so that $\hat{P}$ is fixed), and optimize with respect to the t_h's subject to $\theta_N - (1/n)\sum_{h \in N} t_h d_h = 0$. The Lagrangian is

$$L = \pi_{IS} + \lambda\left[\sum_{h \in N} t_h d_h - n\theta_N\right]$$

Manipulations of the first-order conditions yield

$$t_i = c_S + \left(\frac{d_i}{[-\hat{P}']x_i'}\right)\left[\lambda[-\hat{P}'] - (\hat{P} - d_i c_S)\right]; \quad i \in I \tag{21}$$

and, for all $j \in J$,

$$t_j = c_S + \left(\frac{d_j}{d_j^2 + [-\hat{P}']x_j'}\right)\left[\lambda[-\hat{P}'] + \hat{P} - d_j t_j - [-\hat{P}'](\hat{x}_j/d_j)\right] \tag{22}$$

These formulas are only implicit because the t_i (or t_j) appear on both sides of the equations. One may relate these formulas to the "efficient component pricing rule" (ECPR)[10] derived by Armstrong, Doyles and Vickers (ADV, 1996):

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[10] The ECPR, also known as the Baumol-Willig rule, allows the incumbent to charges access prices equal to her opprtunity cost on the competitive segment. For a concise discussion of the theoretical debate on ECPR, see Laffont and Tirole (2000, p.p 166-7).

Input price (or access price) = direct cost + opportunity cost of providing access.

However, we should note that ECPR was derived by ADV under the objective of maximizing welfare, not maximizing the profit of the integrated firm. Our component pricing rules (21) and (22) are for a monopolist. It contains the Lagrange multiplier λ which is a function of the given θ_N. (See, for example, equation (14) of the preceding section.)

In order to proceed further, let us assume that

$$U_h(x_h) = \frac{u_h x_h^2}{2} \tag{23}$$

then we have

$$t_i = c + \left[\frac{d_i c_S + \lambda[-\hat{P}'] - \hat{P}}{[-\hat{P}']} \right] d_i u_i \quad ; i \in I \tag{24}$$

$$t_j = \frac{1}{2} \left[c_S + \frac{\lambda[-\hat{P}'] + \hat{P}}{d_j^2 u_j + [-\hat{P}']} (d_j u_j) \right] \quad ; j \in J \tag{25}$$

It follows that, for any pair $(i; i')$ of downstrean divisions such that $d_i = d_{i'}$, we have

$$\frac{t_i - c_S}{t_{i'} - c_S} = \frac{u_i}{u_{i'}} \quad ; i; i' \in I$$

and for any pair $(j; j')$ of external downstream firms, we have

$$\frac{t_j - (c_S/2)}{t_{j'} - (c_S/2)} = \frac{\gamma_j}{\gamma_{j'}} \quad ; j; j' \in J$$

where

$$\gamma_j = \frac{d_j u_j}{d_j^2 u_j + [-\hat{P}']}$$

(Note that $\partial\gamma_j/\partial u_j > 0$). Thus we have established the following results:

Proposition 3:

(i) the input prices for external downstream firms (that have the same d_j) are subject to victimization, i.e., firms with a higher u_j (i.e., those firms whose slope of the marginal cost of self-supply is relatively steep) will be charged a higher t_j.

(ii) within the integrated firms, the transfer prices applied to downstream divisions are more favourable to those with lower costs of self-supply. ■

Property (i) is consistent with part (c) of proposition 1. Property (ii) implies that the less efficient divisions of the integrated firm are "penalized", in the sense that the integrated firm cross-subsidizes its more cost-efficient divisions. This is similar to the theory of picking-winner in the strategic trade literature.

Remark 3.1: From (25), we can ask the following question: for a given θ_N, (so that $\hat{Q}$ is fixed), and a given number n of downstream entities, how does t_j change if the set I of downstream divisions expands relative to the set J of independent downstream firms? To simplify, assume that the d_h's are the same for all $h \in N$. Compare the situation where I is the empty set (i.e., the upstream firm is not integrated with any downstream firm) with the situation where I consists of only one firm, which we denote as firm 1. Let $\lambda^*_0(\theta_N)$ and $\lambda^*_1(\theta_N)$ denote the optimal value of the Lagrange multiplier in these two situations respectively. If u_1 is very small, then we can show (see Appendix A4) that

$$\lambda^*_1(\theta_N) < \lambda^*_0(\theta_N) \qquad (26)$$

This inequality implies that, if firm j can self-supply, the input price t_j charged by the upstream firm, given by (25), decreases when the upstream firm becomes vertically integrated with firm 1. Thus, for a given θ_N, vertical integration does not result in a "raising rivals' cost" strategy.

Remark 3.2: From (25) and assuming the concavity[11] of $\hat{\Pi}_{SI}$ with respect to the t_j's, we conclude that, for any pair of identical downstream …rms, the integrated …rm charges them the same input price. Thus "equals are treated equally". As we will see in the following section, this property no longer holds in a model where the upstream …rm S can choose quality levels that it o¤ers to downstream …rms.

Remark 3.3: The second step in the solving the optimization problem of the integrated …rm consists of determining the optimal μ_N. This can be done using the approach taken in Appendix A1.

4. Favoritism in Quality of Access

So far, we have focussed on input price discrimination. As pointed out by Vickers (1995, p. 14), input price is only one of several possible ways that an integrated …rm could use to restrict access. Another dimension of restriction is the quality of access. Quality discrimination gives …rm S an alternative way of raising rivals' costs. According to Vickers, " a possible example is the interconnection of telecommunications networks. Though the pricing terms on which British Telecom was to give access to its rival Mercury were set in 1985, there has been continuing dispute about the quality of that access in terms of delays, the quality of lines and exchanges, etc., and the impact on Mercury's competitive position"(p.14).[12]

In this section, we complement Vickers' informal discussion on quality of access, by developing a formal model of quality discrimination.We do not claim to present here a model that re‡ects the reality of the British communications industry[13]. We will show that input

[11] In a more general case, the function Π_{SI} is strictly concave in the t_j's if $U_h(x_h) = (u_h/^1)x_h^{1}$ where 2 ¸ ¹ ¸ 1.(See the Appendix.)

[12] According to Oftel, in 1996, Mercury had 10.7% of the domestic-call market and 15.6% of the international call market. (O¢ce of Telecommunications, or Oftel, Market Information Update, July, 1977). For most customers, Mercury relies on its rival, British Telecom, for originating calls and terminating calls.

[13] For a brief survey of the regulatory problem in the British communication industry, see La¤ont and Tirole (2000).

quality discrimination exhibits a new feature not encountered in input price discrimination: under certain conditions, the upstream firm will find it profitable to offer identical downstream firms non-identical quality levels.

The downstream sector consists of n firms producing a homogenous final good. Downstream firm i's output is q_i. Its unit production cost is $d_i = d_i(\theta_i)$ where θ_i is the quality level of the access supplied by the upstream firm S to the downstream firm i. We assume that θ_i can be chosen from the range $[\theta_L; \theta_H]$. Firm S is not vertically integrated[14].We assume that

$$d_i(\theta_i) = d_i^o - r_i(\theta_i) \tag{27}$$

where $d_i^o > 0$ and $r_i(\theta_L) = 0$; $r_i' > 0$, $r_i'' < 0$, and $r_i(\theta_H) < d_i^o$. Thus $r_i(\theta_i)$ is the reduction in unit cost when the quality of access θ_i exceeds the minimum level θ_L.

The upstream firm's cost of providing access quality level θ_i to firm i (whose output is q_i) is assumed to be $c_S\theta_i q_i$. This indicates that (i) for a given θ_i, the cost to the upstream firm S is linear in q_i, (ii) for a given output level q_i, the cost to the upstream firm S is proportional to the quality of access that it provides. Let $z_i = \theta_i q_i$. We may interpret z_i as the number of units of a standardized intermediate input that firm i buys from firm S. Firm S announces to firm i that the price of each unit of tandardized intermediate input is t_i.

Firm i's profit function is

$$\pi_i = Pq_i - [d_i^o - r_i(\theta_i)]q_i - t_i\theta_i q_i$$

Clearly, for any given output level q_i, firm i, facing a given t_i, will choose the quality level θ_i to maximize π_i (i.e., to minimize cost.) Thus θ_i^* is given by

$$t_i = r_i'(\theta_i^*) \tag{28}$$

[14]Note that British Telecom is integrated, so our model is not strictly applicable for the dispute between Mercury and British Telecom

(here, we assume an interior solution, which would hold if $r'(\mu_L) = \infty$ and $r'(\mu_H) = 0$.) In what follows, we set $\mu_L = 0$.

The cost function of firm i may thus be written as

$$C_i(q_i; t_i) = [d_i^o - r_i(\mu_i^*) + r'(\mu_i^*)\mu_i^*]\, q_i$$

Define the marginal cost of output q_i as

$$\theta_i(\mu_i^*) \equiv \frac{\partial C_i(q_i; t_i)}{\partial q_i} = [d_i^o - r_i(\mu_i^*) + r'(\mu_i^*)\mu_i^*] \tag{29}$$

Note that, since we assume that $r_i(.)$ is strictly concave, and $r_i(0) = 0$;

$$\theta_i - d_i^o = r'(\mu_i^*)\mu_i^* - r_i(\mu_i^*) < 0 \tag{30}$$

Consider the Cournot equilibrium achieved by the downstream oligopolists, given the t_i's. The first order conditions yield

$$\hat{q}_i = \frac{\hat{P} - \theta_i}{[-\hat{P}']} = \frac{\hat{P} - [d_i^o - r_i(\mu_i^*) + r'(\mu_i^*)\mu_i^*]}{[-\hat{P}']} \tag{31}$$

or

$$\hat{q}_i(\mu_i^*; \hat{P}) = \frac{(\hat{P} - d_i^o) + r_i(\mu_i^*) - r'(\mu_i^*)\mu_i^*}{[-\hat{P}']} \tag{32}$$

where $\hat{P} = P(\hat{Q}(\theta_N))$. From (30), if $(\hat{P} - d_i^o) > 0$ then $\hat{q}_i(\mu_i^*; \hat{P}) > 0$.

The profit of the upstream firm is

$$\hat{\pi}_S = \left[\sum_{i \in N} t_i \mu_i^* \hat{q}_i\right] - c_S \left[\sum_{i \in N} \mu_i^* \hat{q}_i\right] \tag{33}$$

or ,using $t_i = r_i'(\mu_i)$;

$$\hat{\pi}_S = \left[\sum_{i \in N} r'(\mu_i^*)\mu_i^* \hat{q}_i(\mu_i^*; \hat{P})\right] - c_S \left[\sum_{i \in N} \mu_i^* \hat{q}_i(\mu_i^*; \hat{P})\right] = \widehat{\pi}_S - \widehat{C}_S \tag{34}$$

where $\hat{\pi}_S = \sum_{i\in N} r'_i(\mu_i^*)\mu_i^*\hat{q}_i(\mu_i^*;\hat{P})$ is the total revenue, and $\hat{C}_S = c_S\left[\sum_{i\in N}\mu_i^*\hat{q}_i(\mu_i^*;\hat{P})\right]$ the total cost of the upstream supplier.

Thus, the optimization problem of the upstream firm amounts to choosing quality levels μ_i^*'s, via the choice of the t_i's since $t_i = r'_i(\mu_i)$; by (28)) to maximize $\hat{\pi}_S$, subject to the constraint:

$$\left[\sum_{i\in N} r_i(\mu_i^*) - r'_i(\mu_i^*)\mu_i^*\right] = d_N^o - \theta_N \tag{35}$$

for a given θ_N (that is, for a given final price $\hat{P}$).

Example 4.1:(Favoritism among equals)

Take the case of the strictly concave unit cost reduction power function

$$r_i(\mu_i) = (\delta_i\mu_i)^{\alpha}; \quad 0 < \alpha < 1$$

then $r'_i(\mu_i)\mu_i = \alpha r_i(\mu_i)$ .Thus

$$r_i(\mu_i^*) - r'_i(\mu_i^*)\mu_i^* = (1-\alpha)r_i(\mu_i^*) \tag{36}$$

Substituting (36) into (31), we get

$$\hat{q}_i(\mu_i^*;\hat{P}) = \frac{(\hat{P} - d_i^o) + (1-\alpha)r_i(\mu_i^*)}{[-\hat{P}']} \tag{37}$$

For a given θ_N, that is for a given $\hat{P}$, the revenue function of the upstream supplier is

$$= \alpha\sum_{i\in N} r_i(\mu_i^*)\hat{q}_i(\mu_i^*;\hat{P}) = (\alpha/[-\hat{P}'])\sum_{i\in N} r_i(\mu_i^*)\left[(\hat{P} - d_i^o) + (1-\alpha)r_i(\mu_i^*)\right]$$

This is a strictly convex function of the r_i :

$$\hat{\pi}_S = (\alpha/[-\hat{P}'])\sum_{i\in N} r_i\left[(\hat{P} - d_i^o) + (1-\alpha)r_i\right]$$

For a given θ_N the cost function of the upstream supplier is

$$\hat{C}_S = c_S\left[\sum_{i\in N}\mu_i^*\hat{q}_i(\mu_i^*;\cdot)\right] = (c_S/[-\hat{P}''])\sum_{i\in N}\mu_i^*\left[(\hat{P}-d_i^o)+(1-\alpha)r_i(\mu_i^*)\right]$$

Let us invert the function $r_i = r_i(\mu_i) = (\delta_i\mu_i)^{\alpha}$ to get $\mu_i = \mu_i(r_i) = (1/\delta_i)(r_i)^{1/\alpha}$. Then the cost function of the upstream supplier is itself a strictly convex function of the unit cost reductions r_i:

$$\hat{C}_S = (c_S/[-\hat{P}''])\sum_{i\in N}(1/\delta_i)(r_i)^{1/\alpha}\left[(\hat{P}-d_i^o)+(1-\alpha)r_i\right]$$

Define

$$r_N \equiv \frac{1}{n}\sum_{i\in N} r_i \tag{38}$$

From (36) and (38), we can write (35) as

$$(1-\alpha)r_N = d_N^o - \theta_N$$

This gives

$$\theta_N = d_N^o - (1-\alpha)r_N$$

and thus we may write

$$\hat{P}(\theta_N) = \hat{P}(d_N^o - (1-\alpha)r_N) = \hat{P}(r_N)$$

The profit function of the upstream supplier is the sum of a difference between two strictly convex functions of r_i

$$\hat{\Pi}_S = \hat{R}_S - \hat{C}_S = \sum_{i\in N} f_i(r_i;r_N) \tag{39}$$

where

$$f_i(r_i;r_N) = \frac{1}{[-\hat{P}'']}\left[(\hat{P}-d_i^o)+(1-\alpha)r_i\right]\left[\alpha r_i - c_S(1/\delta_i)(r_i)^{1/\alpha}\right]$$

The upstream firm's problem is the to choose, for a given r_N (hence, a given $\hat{P}$), the $r_i \geq 0$ to maximize

$$\hat{\pi}_S = \sum_{i \in N} f_i(r_i; r_N)$$

subject to

$$\sum_{i \in N} r_i = n r_N$$

If c_S is small, or if the δ_i are great, then each $f_i(r_i; r_N)$ is strictly convex in r_i in the interval $0 \leq r_i \leq n r_N$, and so is the sum of them. Then, for a given r_N, the optimum is at a corner. For example, if $n = 2$, one firm will achieve the cost reduction $2r_N$ and the other firm's cost reduction will be zero.

The determination of the optimal $r_i = r_i^*(r_N)$ are injected in the value of the profit function of the upstream supplier $\hat{\pi}_S(r_N) = \sum_{i \in N} f_i(r_i(r_N); r_N)$. The next step is to maximize with respect to r_N to find the optimal r_N^*: The optimal θ_N^* follows.

Proposition 4 (Favoritism among equals): If the function $\hat{\pi}_S$ is strictly convex in the r_i^*'s (for a given r_N), then ex-ante identical firms will be given different quality levels.

Comments on example 4.1: The profit function of the upstream firm can be convex or concave in r_i (cost reductions). Recall that

$$\hat{\pi}_S = \left[\sum_{i \in N} t_i \mu_i^* \hat{q}_i\right] - c_S \left[\sum_{i \in N} \mu_i^* \hat{q}_i\right] = \hat{R}_S - \hat{C}_S \tag{40}$$

The revenue of the upstream supplier $\sum_{i \in N} t_i \mu_i^* \hat{q}_i$ is strictly convex in the unit cost reduction r_i (for a given r_N) because the price of μ_i^* units of quality, per unit of the final good produced, is proportional to the unit cost reduction : $t_i \mu_i^* = \alpha r_i$, and the equilibrium quantity

$\hat{q}_i$ produced by firm i is a linear and increasing function of the unit cost reduction of firm i, for a given r_N:

$$\hat{q}_i(r_i;\hat{P}) = \frac{(\hat{P} - d_i^0) + (1 - \alpha)r_i}{[-\hat{P}']} \tag{41}$$

The cost of the upstream supplier $\hat{C}_S = c_S \sum_{i \in N} \mu_i^* \hat{q}_i$ is also strictly convex in the unit cost reductions r_i, because the μ_i^* units of quality is strictly convex in r_i, $r_i(\mu_i) = (\delta_i \mu_i)^{\alpha} \iff \mu_i = (1/\delta_i)(r_i)^{1/\alpha}$, $\alpha > 1$: If the revenue function is sufficiently greater than the cost function, the difference between these two convex functions is convex.

Remark: Proposition 4 indicates that input quality discrimination is very different from input price discrimination. High level of quality of the input reduces the downstream firms' unit cost (not including the upstream firm"s charge) of producing the final output. A greater μ_i (equivalently, a greater r_i) saves the production cost for firm i, at any given q_i, and increases the production cost for firm S at any given q_i. Thus, unlike the case of input price discrimination considered in Sections 2 and 3, input quality discrimination affects the real cost structure of all firms.

5. Concluding Remarks

We have showed that if downstream firms can, to some extent, self-supply the vital input at relatively low marginal cost, the upstream firm will tend to treat more favorably those firms that are more efficient in the use of the input it supplies. Moreover, if the upstream firm is integrated with one or several downstream firms, then in general it will give discount to larger downstream divisions, and victimize external downstream firms that have high costs of self-supply. Downstream firms with the same characteristics are treated equally with respect to input price: favoritism does not occur among equals.

Quality of access provided by an upstream firm can vary accross downstream firms. We have shown that even when all downstream

…rms are ex-ante identical, favoritism in access quality can occur. Unlike raw materials, which tend to be proportional to …nal output level, quality of access is somewhat like capital equipment that shifts down the marginal cost curves. Thus it may be more e¢cient to concentrate this type of "investment" in one downstream …rm, to exploit a sort of economy of scale.

We have also derived an access pricing rule from the point of view of an upstream …rm that faces heterogenous downstream oligopolists. This rule resembles the "e¢cient component pricing rule" (ECPR) in the regulation literature.

Several extensions can be pursued. First, using our model, an ECPR could be derived from the point of view of a regulator. Second, asymmetric information may be introduced, to explore the cases where the upstream …rm or the regulator does not know the cost structure of the …rms. Third, in the case of quality of access, we must …nd out whether there is a strong incentive for an integrated …rm to raise rivals' costs. Another possible generalization is the case where downstream …rms need several intermediate inputs, each being produced by a distinct upstream …rm..

Appendix A.1: Favoritism under non-linear demand and non-constant input-output ratio

In this Appendix we extend our results to the case of non-constant input-output ratio. The model is similar to that of DeGraba (1990), but we replace his assumption of linear final demand by non-linear final demand, and we assume convex downstream cost instead of constant marginal costs. Furthermore, while DeGraba assumes that, in relation to input prices, downstream firms differ from each other in an additive way (i.e., firm i's per-unit cost of output is $t_i + c_i$, where t_i is the input price for firm i determined by the upstream monopolist, and c_i is an additional marginal cost of production which vary across firms), we assume that, in relation to input costs, downstream firms differ from each other in a multiplicative way (i.e., firm i's per-unit cost of output is $t_i D_i(q_i)/q_i$, where $D_i(q_i)$ is the input level necessary to produce output q_i.) We will show that "discount reversal" (i.e., the upstream firm charges a lower price to smaller downstream firms) occurs in this model, as it does in DeGraba's model.[15]

There are n downstream Cournot oligopolists producing a homogenous good, using an intermediate input produced by an upstream monopolist. The set of downstream firms is $N = \{1, 2, \ldots, n\}$. Let q_i denote the output of (downstream) firm i, and let $Q = \sum_{i \in N} q_i$. In order to produce the quantity q_i, the downstream firm i needs to use z_i units of the intermediate input: $z_i = D_i(q_i)$, where $D_i(0) = 0$, $D_i' > 0$ and $D_i'' \geq 0$. We refer to $D_i(.)$ as the downstream input-requirement function of firm i.The upstream supplier, denoted by S, charges firm i the input price t_i (per unit). Let y_i be the amount of input that downstream firm i buys from firm S. In this Appendix, since we assume that the downstream firms have no alternative sources of input supply, we have $y_i = z_i$.

We consider a two-stage game. In the first stage, the supplier S chooses firm-specific input prices $(t_1, \ldots, t_n)$, $i = 1, \ldots, n$. In the

[15] For the case of constant marginal costs and non-linear demand, see Long and Soubeyran (1997b), where the discount reversal is explained in terms of the "concentration motive theorem."

second stage, the downstream firms choose their outputs, and achieve a Cournot equilibrium.

The inverse demand function for the final good is $P = P(Q)$ with $P'(Q) < 0$. Given $t_1, \ldots, t_n$, we have, at a Cournot equilibrium where all firms produce, the conditions

$$P'(\hat{Q})\hat{q}_i + P(\hat{Q}) = t_i D_i'(\hat{q}_i); \quad i \in N \tag{42}$$

Equilibrium profits are

$$\hat{\pi}_i = P(\hat{Q})\hat{q}_i - \left[\frac{P'(\hat{Q})\hat{q}_i + P(\hat{Q})}{D_i'(\hat{q}_i)}\right] D_i(\hat{q}_i)$$

or, more compactly,

$$\hat{\pi}_i = \left(1 - \frac{1}{\hat{\varepsilon}_i}\right) P(\hat{Q})\hat{q}_i + \frac{1}{\hat{\varepsilon}_i}[-P'(\hat{Q})]\hat{q}_i^2 \tag{43}$$

where $\hat{\varepsilon}_i$ is the elasticity of the downstream input-requirement function of firm i, evaluated at the Cournot equilibrium:

$$\hat{\varepsilon}_i \equiv \frac{\hat{q}_i D_i'(\hat{q}_i)}{D_i(\hat{q}_i)}$$

The profit function of the upstream firm is

$$\Pi_S = \sum_{i \in N} t_i y_i - C(y)$$

where $y = \sum_{i \in N} y_i$ and $C(y)$ is the upstream firm's cost of producing y. Given $t_1, \ldots, t_n$, the supplier's profit at the corresponding (downstream) Cournot equilibrium is

$$\hat{\Pi}_S = \sum_{i \in N} \left[\frac{P'(\hat{Q})\hat{q}_i + P(\hat{Q})}{D_i'(\hat{q}_i)}\right] D_i(\hat{q}_i) - C\left[\sum_{i \in N} D_i(\hat{q}_i)\right]$$

Equivalently,

$$\hat{\Pi}_S = \sum_{i \in N} \frac{1}{\hat{\varepsilon}_i}\left[P'(\hat{Q})\hat{q}_i^2 + P(\hat{Q})\hat{q}_i\right] - C\left[\sum_{i \in N} D_i(\hat{q}_i)\right] \tag{44}$$

In the first stage, the supplier, S, chooses the t_i's to maximize its profit. It is clear that the choice of the t_i's is equivalent to the manipulation of the marginal costs μ_i's of the downstream firms, which in turn is equivalent to choosing the equilibrium outputs $\hat{q}_i$'s. Of course the participation constraints $\hat{\pi}_i \geq 0$ must be satisfied.

In what follows, we focus on the benchmark case where S cannot use two-part tariffs nor other forms of non-linear pricing. We further simplify the problem by assuming that the upstream cost is linear

$$C(y) = c_S y; \quad c_S > 0$$

and that the downtream firms' input requirement functions are convex and exhibit constant elasticity:

$$D_i(q_i) = \frac{d_i q_i^{\tau}}{\tau}; \tau \geq 1; d_i > 0.$$

Then firm S's profit in the (downstream) Cournot equilibrium becomes

$$\hat{\pi}_S = \frac{1}{\tau} P(\hat{Q})\hat{Q} - \frac{1}{\tau}[-P'(\hat{Q})]\hat{Q}^2 \hat{H} - \frac{c}{\tau} \sum_{i \in N} d_i \hat{q}_i^{\tau} \qquad (45)$$

where $\hat{H}$ is the Herfindahl index of concentration of the downstream industry:

$$\hat{H} = \sum_{i \in N} \left[\hat{q}_i / \hat{Q} \right]^2$$

As will be seen below, the discriminatory price structure chosen by S depends on the Herfindahl index of concentration and on the elasticity of the slope of the demand curve.

We now solve firm S's optimization problem. It is convenient to proceed in two steps. In the first step, we temporarily fixed the industry output $\hat{Q}$, and seek to characterize the monopolist's choice of the $\hat{q}_i$'s, conditional on $\sum_{i=1}^{n} \hat{q}_i = \hat{Q}$ (given). In the second step, we determine $\hat{Q}$.

The first step:
We re-write $\hat{\Gamma}_S$ as

$$\hat{\Gamma}_S = \frac{1}{\tau}\left[P(\hat{Q})\hat{Q} - \sum_{i=1} f_i(\hat{q}_i;\hat{Q}) \right] \tag{46}$$

where

$$f_i(\hat{q}_i;\hat{Q}) \equiv [-P'(\hat{Q})]\hat{q}_i^2 + cd_i\hat{q}_i^{\tau}$$

For a given $\hat{Q}$, choose the Cournot equilibrium outputs, the $\hat{q}_i$'s, to maximize (46) subject to $\sum_{i=1}^{n}\hat{q}_i = \hat{Q}$ and the non-negativity of $\hat{q}_i$ and $\hat{\pi}_i$. (We will focus on the case where the solution is an interior solution, i.e., $\hat{q}_i > 0$ and $\hat{\pi}_i > 0$). The Lagrangian is

$$L = \frac{1}{\tau}\left[\{P(\hat{Q}) - \lambda\}\hat{Q} + \sum_{i=1} \{\lambda\hat{q}_i - f_i(\hat{q}_i;\hat{Q})\} \right]$$

and is strictly concave in the $\hat{q}_i$ for a given $\hat{Q}$. Then, at an interior solution,

$$\lambda - \frac{\partial f_i(\hat{q}_i;\hat{Q})}{\partial \hat{q}_i} = 0; \quad i \in N \tag{47}$$

Equation (47) implies

$$\lambda + 2P'(\hat{Q})\hat{q}_i^* = d_i c\tau(\hat{q}_i^*)^{\tau-1} > 0; \quad i \in N$$

It follows from this equation that $\hat{q}_i^* > \hat{q}_j^*$ if and only if $d_i < d_j$: Thus we have established the following result:

Proposition A.1: The monopolist will adopt an input pricing scheme that ensures that low-cost firms (i.e., those with low d_i) produce more than high cost firms. Furthermore, marginal production costs, $d_i\tau(\hat{q}_i^*)^{\tau-1}$ are not equalized across firms.

The results that marginal production costs are not equalized across firms is due to the fact that the monopolist is constrained to use linear pricing for each downstream firm, leaving them with positive profits.[16]

Equation (47) can be inverted[17] to give

$$\hat{q}_i^* = \phi_i(\lambda; \hat{Q}) \tag{48}$$

and the optimal value of λ, denoted by $\lambda^*(\hat{Q})$, can thus be obtained from the condition

$$\sum_{i \in N} \hat{q}_i^* = \sum_{i \in N} \phi_i(\lambda; \hat{Q}) = \hat{Q} \tag{49}$$

(See Appendix A2 for two examples that illustrate this procedure). The optimal firm-specific input prices are

$$t_i^* = \frac{P'(\hat{Q})\hat{q}_i^* + P(\hat{Q})}{d_i(\hat{q}_i^*)^{\tau-1}} \tag{50}$$

which, together with (47), yields the formula for firm S's mark-up

$$t_i^* - c = \frac{(\tau - 2)P'(\hat{Q})\hat{q}_i^* + (\tau P(\hat{Q}) - \lambda)}{\tau d_i(\hat{q}_i^*)^{\tau-1}} \tag{51}$$

The right-hand side of (51) is increasing in $\hat{q}_i^*$ for τ in the interval [1,2], and decreasing in d_i. This fact, together with Proposition A.1 (which says that $\hat{q}_i^*$ is decreasing in d_i) yields the following result:

Proposition A.2: For τ in the interval [1,2], the monopolist will practice "favoritism against the strongs", i.e., firms that are more efficient (those with a smaller d_i) must pay a higher price per unit of input supplied by the monopolist.

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[16] It is easy to verify that if the monopolist could use two-part pricing then T_i would be set so that $\pi_i = 0$, in which case downtream marginal costs would be equalized.

[17] Because $\partial \hat{q}_i / \partial \lambda > 0$.

Another sufficient condition for "discount reversal" is $2P(\hat{Q}) - \lambda^*(\hat{Q}) > 0$ (given that $\tau \geq 1$). To see this, re-write (50) as

$$t_i^* = \frac{\tau \{P'(\hat{Q})\hat{q}_i^* + P(\hat{Q})\}}{2P'(\hat{Q})\hat{q}_i^* + \lambda^*(\hat{Q})}$$

It follows from this equation that, for given $\hat{Q}$, t_i^* is increasing in $\hat{q}_i^*$ if $2P(\hat{Q}) - \lambda^*(\hat{Q}) > 0$.

Proposition A.3: For $\tau \geq 1$, the monopolist will practice "favoritism against the strongs" if $2P(\hat{Q}) - \lambda^*(\hat{Q}) > 0$.

Remark: Proposition A.3 requires the knowledge of $\lambda^*(\hat{Q})$. The examples in the Appendix show how $\lambda^*(\hat{Q})$ can be computed. Alternatively, as Proposition A.4 below indicates, we can find sufficient conditions for $2P(\hat{Q}) - \lambda^*(\hat{Q}) > 0$ in terms of the curvature of the demand curve and the index of concentration of the downstream industry.

It remains to determine the monopolist's optimal $\hat{Q}$. This is done in the second step below.

The second step:

We now try to express the monopolist's profit as a function of $\hat{Q}$, having known how, for a given $\hat{Q}$, the $\hat{q}_i^*$ (and hence t_i^*) are optimally chosen. Following the duality approach used in Rockafellar (1970, Chapter 12), we define the "conjugate function" f^* of the original function $f_i(\hat{q}_i;\hat{Q})$ as follows:

$$f_i^*(\lambda;\hat{Q}) = \sup_{\hat{q}_i} \left[\lambda\hat{q}_i - f_i(\hat{q}_i;\hat{Q}) \right] ; \hat{q}_i \geq 0;$$

where $\hat{Q}$ is given. Then, the profit function of the monopolist, given the maximization performed in Step 1 above, is

$$\Pi_S^*(\hat{Q}) = L^*(\hat{Q}) = \frac{1}{\tau}\left[(P(\hat{Q}) - \lambda^*(\hat{Q}))\hat{Q} + \sum_{i \in N} f_i^*(\lambda^*(\hat{Q});\hat{Q}) \right]$$

Assuming an interior solution, the optimal $\hat{Q}$ must satisfy the first order condition:

$$\tau \frac{d\Pi_S^*(\hat{Q})}{d\hat{Q}} = (P'(\hat{Q}) - \lambda^{*\prime}(\hat{Q}))\hat{Q} + (P(\hat{Q}) - \lambda^*(\hat{Q}))$$

$$+ \sum_{i \in N} \frac{\partial f_i^*}{\partial \lambda}\frac{d\lambda^*}{d\hat{Q}} + \sum_{i \in N} \frac{\partial f_i^*}{\partial \hat{Q}} = 0 \qquad (52)$$

Using the envelope theorem, we have $\partial f_i^* / \partial \lambda = \hat{q}_i^*$, and (52) becomes

$$\tau \frac{d\Pi_S^*(\hat{Q})}{d\hat{Q}} = P(\hat{Q}) - \lambda^*(\hat{Q}) + P'(\hat{Q})\hat{Q}[1 + EH] = 0 \qquad (53)$$

where H is the Herfindahl index of concentration $(1 \geq H \geq (1/n)^2)$ defined as

$$H = \sum_{i \in N} \frac{\hat{q}_i^{*2}}{\hat{Q}^2}$$

and E is the elasticity of the slope of the demand curve at $\hat{Q}$: $E = P''(\hat{Q})\hat{Q}/[-P'(\hat{Q})]$:

Remark: By definition, the Herfindahl index of concentration is at its maximum value ($H = 1$) if the industry output, Q, is produced by one firm, and H is at its minimum ($H = 1/n^2$) if all the n firms have identical market shares.

If $\Pi_S^*(\hat{Q})$ is strictly concave[18] in $\hat{Q}$ and the maximum is an interior one, then equation (53) uniquely determines the optimal $\hat{Q}^*$. At that optimum point,

$$2P(\hat{Q}^*) - \lambda^*(\hat{Q}^*) = P(\hat{Q}^*) - P'(\hat{Q}^*)\hat{Q}^*[1 + E^* H^*] \qquad (54)$$

The right-hand side of this equation is positive if $E^* \geq 0$ (this inequality holds if the demand curve is linear or convex), or if $E^* < 0$ but $E^* H^* \geq -1$. Using this result and Proposition A3, we obtain:

[18] A set of sufficient conditions for this to hold is $\tau = 1$ and $P(Q)$ is linear.

Proposition A.4: For ""favoritism against the strongs"" to occur, it is sufficient that the demand curve is linear or convex (implying $E^* \geq 0$), or that it is not too concave, i.e., $E^*H^* \geq -1$:

The optimal input price that the monopolist charges firm i is obtained from (50), (47), and (54):

$$t_i^* = \frac{c\gamma[P'(\hat{Q}^*)\hat{q}_i^* + P(\hat{Q}^*)]}{2P'(\hat{Q}^*)\hat{q}_i^* + P(\hat{Q}^*) + P'(\hat{Q}^*)\hat{Q}^*[1 + E^*H^*]}$$

where the denominator is positive because it must be the same as the denominator of the right-hand side of (50), the left-hand side being t_i^* in both equations. This input price is dependent on the concentration index of the downstream industry, and on the elasticity of the slope of the demand curve.

Appendix A.2 :Some special cases

We now provide examples illustrating the procedure described by (48) and (49).

Example 1: with $\gamma = 1$ (linear downstream costs), (48) gives

$$\hat{q}_i^* = \phi_i(\theta; \hat{Q}) = \frac{\theta - vb_i}{2[-P'(\hat{Q})]}$$

and (49) gives

$$\theta^*(\hat{Q}) = \frac{2\hat{Q}[-P'(\hat{Q})]}{n} + \frac{v}{n}\sum_{i \in N} b_i > 0$$

Therefore

$$\hat{q}_i^*(\hat{Q}) = \frac{\hat{Q}}{n} - \frac{v}{2[-P'(\hat{Q})]}[b_i - b_N]$$

where $b_N \equiv (1/n)\sum_{i \in N} b_i$. Thus,

$$\text{sign}[\hat{q}_i^*(\hat{Q}) - \hat{q}_j^*(\hat{Q})] = -\text{sign}[b_i - b_j]$$

that is, the firm with lower cost will produce more, confirming Proposition A.1.

Example 2: with $\tau = 2$ (quadratic downstream costs), (48) gives

$$\hat{q}_i^* = \phi_i(\lambda; \hat{Q}) = \frac{\lambda}{2\{[-P'(\hat{Q})] + v b_i\}} \equiv \lambda \psi_i(\hat{Q})$$

and (49) gives

$$\lambda^*(\hat{Q}) = \frac{\hat{Q}}{\sum_{i \in N} \psi_i(\hat{Q})}$$

Therefore

$$\hat{q}_i^*(\hat{Q}) = \frac{\psi_i(\hat{Q})\hat{Q}}{\sum_{i \in N} \psi_i(\hat{Q})}$$

Here, also, we obtain

$$\text{sign}\{\hat{q}_i^*(\hat{Q}) - \hat{q}_j^*(\hat{Q})\} = \text{sign}\{\psi_i(\hat{Q}) - \psi_j(\hat{Q})\} = -\text{sign}[b_i - b_j]$$

APPENDIX A.3: The determination of λ^* in Section 3.

Recall that $\sum_{h \in N} d_h t_h = n\theta_N$. Substituting (24) and (25) into the equation

$$\theta_N = \frac{1}{n} \sum_{h \in N} t_h d_h$$

we obtain

$$\lambda^*(\theta_N) = \frac{A + B}{D} \tag{55}$$

where

$$A = n\theta_N - \sum_{i \in I} d_i c_S - \frac{1}{2} \sum_{j \in J} d_j c_S$$

$$B = \frac{1}{[-P'(\hat{Q})]} \sum_{i \in I} [\hat{P} - d_i c_S] d_i^2 u_i - \frac{\hat{P}}{2} \sum_{j \in J} \gamma_j$$

$$D = \frac{[-P'(\hat{Q})]}{2} + \sum_{i \in I} d_i^2 u_i$$

with

$$\gamma_j = \frac{d_j^2 u_j}{d_j^2 u_j + [-P'(\hat{Q})]}$$

APPENDIX A.4: Proof of (26)

Let $d_h = 1$ for all $h \in N$. Then $\lambda^*(\theta_N)$ in (55) becomes

$$\lambda^*(\theta_N) = \frac{A' + B'}{D'}$$

where

$$A' = n\theta_N - (c_S/2)(n + n_I)$$

$$B' = \frac{[\hat{P} - c_S]}{[-P'(\hat{Q})]} \sum_{i \in I} u_i - \frac{\hat{P}}{2} \sum_{j \in J} \gamma_j$$

$$D' = \sum_{i \in I} u_i + \frac{[-P'(\hat{Q})]}{2}$$

where

$$\gamma_j = \frac{u_j}{u_j + [-P'(\hat{Q})]}$$

When $n_I = 0$; we have $\lambda^*(\theta_N) = \lambda_0^*(\theta_N)$ where

$$\lambda_0^*(\theta_N) = \frac{E}{F}$$

with

$$E = n\theta_N - (nc_S/2) - \frac{\hat{P}}{2}\sum_{j\in N}\gamma_j$$

and

$$F = \frac{[-P'(\hat{Q})]}{2}$$

When $n_I = 1$; we have $\lambda^*(\theta_N) = \lambda_1^*(\theta_N)$ where

$$\lambda_1^*(\theta_N) = \frac{E+G}{F+u_1}$$

where

$$G = u_1\left[\frac{[\hat{P} - c_S]}{[-P'(\hat{Q})]} + \frac{\hat{P}}{2\{u_1 + [-P'(\hat{Q})]\}}\right] - \frac{c_S}{2}$$

If u_1 is very small, then $\lambda_1^*(\theta_N) < \lambda_0^*(\theta_N)$. Thus, at an unchanged θ_N, the integration of an upstream firm with a downstream firm reduces the t_j's, $j \in J$. (However, θ_N would not be unchanged when the integration occurs.)

APPENDIX A.5: The concavity of Π_{IS}

$$\frac{\partial \Pi_{IS}}{\partial t_i} = -\left[\frac{d_i(\hat{P} - d_i c_S)}{[-\hat{P}']}\right] - (t_i - c_S)\hat{x}_i'$$

$$\frac{\partial \Pi_{IS}}{\partial t_j} = \hat{y}_j + (t_j - c_S)\frac{\partial \hat{y}_j}{\partial t_j}$$

$$\frac{\partial^2 \Pi_{IS}}{\partial t_i^2} = -\hat{x}_i' - (t_i - c_S)\hat{x}_i''$$

$$\frac{\partial^2 \Pi_{IS}}{\partial t_j^2} = 2\frac{\partial \hat{y}_j}{\partial t_j} + (t_j - c_S)\frac{\partial^2 \hat{y}_j}{\partial t_j^2}$$

where, from $t_h = U_i'(\hat{x}_i)$;

$$\hat{x}_i' = \frac{1}{U_i''(\hat{x}_i)} > 0$$

and

$$\hat{x}_i'' = -\frac{U_i'''(\hat{x}_i)}{[U_i''(\hat{x}_i)]^3} > 0$$

if $U_i'''(\hat{x}_i) < 0$.

We also have

$$\frac{\partial \hat{y}_j}{\partial t_j} = -\frac{d_j^2}{[-\hat{P}']} - \hat{x}_j' < 0$$

and

$$\frac{\partial^2 \hat{y}_j}{\partial t_j^2} = -\hat{x}_j''$$

It follows that

$$\frac{\partial^2 \Pi_{IS}}{\partial t_j^2} = -2\left[\frac{d_j^2}{[-\hat{P}']} + \hat{x}_j'\right] - (t_j - u_j)\hat{x}_j''$$

Let us specify

$$U_h(x_h) = \left[\frac{u_h}{\sigma}\right] x_h^{\sigma} \quad ; \sigma \geq 1$$

then $U_h''' \leq 0$ if and only if $2 \geq \sigma \geq 1$. In this case, $\hat{x}_j'' > 0$ and $\partial^2 \Pi_{IS}/\partial t_j^2 < 0$ if $t_j \geq c_S$.

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