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Voluntary Contributions to a Public Good: Non-neutrality Results

Ngo Van Long [*], *Koji Shimomura* [†]

Résumé / *Abstract*

Nous démontrons que le théorème sur l'invariance du stock total d'un bien public par rapport à la distribution de revenus n'est valable que si les contributeurs ignorent l'impact de leurs contributions sur le prix relatif des biens privés. Par conséquent, le résultat de Warr, Kemp, Bergstrom, Blume et Varian n'a qu'une sphère d'application limitée. Nos résultats sur le manque de neutralité sont valables même si les préférences, les technologies, et les dotations de ressources de tous les pays sont identiques.

Mots clés : contributions volontaires, biens publics

We show that the famous neutrality result in the theory of public good contributions (Warr, Kemp, Bergstrom, Blume and Varian) depends crucially on the assumption that agents do not take into account the effect of their public good contribution decisions on the relative price of the private goods. Thus, the scope of applicability of their result is not as large as one might at first think. Our non-neutrality results hold even if all countries are identical in technology, preferences, and endowments.

Keywords: *public goods, voluntary contributions*

Codes JEL : C73, H41, D60

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1 Introduction

According to a well known proposition in the literature on voluntary contributions to a public goods[1], small transfers of income (or wealth) among active contributors to a public good will have no effects on the total amount of the public good, and no effects on the welfare level of each participant. We shall refer to this result as "the WKBBV neutrality theorem"[2].The intuition behind this neutrality result is as follows. Given the initial income distribution, the Nash equilibrium contribution level of each participant is determined. If initially there are n active participants, each contributing in the initial Nash equilibrium a strictly positive fraction, but not the whole, of his income to the public good, then any mean-preserving change in income distribution among these participants will simply increase (or decrease) each person's contribution to the public good by an amount which is just equal to his increase (or decrease) in income. Each individual's consumption of private goods will remain unchanged, and so will his total consumption of the public good. It is easy to verify that this allocation satisfies the necessary conditions of the (new) Nash equilibrium (with changes in individual contributions, but no change in total contribution and in individual welfare).

The articles in the above literature share a common assumption: all the players of the public-good contribution game are price-takers. That assumption restricts the scope of application of the theory. In many real-world situations, some (or all) contributors to a public good are large enough to influence prices. Large countries such as Japan, Germany, the USA etc., contribute to many international public good projects. Surely these large players are not price-takers. In this paper we show that if countries take into account the effect of its public good production on the relative prices of private goods, then the standard WKBBV neutrality theorem ceases to hold in our general

[1] See the articles by Warr (1983), Kemp (1984), Bergstrom, Blume and Varian (1986), and the textbook exposition by Cornes and Sandler (1986).

[2] WKBBV is the acronym for Warr, Kemp, Bergstrom, Blume, and Varian.

setting, unless some very restrictive assumptions are added. We also state some additional assumptions that would restore the validity of the WKBBV neutrality theorem.

In this paper we intentionally restrict the scope of our analysis in order to focus on the non-neutrality issue, in particular, on the role of public good supplies on relative prices of private goods. Because of this, we will refrain from reporting in detail other important -but not closely related- contributions to the theory of voluntary contribution to a public good. While our paper deals with comparative statics in a general equilibrium model, we would like to draw attention to papers on dynamic aspects of contribution to public goods. This literature typically assumes that there is a growing stock of public good that enters the utility function. Fershtman and Nitzan (1991) modelled public good as a stock that grows with addtional contributions. They assumed agents condition their additional contributions on the current level of the stock of the public good. They showed that the free riding problem is worse, compared with the case where agents are able to commit to a time path of contributions. Wirl (1996) showed that if non-linear strategies are admitted, then the outcome can be better than that predicted by Fershtman and Nitzan. Itaya and Shimomura (2001) obtained results similar to that of Wirl, but in a more general setting. Marx and Matthews (2000) introduced imperfect information and focussed on Baysesian equilibria. In all these models, the public good is a stock that grows over time. Benchekroun and Long (2005) modelled the public good as a flow, so that the only stock is the intangible stock of cooperation (which does not exist in the models mentioned above).

2 The Model

2.1 Notations and assumptions

In our model, the players (the contributors to the public good) are not individuals, but are governments who care about the welfare of their country.

There are n countries. Each country is inhabited by a continuum of identical individuals, uniformly distributed over the unit interval $[0, 1]$. Each individual in country i possesses K_i units of capital and L_i units of labor (measured in efficiency units). The population of each country is normalised to unity. All individuals have identical preferences over three goods: two private goods, called good A and good B and the third good which is a pure public good, whose aggregate supply is denoted by G.(For example, G may represent the world's total scientific research output on global warming). Let g_i denote country i's production of the public good, and $G = \sum_i g_i$.

Let x_i^j denote the private consumption of good j ($j = A, B$) by the representative individual in country i. The preference ordering of individual i is represented by the *"felicity"* function[3] ϕ :

$$\mathcal{U}_i = \phi^i(u(x_i^A, x_i^B), G)$$

where ϕ is increasing in the two arguments (u, G) and $u(x_i^A, x_i^B)$ is the *"utility"* derived from consuming the two private goods. We shall refer to $u_i = u(x_i^A, x_i^B)$ as individual i's *utility level*, and $\mathcal{U}_i$ as his *felicity level.* We make the following assumptions:

Assumption 1: $u(.)$ is homogeneous of degree one, twice differentiable, strictly quasi-concave, and strictly increasing in the consumption levels of the two private goods. The felicity function ϕ^i is twice differentiable and increasing in u and G.

Remark 1: The assumption that "$u(.)$ is homogeneous of degree one" could be replaced by " $u(.)$ is homothetic" and our results remain essentially unchanged. We assume homogeneity of degree 1 for $u(.)$ only to lighten notation.

Assumption 2: All three goods are produced under constant returns to scale, using capital and labor as inputs. Production functions of the private goods are twice differentiable, strictly quasi-concave, and strictly increasing in

[3] We borrow the word "felicity" from Arrow and Kurz (1970, pages xviii and xx).

both inputs. Good B is more capital intensive than good A. All countries have identical technologies, and both private goods are produced in each country.

Remark 2: We do not specify at this stage whether the public good production function exhibits the neoclassical technology (i.e., diminishing marginal rate of technical substitution between capital and labor), or exhibits the Leontief technology (i.e., fixed coefficients).

Goods A and B are internationally traded goods, and the public good is non-traded. Let A be the numeraire good. Let p, w, r denote the price of good B, the wage rate, and the rental rate in terms of the numeraire good. Let $C^j(w,r)$ be the unit cost function of good j ($j = A, B$). Individuals and firms take prices as given, and perfect competition prevails. We have (under the assumption that) the following price-equals-cost equations

$$1 = C^A(w,r) \tag{1}$$

$$p = C^B(w,r) \tag{2}$$

It well known that the unit cost functions are concave and homogeneous of degree one in (w,r). We make the following assumptions on the unit cost functions of the private goods:

Assumption 3: For any $p > 0$, the pair of equations (1) and (2) yields a unique pair $(w(p), r(p)) > (0,0)$.

It is well known that an increase in p (the relative price of the capital intensive good) will raise the real rental rate and depress the real wage rate, in terms of good A (also in terms of good B). Thus

$$r'(p) > 0 \text{ and } w'(p) < 0.$$

The cost of production of a unit of the public good is $C^g(w,r)$. We consider both the neoclassical case, and the Leontief case. In the Leontief case,

$$C^g(w,r) = wa_L + ra_K$$

where a_L and a_K are non-negative constants. In the neoclassical case, $C^g(.)$ displays continuous cross partial derivatives. Since $w = w(p)$ and $r = r(p)$, it is convenient to define (in both cases):

$$c(p) = C^g(w(p), r(p))$$

Concerning the supply of the public good, it is convenient, though not at all necessary, to suppose that it is done in the following manner. The government of country i procures g_i units of the public good from domestic firms, at the price p^G which is equal to unit cost. Its total expenditure on the public good is $p^G g_i \equiv T_i$. To finance this expenditure, the government imposes a lump sum tax T_i on the representative individual. (Recall that the population of each country is normalised to unity.)

The (factor) income of the representative individual in country i is $rK_i + wL_i$ Since he is required to pay the lump sum tax T_i, his disposable income is

$$y_i^D = rK_i + wL_i - c(p)g_i \tag{3}$$

The consumer uses his disposable income to buy the two private goods so as to maximize $u(x_i^A, x_i^B)$ subject to the budget constraint $x_i^A + px_i^B = y_i^D$.This yields his indirect utility function $v_i = v(p, y_i^D)$. Because of the assumed linear homogeneity of $u(.)$, the indirect utility function and the expenditure function take the forms[4]

$$v(p, y_i^D) = \frac{y_i^D}{e(p)}$$
$$E(p, u_i) = e(p)u_i$$

where $e(p) > 0$ is the cost (in terms of good A) of achieving one unit of utility. It is well known that the function $e(p)$ is concave and increasing in the relative price p of good B. Roy's identity yields the Marshallian demand function for good B :

$$x_i^B = -\frac{\frac{\partial v}{\partial p}}{\frac{\partial v}{\partial y_i}} = \left[\frac{e'(p)}{e(p)}\right] y_i^D \tag{4}$$

[4] See, for example, Varian (1992, p. 147).

Remark: The elasticity of $e(p)$, denoted by η, is the budget share of good B

$$\eta \equiv \frac{pe'(p)}{e(p)} = \frac{px_i^B}{y_i^D} \tag{5}$$

2.2 The two-stage game

We consider the following two-stage game. In the first stage, each government i announces (non-cooperatively, and independently of other governments) the amount of public good g_i that it will contribute to the world total provision of the public good. It informs its citizens that they must pay a lump-sum tax T_i to finance the procurement of g_i.

In stage two, given the announced vector $\mathbf{g} \equiv (g_1, g_{2,} ..., g_n)$, the competitive market solves the problem of allocation of factors of production in each country, and individuals decide how to allocate their disposable incomes between the two private goods.

We shall solve this game backward. That is, for any given vector $\mathbf{g}$ announced in stage one, we must solve for the resulting equilibrium market price p (and the associated equilibrium value of r, w, and utility levels that result from the consumption of the two private goods). Everyone knows that the outcome of stage 2 game is a function of the announced vector $\mathbf{g}$. In stage 1, each government i announces the quantity g_i. The objective of each government is to maximize the welfare level (i.e., fecility) of its representative citizen.

3 Analysis of stage-2 equilibrium allocation and prices

We have explained the consumer's problem in section 2. We now turn to the production side. Given p, the factor prices w and r are determined[5] by the

[5]Throughout the paper, we maintain the assumption that each country produce both private goods.

equations (1) and (2). Using Shephard's lemma, the amounts of capital and labor required to produce one unit of the public good are, respectively

$$\frac{\partial C^g}{\partial r} \text{ and } \frac{\partial C^g}{\partial w}$$

Thus, given g_i, the remaning amounts of capital and labor in country i that can be used for the production of private goods are

$$\mathcal{K}_i = K_i - g_i\frac{\partial C^g}{\partial r} \text{ and } \mathcal{L}_i = L_i - g_i\frac{\partial C^g}{\partial w}$$

Let $F^i(.)$ denote the production function for good i ($i = A, B, g$). Following standard analysis of competitive allocation using duality theory, we define the private-goods revenue function for country i as the value function of the following maximization problem. For any given g_i and given p, the competitive firms behave as if they collectively choose $(K_i^A, K_i^B, L_i^A, L_i^B)$ to maximize the country's total value of private-good outputs,

$$R_i = \max F^A(K_i^A, L_i^A) + pF^B(K_i^B, L_i^B)$$

subject to

$$L_i^A + L_i^B + L_i^g \leq L_i$$

$$K_i^A + K_i^B + K_i^g \leq K_i$$

$$F^g(K_i^g, L_i^g) = g^i$$

The solution of this problem yields the private-good revenue function

$$R_i = R(p, K_i, L_i, g_i)$$

As a consequence of the envelope theorem, the partial derivative of the revenue function with respect to p is the output of good B, denoted by q_i^B :

$$\frac{\partial R(p, K_i, L_i, g_i)}{\partial p} = F^B(K_i^B, L_i^B) \equiv q_i^B \tag{6}$$

On the other hand, the revenue from private goods production can also be expressed as the sum of payments to factors of productions used in private good production:

$$R_i = r(p)\left[K_i - g_i\frac{\partial C^g}{\partial r}\right] + w(p)\left[L_i - g_i\frac{\partial C^g}{\partial w}\right]$$

$$= r(p)K_i + w(p)L_i - g_i\left[r\frac{\partial C^g}{\partial r} + w\frac{\partial C^g}{\partial w}\right]$$

$$= r(p)K_i + w(p)L_i - g_iC^g(w,r)$$

Thus

$$R_i = R(p,K_i,L_i,g_i) = r(p)K_i + w(p)L_i - g_ic(p)$$

and hence, recalling (6), the country's output of good B is

$$q_i^B = \frac{\partial R(p,K_i,L_i,g_i)}{\partial p} = r'(p)K_i + w'(p)L_i - g_ic'(p) \tag{7}$$

It is a well known result that the revenue function is convex in the price p. Thus

$$\frac{\partial^2 R(p,K_i,L_i,g_i)}{\partial p^2} = r''(p)K + w''(p)L - g_ic''(p) \geq 0 \tag{8}$$

This implies that, given g_i, the output q_i^B is a non-decreasing function of its price.

From (3), (4) and (7), country i's excess demand for good B is

$$M_i^B \equiv x_i^B - q_i^B$$

$$= \left[\frac{e'(p)}{e(p)}\right][rK_i + wL_i - c(p)g_i] - [r'(p)K_i + w'(p)L_i - g_ic'(p)] \tag{9}$$

Lemma 1: The world uncompensated excess demand for good B is a a linear function of K, L and G for given p :

$$M^B(p,K,L,G) = \frac{e'(p)}{e(p)}[r(p)K + w(p)L - c(p)G] - [r'(p)K + w'(p)L - c'(p)G]$$

where

$$K = \sum_{i=i}^{n} K_i, \qquad L = \sum_{i=i}^{n} L_i, \qquad G = \sum_{i=i}^{n} g_i$$

Proof: Sum the equation (9) over all i.■

Thus, the free trade equilibrium price p, denoted by $\widehat{p}$, must satisfy the condition

$$e'(\widehat{p})\left[r(\widehat{p})K + w(\widehat{p})L - c(\widehat{p})G\right] - e(\widehat{p})\left[r'(\widehat{p})K + w'(\widehat{p})L - c'(\widehat{p})G\right] = 0 \quad (10)$$

It follows that $\widehat{p}$ is a function of (G, K, L) and is independent of their distributions.

Remark: The slope of the world excess demand curve for good B, evaluated at $\widehat{p}$, is negative. To see this, note that

$$\begin{aligned}\frac{\partial M^B}{\partial p} = \frac{1}{e(\widehat{p})}\left\{e''(\widehat{p})\left[r(\widehat{p})K + w(\widehat{p})L - c(\widehat{p})G\right] + e'(\widehat{p})\left[r'(\widehat{p})K + w'(\widehat{p})L - c'(\widehat{p})G\right]\right\} \\ - \left[\frac{e'(\widehat{p})}{e(\widehat{p})}\right]^2\left[r(\widehat{p})K + w(\widehat{p})L - c(\widehat{p})G\right] \\ - \left[r''(\widehat{p})K + w''(\widehat{p})L - c''(\widehat{p})G\right]\end{aligned}$$

By making use of (10), the above expression reduces to

$$\frac{\partial M^B}{\partial p} = \frac{1}{e(\widehat{p})}\left\{e''(\widehat{p})\left[r(\widehat{p})K + w(\widehat{p})L - c(\widehat{p})G\right] - e(\widehat{p})\left[r''(\widehat{p})K + w''(\widehat{p})L - c''(\widehat{p})G\right]\right\} \quad (11)$$

The right-hand side of (11) is negative because (i) the expenditure function is concave in price, and (ii) equation (8) holds. Note that $\partial M^B/\partial p < 0$ is the Walrasian stability condition.

Lemma 2: Given the assumption that all countries produce both goods, the equilibrium world price of good B is a function of the sum of the K_i's, the sum of the L_i's, and the sum of the g_i's.

$$\widehat{p} = \widehat{p}(K, L, G)$$

In particular, (i) given G and L, any (small) redistribution of the K_i's among countries (keeping the total K constant) will have no effects on the equilibrium relative price $\widehat{p}$, (ii) an increase in G will increase (decrease) the equilibrium relative price $\widehat{p}$ if and only if the budget share of good B in the private goods expenditure exceeds (falls short of) the elasticity of the unit cost of the public good with respect to the price of good B.

Proof: Equation (10) defines an implicit function

$$\widehat{p} = \widehat{p}(K, L, G) \tag{12}$$

Let

$$\Delta \equiv e''(\widehat{p})\left[r(\widehat{p})K + w(\widehat{p})L - c(\widehat{p})G\right] - e(\widehat{p})\left[r''(\widehat{p})K + w''(\widehat{p})L - c''(\widehat{p})G\right] < 0$$

Differentiating (10) totally, we get

$$\begin{aligned}\Delta d\widehat{p} + \left[e'(\widehat{p})r(\widehat{p}) - e(\widehat{p})r'(\widehat{p})\right] dK + \left[e'(\widehat{p})w(\widehat{p}) - e(\widehat{p})w'(\widehat{p})\right] dL \\ - \left[e'(\widehat{p})c(\widehat{p}) - e(\widehat{p})c'(\widehat{p})\right] dG = 0\end{aligned} \tag{13}$$

Therefore, using (13) and (5), we get

$$\frac{\partial \widehat{p}}{\partial G} = \frac{\left[e'(\widehat{p})c(\widehat{p}) - e(\widehat{p})c'(\widehat{p})\right]}{\Delta} = \frac{c(\widehat{p})}{\widehat{p}e(\widehat{p})\Delta}\left[\eta - \frac{\widehat{p}c'(\widehat{p})}{c(\widehat{p})}\right] \tag{14}$$

■

Thus each country knows that if it increases its contribution to the global public good, the price of good B will fall if and only if the budget share of good B in the private goods expenditure exceeds the elasticity of the unit cost of the public good with respect to the price of good B.

Lemma 3: Let β_i denote the following vector of parameters

$$\beta_i \equiv (K, L, G, K_i, L_i, g_i)$$

Then, at the free trade equilibrium, the disposable income of country i's representative individual is

$$y_i^D(\beta_i) = r(\widehat{p}(K, L, G))K_i + w(\widehat{p}(K, L, G))L_i - c(\widehat{p}(K, L, G))g_i \tag{15}$$

and his indirect utility is

$$u_i = v(\widehat{p}(K, L, G), y_i^D(\beta_i)). \tag{16}$$

Lemma 4: If country i's is a net importer of good B, an increase in $\widehat{p}$ will reduce its utility by $M_i^B / e(\widehat{p}(K, L, G))$.

Proof:

Differentiating the equilibrium indirect utility $u_i = v(\widehat{p}(K, L, G), y_i^D(\beta_i))$ with respect to $\widehat{p}$ yields

$$\frac{du_i}{d\widehat{p}} = v_p + v_y \frac{\partial y_i^D}{\partial \widehat{p}}$$

or

$$\frac{1}{v_y} \frac{du_i}{d\widehat{p}} = \left[\frac{\partial y_i^D}{\partial \widehat{p}} - \left(\frac{-v_p}{v_y} \right) \right] = q_i^B - x_i^B \equiv -M_i^B$$

where M_i^B denotes country i's import of good B. For later reference, we note that

$$v_p = \frac{-e'(\widehat{p}) y_i^D}{e(\widehat{p}(K, L, G))^2} \text{ and } v_y = \frac{1}{e(\widehat{p}(K, L, G))}$$

and

$$\frac{\partial y_i^D}{\partial \widehat{p}} = K_i r' + L_i w' - g_i c'$$

Thus

$$\frac{du_i}{d\widehat{p}} = -\frac{1}{e(\widehat{p})} \left\{ \frac{e'}{e} \left[rK_i + wL_i - cg_i \right] - \left[K_i r' + L_i w' - g_i c' \right] \right\} \equiv -\frac{M_i^B}{e(\widehat{p})} \tag{17}$$

where the term inside the curly brackets is country i's import of good B, being the difference between consumption and production.■

Remark: Keeping $\widehat{p}$ constant, if a country increases g_i, its utility derived from private goods will change by

$$\frac{\partial u_i}{\partial g_i} = v_y \frac{\partial y_i^D}{\partial g_i} = -\frac{c(\widehat{p})}{e(\widehat{p})} < 0 \tag{18}$$

4 Analysis of Stage-1 Game

Now consider stage 1. The government of country i knows the indirect utility function $u_i = v(\widehat{p}(K, L, G), y_i^D(\beta_i))$. Its objective is to choose g_i to maximize

$$\phi^i(v(\widehat{p}(K, L, G), y_i^D(\beta_i)), G)$$

where $G = G_{-i} + g_i$. The first-order condition for an interior maximum is

$$\frac{\partial \phi^i}{\partial u_i}\frac{du_i}{dg_i} + \frac{\partial \phi^i}{\partial G} = 0 \tag{19}$$

This first-order condition can also be expressed as

$$\phi_G^i = -\phi_v^i \left[\left(v_p + v_y \frac{\partial y_i^D}{\partial \widehat{p}} \right) \frac{\partial \widehat{p}}{\partial G} \right] - \phi_v^i \left[v_y \frac{\partial y_i^D}{\partial g_i} \right] \tag{20}$$

This condition shows that, in choosing the best reply g_i to a given G_{-i}, country i should equate the marginal gain from having a larger total supply of the public good, ϕ_G^i, with the marginal loss, $-\phi_v^i \frac{du_i}{dg_i}$, from a fall in utility derived from private goods. The magnitude of $\phi_v^i \frac{du_i}{dg_i}$ depends on two terms:

(i) The first term represents the terms-of-trade effect, i.e., the first expression on the right-hand side of (20), which is in general non-zero, unless *either* $v_p + v_y \frac{\partial y_i^D}{\partial \widehat{p}} = 0$ (i.e., the country's net import of good B is zero), *or* $\frac{\partial \widehat{p}}{\partial G} = 0$ (i.e., the razor's edge case where the right-hand side of (14) just happens to be zero.)

(ii) The second term is the direct income loss, i.e., the second expression on the right-hand side of (20), which is always positive.

The right-hand side of (20) may also be written as

$$-\phi_v^i \frac{du_i}{dg_i} = -\phi_v^i \left[\frac{du_i}{d\widehat{p}} \frac{\partial \widehat{p}}{\partial G} + \frac{\partial u_i}{\partial g_i} \right] = (\phi_v^i) \left[\frac{M_i^B}{e(\widehat{p})} \left(\frac{c(\widehat{p})\Omega}{\widehat{p}e(\widehat{p})\Delta} \right) + \frac{c(\widehat{p})}{e(\widehat{p})} \right] \tag{21}$$

where

$$\Omega \equiv \left[\eta - \frac{\widehat{p}c'(\widehat{p})}{c(\widehat{p})} \right]$$

$$M_i^B \equiv \frac{e'}{e}\left[rK_i + wL_i - cg_i\right] - \left[K_i r' + L_i w' - g_i c'\right] \tag{22}$$

We assume that given G_{-i}, the first-order condition (19) yields a unique g_i and that the second-order condition is satisfied.

Then the Nash equilibrium vector of contributions $(g_1, ..., g_n)$ is the solution of the system of n first-order conditions (19) for $i = 1, ..., n$.

It will be convenient to write the first-order condition as

$$\frac{\phi_G^i}{\phi_v^i} = \frac{c(\widehat{p})}{e(\widehat{p})}\left\{1 + \frac{M_i^B}{e(\widehat{p})}\left(\frac{\Omega}{\widehat{p}\Delta}\right)\right\} \tag{23}$$

where the left-hand side is the marginal rate of subsitution of G for u_i along a constant-fecility curve, $\phi^i(u_i, G) =$ constant. The right-hand side can be interpreted as the marginal rate of transformation (taking into account the response of equilibrium terms of trade to changes in public good contributions).

5 Non-neutrality result with respect to redistribution

Consider a redistribution of capital among the n countries. Let dK_i be a small change in country i's capital stock. Assume $\sum_i dK_i = 0$. Under what conditions will there be no change in the total provision G of public good nor in the private-goods utility levels u_i? That is, under what conditions can one carry over the neutrality result of Warr-Kemp-Bergstrom-Blume-Varian (WKBBV) to our world with endogenous terms of trade? If the WKBBV result holds in our world, G will be unchanged, and the country for which $dK_i > 0$ will supply more public good ($dg_i > 0$), but its private utility u_i will be unchanged. And, since both G and u_i are unchanged, its felicity level $\phi_i(u_i, G)$ will also be unchanged.

5.1 A General Result

Suppose the WKBBV neutrality result holds.Then u_i and G are unchanged, hence the value of the left-hand side of (23) is not affected by a redistribution of capital. It follows that the right-hand side of (23) must be unaffected. With G and K unchanged, $\widehat{p}$ will be unchanged, by Lemma 2. Hence the right-hand side of (23) is unaffected if and only if $\Omega M_i^B/\Delta$ is unchanged, which is in turn equivalent to the invariance of

$$\frac{\Omega e'}{e}\left[(K_i + dK_i)r + wL_i - (g_i + dg_i)c\right] - \Omega\left[(K_i + dK_i)r' + L_i w' - (g_i + dg_i)c'\right]$$

This invariance obtains if and only if

$$\Omega\Gamma = 0$$

where

$$\Gamma \equiv [\frac{e'}{e}r - r']dK_i - [\frac{e'}{e}c - c']dg_i \tag{24}$$

On the other hand since $u_i = v(p, y_i^D)$, if both u_i and p are unchanged, then y_i^D must be unchanged. Now, before the redistribution of capital, y_i^D is

$$y_i^D = rK_i + wL_i - c(p)g_i$$

and after redistribution y_i^D is

$$y_i^D = r(K_i + dK_i) + wL_i - c(p)(g_i + dg_i)$$

Thus y_i^D is unchanged if and only if

$$rdK_i - c(p)dg_i = 0$$

i.e.,

$$dg_i = \frac{r}{c}dK_i$$

Substituting this equality into (24), we have

$$\begin{aligned}\Gamma &\equiv [(\frac{e'}{e}r - r') - \frac{r}{c}(\frac{e'}{e}c - c')]dK_i \\ &= [r(\frac{e'}{e}\frac{r}{r} - \frac{r'}{r}) - r(\frac{e'}{e}\frac{c}{c} - \frac{c'}{c})]dK_i \\ &= r[\frac{c'}{c} - \frac{r'}{r}]dK_i\end{aligned}$$

Considering $c(p) \equiv C^g(w(p), r(p))$, and making use of the fact that $C^g(w,r)$ is homogenous of degree one, we can write

$$\frac{c'}{c} = \frac{w}{C^g}\frac{\partial C^g}{\partial w}\frac{w'}{w} + \frac{r}{C^g}\frac{\partial C^g}{\partial r}\frac{r'}{r} = \frac{w}{C^g}\frac{\partial C^g}{\partial w}\frac{w'}{w} + \left[1 - \frac{w}{C^g}\frac{\partial C^g}{\partial w}\right]\frac{r'}{r}$$

thus

$$\Gamma = \frac{rw}{C^g}\frac{\partial C^g}{\partial w}[\frac{w'}{w} - \frac{r'}{r}]dK_i \tag{25}$$

Now, since the sign of $w'(p)$ is always opposite to the sign of $r'(p)$ (that is, in the absence of technical progress, it is not possible for both factor prices to move in the same direction), we know that the right-hand side of (25) is non-zero, unless $\frac{\partial C^g}{\partial w} = 0$. It follows that is impossible to have $\Gamma = 0$, unless labor is not used in the production of the public good.

Proposition 1: Assume labor is an input used in strictly positive amount (possibly together with capital) in the production of the public good. Then the WKBBV neutrality result does not hold in our model, except in the extremely special case where the budget share of good B in the private goods expenditure just happens to be equal to the elasticity of the unit cost of the public good with respect to the price of good B, i.e., when

$$\left[\eta - \frac{\widehat{p}c'(\widehat{p})}{c(\widehat{p})}\right] = 0 \tag{26}$$

Remark: The intuition behind our non-neutrality result is as follows. When a country i decides on the amount of public good g_i it should contribute, it takes into account two factors. First, at constant price, an increase in g_i by dg_i will increase the total supply of public good G by $dG = dg_i$ (Nash behavior), and reduce its disposable income (for private good consumption) by $c(p)dg_i$. Second, the change in g_i , and hence in G (given the contributions of other countries) will have a terms of trade effect (unless the budget share of good B in the private goods expenditure equals the elasticity of the unit cost of the public good with respect to the price of good B). More precisely, suppose that, as a result of an exogenous redistribution of capital, country i receives dK_i units of capital, and knows that the sum total of redistribution to other countries equals $-dK_i$. Suppose it knows that other countries change their contribution to the public good by $dG_{-i} = -rdK_i/c(\widehat{p})$. Should country i increase its contribution by $dg_i = rdK_i/c(\widehat{p})$?Such an increase would imply

$$dK_i^g = a_K dg = a_K \left[\frac{rdK_i}{c(\widehat{p})}\right] \text{ where } a_K = \frac{\partial C^g}{\partial r}$$

$$dL_i^g = a_L dg = a_L \left[\frac{rdK_i}{c(\widehat{p})}\right] \text{ where } a_L = \frac{\partial C^g}{\partial w}$$

This in turn would imply that the change in capital and labor available for private goods productions are

$$d\mathcal{K}_i = dK_i - dK_i^g = dK_i \left[1 - \frac{ra_K}{c(\widehat{p})}\right] = dK_i \left[\frac{ra_K + wa_L - ra_K}{c(\widehat{p})}\right]$$

$$d\mathcal{L}_i = -dL_i^g = -dK_i \left[\frac{ra_L}{c(\widehat{p})}\right]$$

Thus, if $a_L > 0$, we would have $d\mathcal{K}_i > 0$ and $d\mathcal{L}_i < 0$. It follows (from Rybczynski theorem) that q_i^A would decrease, and q_i^B would increase. This would have a terms-of-trade effect, which must be taken into account.

5.2 Degenerate Cases

We now look at special cases in which the WKBBV neutrality results hold.

The first case may be called the "small country case". Recall that in the general case, we have the terms of trade effect, which is the term

$$\frac{du_i}{d\widehat{p}}\frac{\partial \widehat{p}}{\partial g_i} = \frac{du_i}{d\widehat{p}}\frac{\partial \widehat{p}}{\partial G} = -\frac{M_i^B}{e(\widehat{p})}\left(\frac{c(\widehat{p})\Omega}{\widehat{p}e(\widehat{p})\Delta}\right) \tag{27}$$

in equation (21). If a country is small, it will think that its changes in g_i will have no effect on the world equilibrium price of good B. That is, it behaves as if $\partial\widehat{p}/\partial G = 0$, i.e., as if $\Delta = \infty$, i.e, as if it faces an infinitely elastic supply curve from the rest of the world. In this case, expression (23) becomes

$$\frac{\phi_G^i}{\phi_v^i} = \frac{c(\widehat{p})}{e(\widehat{p})}$$

and thus G is unchanged (the sum $G_{-i} + g_i$ is constant).

The second special case has been mentioned earlier. If labor is not an input in the production of the public good, then countries that receive $dK_i > 0$ will use this additional input to produce additional units of public good, leaving the private-sector allocation of capital and labor unchanged, and countries for which $dK_i < 0$ will reduce its output of the public good accordingly, again leaving the private-sector allocation of capital and labor unchanged. (This argument relies on the assumption that dK_i is small relative to the pre-redistribution amount of capital used in the production of the public good g_i.) Only in the case where labor is an input in the production of the public good will the terms of trade consideration comes into play, because countries know that once the private-sector allocation of capital and labor is changed, the Rybczinsky effect implies, at any given price ratio, changes in relative supply of the two private goods.

Proposition 2: If all countries are small, or if labor is not an input in the production of the public good, then the WKBBV neutrality theorem holds.

5.3 Other Considerations

We now turn to two additional considerations: tradeable public good, and corner solutions. First, let us examine the role of the assumption that the public good is not internationally traded (in exchange for private goods). Under the assumptions of our model, under diversification of private good production in each country, factor prices are equalised, and hence the unit costs of production of the public good are the same in all countries. Under these conditions, one might think that there are no reasons why the public good would be internationally traded even if such trade is feasible. But upon reflection, recalling the Rybczinsky effect mentioned in the previous sub-sections, a government might have an incentive to outsource (to a foreign country) the production of its public good contribution. This can cause the foreign country to change its private-sector allocation of capital and labor, and hence this can impact the terms of trade. In other words, in a two-country world, if tariffs and quotas are not permitted, a country can still influence the terms of trade by outsourcing to the foreign country its public good contribution.

The second issue we want to discuss is the case of corner solutions. This matter is too complicated to deal with in full generality. We will therefore restrict attention to one result. In the BBV model, if one player, say player 1, does not contribute to the public good, then a redistribution of endowment away from that player will increase the total supply of public good. Would this result also hold in our model, where countries can influence the terms of trade?

For simplicity, let us consider a special case. Suppose for all countries,

$$\phi^i(u, G) = u + \psi^i(G)$$

Then the first order condition for the maximization problem of the government i is (taking into account the constraint $g_i \geq 0$),

$$\psi^i_G(G) - \left[\frac{M_i^B}{e(\widehat{p})}\left(\frac{c(\widehat{p})\Omega}{\widehat{p}e(\widehat{p})\Delta}\right) + \frac{c(\widehat{p})}{e(\widehat{p})}\right] + \lambda_i = 0$$

where

$$\lambda_i \geq 0,\ g_i \geq 0,\ \lambda_i g_i = 0,\ i = 1, 2, 3, ..., n.$$

Suppose country 1 is a non-contributor, while all other countries ($j =$

$2, 3, ..., n$) are contributors. Then

$$\psi_G^j(G) - \left[\frac{M_j^B}{e(\widehat{p})}\left(\frac{c(\widehat{p})\Omega}{\widehat{p}e(\widehat{p})\Delta}\right) + \frac{c(\widehat{p})}{e(\widehat{p})}\right] = 0,\ j \neq 1. \tag{28}$$

Consider a redistribution of capital away from country 1. To fix ideas, suppose that originally $\Omega/\Delta > 0$ and $M_1^B > 0$. At given $\widehat{p}$, a redistribution with $dK_1 < 0$ will make country 1 worse off, and M_1^B falls. Suppose the total supply of public good G remains unchanged. Then $\widehat{p}$ will be unchanged. So M^B is unchanged, implying that

$$\sum_{j=2}^{n} dM_j^B = -dM_1 > 0$$

Summing (28) over $j = 2, 3, ..., n$, we get, before the redistribution,

$$\sum_{j=2}^{n} \psi_G^j(G) = \left[\frac{M_1^B}{e(\widehat{p})}\left(\frac{c(\widehat{p})\Omega}{\widehat{p}e(\widehat{p})\Delta}\right) + \frac{c(\widehat{p})}{e(\widehat{p})}\right] \tag{29}$$

and, after the redistribution,

$$\sum_{j=2}^{n} \psi_G^j(G) = \left[\frac{(M_1^B + dM_1^B)}{e(\widehat{p})}\left(\frac{c(\widehat{p})\Omega}{\widehat{p}e(\widehat{p})\Delta}\right) + \frac{c(\widehat{p})}{e(\widehat{p})}\right] \tag{30}$$

Equations (29) and (30) are mutually inconsistent. It follows that G must change after the redistribution.

Proposition 3: In general, a redistribution away from a non-contributing country will lead to a change in total contribution of the public good.

5.4 Additional Results

We can calculate $\frac{\widehat{p}c'(\widehat{p})}{c(\widehat{p})}$ in (26). The result recorded in the following Lemma.

Lemma 5: Let θ^j denote the labour share in the unit cost of good j (where $j = A, B, g$). Then

$$\frac{\widehat{p}c'(\widehat{p})}{c(\widehat{p})} = \frac{\theta^g(1-\theta^A) - (1-\theta^g)\theta^A}{\theta^B - \theta^A} \tag{31}$$

Proof: See the Appendix.

The following results are corollary of Lemma 5:

Corollary 1: $\widehat{p}c'(\widehat{p})/c(\widehat{p})$ is positive if the numeraire good is either the most labor intensive good, or the least labor intensive good.

Corollary 2: A necessary condition for $\widehat{p}c'(\widehat{p})/c(\widehat{p})$ to be equal to η is that θ^g is intermediate between θ^A and θ^B.

Proof:

$$\frac{\widehat{p}c'(\widehat{p})}{c(\widehat{p})} - \eta = \left(\theta^B - \theta^A\right)\left\{\theta^g - \left[\eta\theta^B + (1-\eta)\theta^A\right]\right\}$$

The term inside the curly brackets can be equal to zero only if θ^g is a weighted average of θ^A and θ^B.

Corollary 3: Suppose θ^g is greater than both θ^a and θ^B . Then $\widehat{p}c'(\widehat{p})/c(\widehat{p}) > \eta$ if and only if $\theta^B > \theta^A$.

6 Concluding Remarks

We have shown that the famous neutrality result of Warr, Kemp, Bergstrom, Blume and Varian depends crucially on the assumption that agents do not take into account the effect of their public good contribution decisions on the relative price(s) of the private goods. Thus, the scope of applicability of their result is not as large as one might at fisrt think. Our non-neutrality results hold even if all countries are identical in technology, preferences, and endowments.

Our framework of analysis can be extended to the case where public goods are not "pure". In such an extended framework, the relevant question would no longer be the neutrality with respect to redistribution, but rather how public good decisions impact on the trade patterns and welfare levels of individual countries, as well as world welfare. This is part of our research agenda.

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APPENDIX

Proof of Lemma 5

Part 1: The effect of an increase in the relative price of good B on the wage and rental rates (expressed in terms of good A)

From the price-equals-cost equations,

$$p = C^B(w, r)$$

$$1 = C^A(w, r)$$

Differentiating these two equations with respect to p

$$dp = C_w^B dw + C_r^B dr$$

$$0 = C_w^A dw + C_r^A dr$$

Rearranging terms (noting that $C^B = p$ and $C^A = 1$) to get

$$1 = \left(\frac{wC_w^B}{C^B}\right)\left[\frac{p}{w}\frac{dw}{dp}\right] + \left(\frac{rC_r^B}{C^B}\right)\left[\frac{p}{r}\frac{dr}{dp}\right] \tag{32}$$

$$0 = \left(\frac{wC_w^A}{C^A}\right)\left[\frac{1}{w}\frac{dw}{dp}\right] + \left(\frac{rC_r^A}{C^A}\right)\left[\frac{1}{r}\frac{dr}{dp}\right] \tag{33}$$

Multiply both sides of equation (33) by p to get

$$0 = \left(\frac{wC_w^A}{C^A}\right)\left[\frac{p}{w}\frac{dw}{dp}\right] + \left(\frac{rC_r^A}{C^A}\right)\left[\frac{p}{r}\frac{dr}{dp}\right] \tag{34}$$

Define the elasticity of wage and elasticity of rental with respect to p by

$$\varepsilon_p^w = \left[\frac{p}{w}\frac{dw}{dp}\right] \text{ and } \varepsilon_p^r = \left[\frac{p}{r}\frac{dr}{dp}\right]$$

Define the labour share in good j by θ^j

$$\theta^B = \left(\frac{wC_w^B}{C^B}\right) \text{ and } \theta^A = \left(\frac{wC_w^A}{C^A}\right)$$

Using these shares and elasticities in equations (32) and (34) we get the matrix equation

$$\begin{bmatrix} \theta^B & 1-\theta^B \\ \theta^A & 1-\theta^A \end{bmatrix} \begin{bmatrix} \varepsilon_p^\omega \\ \varepsilon_p^r \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Solving, we get

$$\varepsilon_p^w = \frac{1-\theta^A}{\theta^B-\theta^A}$$

$$\varepsilon_p^w = \frac{-\theta^A}{\theta^B-\theta^A}$$

Thus, if good B is more labour intensive than good A, then an increase in the price of good B will increase the real wage in terms of good A (and $dw/w > dp/p$-the magnification effect- so the real wage in terms of good B also rises), and reduce the rental rate, in terms of both goods.

Part 2: solving for $c'(p)p/c(p)$

Recall that

$$c(p) \equiv C^g(w(p), r(p))$$

Thus

$$c'(p) = C_w^g \frac{dw}{dp} + C_r^g \frac{dr}{dp} = \frac{C^g}{p}\left(\frac{wC_w^g}{C^g}\right)\left[\frac{p}{w}\frac{dw}{dp}\right] + \frac{C^g}{p}\left(\frac{rC_r^g}{C^g}\right)\left[\frac{p}{r}\frac{dr}{dp}\right]$$

$$= \frac{C^g}{p}\left[\theta^g \varepsilon_p^w + (1-\theta^g)\varepsilon_p^r\right] = \frac{C^g}{p}\left[\theta^g\left(\frac{1-\theta^A}{\theta^B-\theta^A}\right) + (1-\theta^g)\left(\frac{-\theta^A}{\theta^B-\theta^A}\right)\right]$$

Therefore

$$\frac{pc'(p)}{c(p)} = \theta^g\left(\frac{1-\theta^A}{\theta^B-\theta^A}\right) + (1-\theta^g)\left(\frac{-\theta^A}{\theta^B-\theta^A}\right)$$

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