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Industry Investments

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The Dynamics of Industry Investments[*]

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Résumé / *Abstract*

Nous étudions le développement d'un duopole dans un modèle en temps continu d'investissement en capacité sans engagement des firmes quant à leurs actions futures. Bien que les unités de capacité soient coûteuses, indivisibles, durables et de taille non négligeable par rapport au marché, l'entrée hâtive ne peut conférer d'avantage durable et à partir d'un certain niveau de développement du marché, les deux firmes sont en activité. Nous évaluons les options réelles d'investissement dans ce contexte. Initialement, le seul équilibre Markovien parfait (ÉMP) est un équilibre de préemption dans lequel le premier investissement en capacité se produit plus tôt et comporte un risque plus élevé que socialement désirable. Une collusion tacite pour retarder les augmentations de capacité subséquentes peut devenir possible en ÉMP. La volatilité du marché et sa vitesse de croissance jouent un rôle crucial : l'émergence d'équilibres de collusion tacite est favorisée par une volatilité plus grande, une croissance plus rapide et un taux d'intérêt ou d'actualisation plus faible.

Mots clés : options réelles, duopole, préemption, collusion, investissement en bloc

We study the development of a duopoly in a continuous-time model of capacity investment under no commitment by firms regarding future actions. While capacity units are costly, indivisible, durable, and large relative to market size, early entry cannot secure a first-mover advantage and both firms are active beyond some level of market development. We evaluate the investment real options in that context. In the early industry development phase, the sole Markov Perfect Equilibrium (MPE) is a preemption equilibrium with the first industry investment occurring earlier (hence being riskier) than socially optimal. Once both firms hold capacity, tacit collusion, taking the form of postponed capacity investment, may occur as a MPE. Volatility and the expected speed of market development play a crucial role in competitive behavior: we show that the emergence of tacit collusion equilibria is favored by higher demand volatility, faster market growth, as well as by lower discount rate.

Keywords: *real options, duopoly, preemption, collusion, lumpy investment*

Codes JEL : C73, D43, D92; L13

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1. Introduction

"Real life" investment games played between competing firms in oligopolistic markets typically share the following characteristics. The environment (demand, information and knowledge, supply of inputs) is uncertain in many ways and evolves over time. The investment units come in finite discrete sizes, that is, investments are indivisible and lumpy. Undoing an investment strategy is costly, that is, investments are in part irreversible. Capacity is built or technology adoption is achieved in multiple discrete and separable steps engaged at different times without commitments as to future actions or investment levels and timing. As capacity is built over many periods, firms keep producing and competing, that is, their long term and short term decisions are intertwined. Firms have some (endogenous) flexibility to adapt the course of their investment strategy to exogenous changes in their environment as those strategies are implemented. At the industry level, investments come in waves, with all firms investing simultaneously, or in sequences, with firms investing at different times. Typically, investment (capacity building) games eventually come to an end as the relevant market matures.

Although uncertainty is a common feature of the economic modeling of investment games, other stylized facts are less frequently modelled. Typical models assume that firms make a unique decision and must live with that decision afterwards. Such models include models of technology adoption, models of entry, and numerous forms of two stage models where firms first make and commit to long term decisions (stage one) before competing in short term decisions (stage two).

Using a strategic real option approach, we develop a model of investment decision making incorporating some of the stylized features identified above: uncertainty, indivisibility, irreversibility, flexibility, dynamic choices of capacity building, no commitments on future actions and strategies, endogenous end to the investment game, and both investment waves and sequential investment timings over time.

The analysis of strategic considerations, in a game theoretic sense, is still in its infancy and should be high in the real option research agenda.[1] The real option approach

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[1] Among investment evaluation methods, the real option approach is reaching advanced textbook status and is rapidly gaining reputation and influence among practitioners. Although both academics and practitioners warn against its sometimes daunting complexity, they also stress its unique ability to take account of flexibility in managing ongoing projects, which is a significant but often neglected

emphasizes the irreversibility, hence indivisibility, of investments.[2] Indivisibility often imply a limited number of players, hence imperfect competition. Yet, while it is often stressed that the real option approach is best to analyze investments of strategic importance – the word 'strategic' appears repeatedly in the real option literature – the bulk of that literature involves decision makers confronted with a stochastic but non-reacting nature rather than reacting competitors.[3]

The present paper extends the recent contributions in different ways while bringing to bear the older literature on strategic investment,[4] in addressing issues such as the role of investment decisions in shaping the structure of a developing sector, the emergence of preemption with rent equalization and dissipation[5] versus tacit collusion with rent maintenance, the existence or not of a first-mover advantage,[6] and the effect of strategic competition on real option exercise rules.[7] We consider dynamic investments without commitment in an homogeneous product duopoly where rival firms face market development uncertainty and invest in lumpy increments of capacity. Typically, firms hold investment options. We find optimal exercise rules and determine the value of the corresponding options as well as of the firms holding them.

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source of value.

[2]Henry (1974) and Arrow and Fisher (1974) were precursors of the approach. In his treatment of the cost benefit analysis of a "new circumferential highway" around Paris, Claude Henry showed that using the Simon-Theil-Malinvaud certainty equivalent approach "will here, systematically and unduly, favor irreversible decisions, for example, destroying the forests and building the highway."

[3]There are notable exceptions. Grenadier (1996) uses a game-theoretic approach to option exercise in the real estate market; Smets (1995) provides a treatment of the duopoly in a multinational setup, which serves as a basis for the oligopoly discussion in Dixit and Pindyck (1994, pp. 309-14); Lambrecht and Perraudin (1996) and Décamps and Mariotti (2004) investigate the impact of asymmetric cost information on firms' investment strategies; Baldrusson (1998) considers a duopoly model where firms make continuous incremental investments in capacity showing that when firms differ in size initially, substantial time may pass until they are of the same size; Grenadier (2002) provides a general solution approach for deriving the equilibrium investment strategies of symmetric firms, in a Cournot-Nash framework, facing a sequence of investment opportunities with incremental capacity investments, showing that competition may destroy in part the value of the option to wait; Weeds (2002), Huisman (2001), Huisman and Kort (2003) study option games in a technology adoption context; Boyer *et al.* (2004) study a duopoly with multiple investments under Bertrand competition; Smit and Trigeorgis (2004) discuss different strategic competition models in the context of real options.

[4]Most notably Gilbert and Harris (1984), Fudenberg and Tirole (1985), and Mills (1988).

[5]As in Fudenberg and Tirole (1985).

[6]As in Gilbert and Harris (1984) or Mills (1988).

[7]The recent synthetic work of Athey and Schmutzler (2001) brings some generality and clarity to our understanding of the role of investment in market dominance. They show in particular that, when firms are farsighted and not committed to strategic investment plans, there is little hope to obtain definitive predictions outside specific models.

Our main results are as follows.

Two types of equilibria may arise. In preemption equilibria, which always exist, firms invest at different market development thresholds, hence at different times. In "tacit collusion" equilibria, which only exist under some conditions, firms invest at the same market development threshold, hence simultaneously, and thus such equilibria correspond to "investment wave" equilibria. In preemption equilibria, rents are equalized and partly dissipated while in tacit collusion equilibria, firms exercise market power by implicitly agreeing to postpone their respective investments in capacity building. If firms have equal positive capacities, then the preemption equilibrium exhibits different but uniquely determined market development investment thresholds, hence uniquely specified but stochastic investment timings with either firm moving first. If firms have different capacities and end game conditions are close to be met, the smaller firm acts as first mover. When they exist, tacit collusion or investment wave equilibria are typically numerous and Pareto superior to preemption equilibria from the firms' viewpoint. Tacit collusion is more profitable when firms have equal capacity in the sense that, when tacit collusion equilibria exist, the joint investment (stochastic) date that maximizes combined profits is an equilibrium if and only if firms are of equal size. Moreover, firms may be able to tacitly collude at some stages of market development but not at others.

Low initial capacities are of particular interest in the case of emerging sectors. When at least one firm has no capacity, preemption is the sole equilibrium as tacit collusion cannot then be enforced since the firm the firm cannot be threatened with the loss of an existing rent. Hence, even though the (preemption) equilibrium is characterized by the presence of only one active firm at first, the initial development of the industry is highly competitive as rents are equalized and partly dissipated. Paradoxically, once both firms are active, the industry may become less competitive as tacit collusion equilibria become possible.

It is well known that higher volatility raises the value of investment options because a flexible decision maker can achieve higher exposure to upside movements and lower exposure to downside ones. In a strategic setup, higher market volatility also favors also the emergence of tacit collusion equilibria. Similarly, higher expected market growth as well as a lower cost of capital favor the emergence of tacit collusion equilibria. Hence, our results suggest that investment waves (joint investment timings) may signal the exercise

of market power and are more likely when firms have similar positive capacities, market growth is high, volatility is high, and/or interest rates are low.

After presenting the model, the competition framework, and the investment game in Section 2, we proceed in Section 3 with the analysis of essentially all possible industry development histories, more specifically with the explicit analysis of three different situations that essentially cover all relevant ones. We conclude in Section 4. Detailed proofs are provided in the Appendix.

2. The model

2.1 Industry characteristics

We consider the development of an industry where demand is affected by multiplicative random shocks. The inverse demand function at time $t \geq 0$ is given by:

$$P(t, X_t) = Y_t \, D^{-1}(X_t), \tag{1}$$

where $X_t \geq 0$ is aggregate output, $Y_t \geq 0$ is a random shock, and $D : \mathbb{R}_+ \to \mathbb{R}_+$ is the non-stochastic component of demand.

Assumption 1
Demand $D(\cdot)$ is strictly decreasing, continuously differentiable and integrable on $\mathbb{R}_+$ and $D(0) = \lim_{p \downarrow 0} D(p) < \infty$; the mapping $x \mapsto x D^{-1}(x)$ is strictly concave on $(0, D(0))$; aggregate shocks $(Y_t)_{t \geq 0}$ follow a geometric Brownian motion:

$$dY_t = \alpha Y_t dt + \sigma Y_t dZ_t \tag{2}$$

with $Y_0 > 0$, $\alpha > 0$, $\sigma > 0$, and $(Z_t)_{t \geq 0}$ a standard Brownian motion with respect to the complete probability space $(\Omega, \mathcal{F}, P)$.[8]
Firms are risk neutral and discount future revenues at the same rate $r > \alpha$. Investment takes place in a lumpy way. Each capacity unit costs I, which is constant over time,

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[8]Thus market demand is driven by consumers' tastes for the output, not by replication of the initial consumers as in Gilbert and Harris (1984).

produces at most $Q = 1$ unit of output, does not depreciate, and has no resale value.

2.2 Competition, output, and investment

We consider a duopoly. At any date t, firms first take their investment decisions and then compete in quantities (*à la Cournot*) subject to capacity constraints.[9] Specifically, within the instant $[t, t + \tau)$, the timing of the game is as follows: (i) first, each firm f chooses how many capacity units ν_t^f to invest in, given the realization of the demand shock Y_t and the existing capital stocks (k_t^f, k_t^{-f}); (ii) next, each firm selects an output level within its capacity, $x_t^f \leq k_t^f + \nu_t^f$; (iii) last, market price is determined according to (1), with $X_t = x_t^f + x_t^{-f}$.

The specification of inverse demand (1) implies that the short-run Cournot game is independent of the realization of the current industry-wide shock. We can assume that, in the absence of capacity constraints, this game has a unique equilibrium (x^c, x^c). Let $k^c = \lceil x^c \rceil$ be the minimum capital stock required to produce x^c. It is then easy to check that, with given capacities $k^f \leq k^{-f}$, only three Cournot equilibrium outcomes can occur: (i) both firms are constrained, so that $x^f = k^f$ and $x^{-f} = k^{-f}$; (ii) the smaller firm is constrained, so that $x^f = k^f$, while the bigger firm is not and reacts optimally by choosing x^{-f} on its reaction function; (iii) both firms are unconstrained, so that $x^f = x^{-f} = x^c$. The corresponding instantaneous profit of a firm with capacity k when its competitor holds ℓ capacity units can be conveniently denoted $Y_t \pi_{k\ell}$, where $\pi_{k\ell}$ depends on capacities only.

2.3 Markov strategies

A key assumption of our model is that firms cannot (credibly) commit to future investment and output decisions. The game typically generates several investments occurring in endogenous order at endogenous dates. There is no commitment by the firms with respect to their role as first or second investor or to the number of units they will acquire. The natural equilibrium concept here is the Markov perfect equilibrium (*MPE*), in which firms' investment and output decisions at each date depend only on the firms' capital stocks measured in capacity units, (k^f, k^{-f}), as well as on the current level of

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[9]See Boyer et al. (2004) for a related preemption model with instantaneous competition in prices (Bertrand).

the industry-wide shock y. This rules out implicit collusion between firms when deciding on output: at each date, and given their current capacities, firms play the unique equilibrium of the static Cournot game described above.

Our definition of Markov strategies and of the resulting payoffs is in line with the Fudenberg and Tirole (1985) concept of mixed strategies for timing games in continuous time. The main difference is that, while they focus on deterministic environments, demand fluctuates randomly in our model.[10] The basic idea is to construct an adequate continuous time representation of limits of discrete time mixed strategy equilibria by defining a strategy for firm f as a function s^f specifying the *intensity* $s^f_{\nu f}(k^f, k^{-f}, y) \in [0, 1]$ with which firm f invests in ν^f capacity units given the capital stocks (k^f, k^{-f}) and the industry-wide shock $Y_t = y$. Given a strategy profile (s^f, s^{-f}), $U^f_{(s^f, s^{-f})}(k^f, k^{-f}, y)$ denotes firm f**'s** expected discounted profit in state (k^f, k^{-f}, y).[11] In the rest of the paper, we will omit firm and strategy profile indices in the expression of value functions when no ambiguity arises.

2.4 Firm valuation

Since $(Y_t)_{t \geq 0}$ is a time homogenous Markov process, an outcome may be described as an ordered sequence of investment triggers together with the short-run instantaneous profits of both firms $Y_t \pi_{k\ell}$ and $Y_t \pi_{\ell k}$ between investments. Let y_{ij} (with $y_{ij} = y_{ji}$), where i and j refers to the firms' capacities immediately before Y_t reaches y_{ij} for the first time,[12] denote the value of Y_t that triggers a new investment when total industry capacity is $i + j$. If the game is over, then $y_{ij} = \infty$.

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[10]A precise definition of Markov strategies and payoffs under uncertainty can be found in Boyer et al. (2004, Appendix A).

[11]The important intuitions that our paper will convey can be grasped in terms of pure strategies. However, in symmetric cases, there will be situations where two pure strategy equilibria exist, where either firm invests first and the other firm second, for identical payoffs. Then there is a possibility, if firms use pure strategies, of both firms investing simultanously by mistake, a sort of coordination failure. Indeed, a firm prefers to invest if its opponent does not but prefers not to invest if its opponent does. Hence both firms wish to avoid the worst of all cases, namely simultaneous investments. Under the foregoing definition of Markov strategies, strategies can be designed such that no entry mistakes can occur. In the first symmetric MPE we construct in the Appendix, $s^f_1(0, 0, y^P) = s^{-f}_1(0, 0, y^P) = 0$, where $k^f = k^{-f} = 0$, $\nu^f = \nu^{-f} = 1$, and y^P is the level of Y_t that triggers the first firm investment in the equilibrim. This acts as a correlation device: each firm is equally likely to invest in state $(0, 0, y^P)$, but the probability of simultaneous entry is zero. This formally jsutifies the usual less rigorous approach consisting in determining at random a (lucky) first mover.

[12]Since capacity units do not depreciate, higher triggers *along a given development path* correspond to higher industry capacity levels: $y_{ij} \leq y_{k\ell} \Leftrightarrow i + j \leq k + \ell$.

Suppose $Y_t = y$ and let us consider, for simplicity, investments of one single capacity unit only ($\nu = 1$), as investments in multiple capacity units can be treated as one-unit investments occurring at the same time. Let $L(i, j, y)$ denote the current value of the firm of capacity i if it carries out an investment immediately, while its opponent has capacity j. Let $F(i, j, y)$ be the current value of the firm of capacity i when its competitor with capacity j carries out an investment immediately. Let $S(i, j, y)$ denote the current value of the firm of capacity i, with its competitor holding capacity j, if both firms make a simultaneous investment at some future date when Y_t reaches say y_{ij}.

The following lemma gives analytical expressions for the L, F, and S functions. The expressions are divided into a first part corresponding to the current investment and a second part corresponding to the continuation of the game. The latter part is not fully specified at this stage; it will be determined recursively by backward induction, starting from the 'horizon', defined in state space as the first (stochastic) time a situation (or capacity combination) is reached such that it is certain that no more investment will take place.

Lemma 1 *let $Y_t = y$. The value of the firm of capacity i, when it invests immediately while the firm of capacity j does not, is given by the following, where $k = i + 1$:*

$$L(i, j, y) = \frac{\pi_{kj}}{r - \alpha} y - I + \left(\frac{y}{y_{kj}}\right)^{\beta} \left[c(k, j, y_{kj}) - \frac{\pi_{kj}}{r - \alpha} y_{kj}\right]$$

where $\beta = \frac{1}{2} - \alpha/\sigma^2 + \sqrt{(\alpha/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2} > 1$ and $c(k, j, y)$ is the continuation value of the same firm at the time of the next industry investment, if any.

Its value, when it stays put while its competitor of capacity j invests now, is given by the following, where $k = j + 1$:

$$F(i, j, y) = \frac{\pi_{ik}}{r - \alpha} y + \left(\frac{y}{y_{ik}}\right)^{\beta} \left[c(i, k, y_{ik}) - \frac{\pi_{ik}}{r - \alpha} y_{ik}\right].$$

Its value, when both firms invest simultaneously at some future trigger value y_{ij}, is

given by the following, where $k = i+1$ and $\ell = j+1$:

$$S(i,j,y) = \frac{\pi_{ij}}{r-\alpha}y + \left(\frac{y}{y_{ij}}\right)^{\beta}\left(\frac{\pi_{k\ell} - \pi_{ij}}{r-\alpha}y_{ij} - I\right) + \left(\frac{y}{y_{k\ell}}\right)^{\beta}\left[c(k,l,y_{k\ell}) - \frac{\pi_{k\ell}}{r-\alpha}y_{k\ell}\right].$$

Consider the expression for $L(i,j,y)$. The first part $\frac{\pi_{kj}}{r-\alpha}y - I$ gives the expected net present value of the profit flows achieved by increasing capacity from i to $k = i+1$ at a cost of I, assuming that no more investment is made. The second part $\left(\frac{y}{y_{kj}}\right)^{\beta}\left[c(k,j,y_{kj}) - \frac{\pi_{kj}}{r-\alpha}y_{kj}\right]$ adjusts the first one for the effect of subsequent investments, that is for the (equilibrium) exercise by both firms of their investment options. Indeed, $\left(\frac{y}{y_{ij}}\right)^{\beta}$ may be viewed as a discount factor defined over the state space rather than the time space[13] and the function $c(k,j,y_{kj})$ is the continuation value function when $Y_t = y_{kj}$.[14] The expressions for $F(i,j,y)$ and $S(i,j,y)$ can be similarly understood.

2.5 Endgame conditions

Although the investment game imposes no restrictions on capacities, we can characterize endgame conditions: the investment game is over if and only if it is known with certainty that no firm will ever invest in additional capacity. The following proposition gives two conditions, one necessary, one sufficient, for the investment game to be over.

Proposition 1 *The investment game is over only if (necessity) either condition* A *or condition* B *is satisfied, implying that both firms hold a strictly positive capacity; more-*

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[13]The expression $\left(\frac{y}{y_{ij}}\right)^{\beta}$ gives the expected discounted value when $Y_t = y$ of receiving one dollar the first time Y_t reaches $y_{ij} > y$, the length of time necessary to go from y to y_{ij} being random. If there is no subsequent investment, so that $y_{ij} = \infty$, the second term in L vanishes.

[14]To emphasize that c is a continuation function by definition, $L(i,j,y)$ can be written as

$$L(i,j,y) = -I + \left[\frac{\pi_{kj}}{r-\alpha}y - \left(\frac{y}{y_{kj}}\right)^{\beta}\left(\frac{\pi_{kj}}{r-\alpha}y_{kj}\right)\right] + \left(\frac{y}{y_{kj}}\right)^{\beta}c(k,j,y_{kj}),$$

where $\left[\frac{\pi_{kj}}{r-\alpha}y - \left(\frac{y}{y_{kj}}\right)^{\beta}\left(\frac{\pi_{kj}}{r-\alpha}y_{kj}\right)\right]$ is the expected present value of a random annuity $Y_t\pi_{kj}$ lasting between today (when $Y_t = y$) and the random date at which Y_t will reach y_{kj}. Suppose that the investment occurring at y_{kj} is made by the firm with capacity j and no further industry investment occurs afterwards. Then at y_{kj}, the new profit flow of the firm of capacity k becomes $\pi_{k(j+1)}Y_t$ and $c(k,j,y_{kj})$ is the expected present value of that profit flow, namely $\frac{\pi_{k(j+1)}}{r-\alpha}y_{kj}$. The value function $L(i,j,y)$ is then completely defined, provided the trigger value y_{kj} has been determined: $L(i,j,y) = \frac{\pi_{kj}}{r-\alpha}y - I + \left(\frac{y}{y_{kj}}\right)^{\beta}\frac{\pi_{k(j+1)} - \pi_{kj}}{r-\alpha}y_{kj}$.

over, the investment game is over if (sufficiency) Condition A *is satisfied:*

(A) *Neither capacity constraint is binding in the short-run Cournot game, that is,*
$k^f \geq k^c = \min\{k \in \mathbb{N} \,|\, k \geq x^c\}, \; f \in \{1, 2\}.$

(B) *Both capacity constraints are binding in the short-run game and would remain binding in case of a unit investment by any one firm.*

Proposition 1 indicates (i) that no firm can keep its opponent out of the market in the long run, and (ii) that a firm cannot use excess capacity in order to maintain a dominant position in the long run.[15] Condition (A) falls short of implying equal capacities for both firms. However it implies that, if capacities are not equal at the end of the game, the number of units used by each firm is the same. If capacities are not equal, some capacity is idle.

Condition (A) is not necessary; however if it is not satisfied at the end of the game, Condition (B) must hold. That condition pertains to tacit collusion. It describes a situation where each firm could still profitably increase its capacity if its rival did not react. For such a situation to last forever (game over), it must be the case that firms restrict capacity, hence output, in equilibrium. Such an equilibrium can hold only if any deviation is adequately punished. Condition (B) describes a situation where a firm can inflict a punishment on its competitor if the latter deviates. If the former firm were no longer capacity constrained following an investment by its opponent (Condition (B) not satisfied), then it would not be able to retaliate to the deviation. It is then certain that the opponent would invest at some date t. The ability to retaliate is however not sufficient to sustain a tacit collusion equilibrium. We characterize below the conditions under which the retaliatory power is sufficient to offset the gain from deviating. If the firm to be punished is small, it does not lose as much from an increase in the capacity of its opponent as if it were bigger. This implies that retaliation, hence collusion, is likely

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[15]The conditions spelled out in Proposition 1 are not compatible with the situation found in Gilbert and Harris (1984) where, in equilibrium, one duopolist concentrates the totality of industry capacity, while the other firm holds no capacity. Their result can be traced to a technical assumption, claimed to be "trivial in that both firms will earn zero profits on new investments in a preemption equilibrium" (p. 206), that gives a first-mover advantage to one firm in order to rule out (mistaken) simultaneous investments. The strategies and equilibrium concept defined above avoid the necessity of any asymmetric treatment.

to be easier between firms of similar size and explains why the investment game cannot be over unless both firms hold strictly positive capacities.

In what follows, firm asymmetry can only take the form of differences in current capacities and may be thought of as inherited from past moves in the industry development game. As discussed above, Lemma 1 provides only a partial characterization of value functions under alternative investment strategies. Completing the characterization requires knowledge of the continuation function $c(\cdot)$ and the appropriate trigger values. These can be determined when the game between the two firms is sufficiently near its end, in the sense of Proposition 1. Once the continuation value function is known in such situations, it is possible to characterize recursively the value function corresponding to previous steps.

3. INDUSTRY DEVELOPMENT

Industry development proceeds by successive capacity acquisitions by one of the firms or both. The particular demand function we use guarantees that the number of capacity units that will eventually be installed is finite and that the industry development game has an end. This is in contrast with most papers on related subjects (investment games; R&D games, etc.) where it is assumed either that the players play only once or that the game goes on indefinitely.

Industry development possibilities may be represented as a tree whose nodes give the number of capacity units held by each firm (Figure 1). While the figure indicates possible sequences of capacity investments, it does not provide any indication about the speed at which investments occur and nodes are reached.[16]

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[16]In particular this representation is compatible with a firm acquiring more than one unit simultanously, or with both firms investing simultaneously, in which case no time is spent on intermediary nodes.

$(k'+3, k'+1)$

$(k'+2, k'+1)$

...

$(k'+1, k'+1)$ $(k'+2, k'+2)$

$(2,0)$ ... $(k', k'+1)$ $(k'+1, k'+2)$

$(1,0)$ $(k', k'+2)$ $(k'+2, k'+2)$

$(0,0)$ ...   ... $(k'+1, k'+3)$

$(1,1)$ ... $(k', k'+3)$

$(0,1)$

... $(k', k'+4)$

$(0,2)$

...

... $(k+2, k)$

...   ... $(k+1, k)$

... (k, k) $(k+1, k+1)$

$(k, k+1)$

$(k, k+2)$
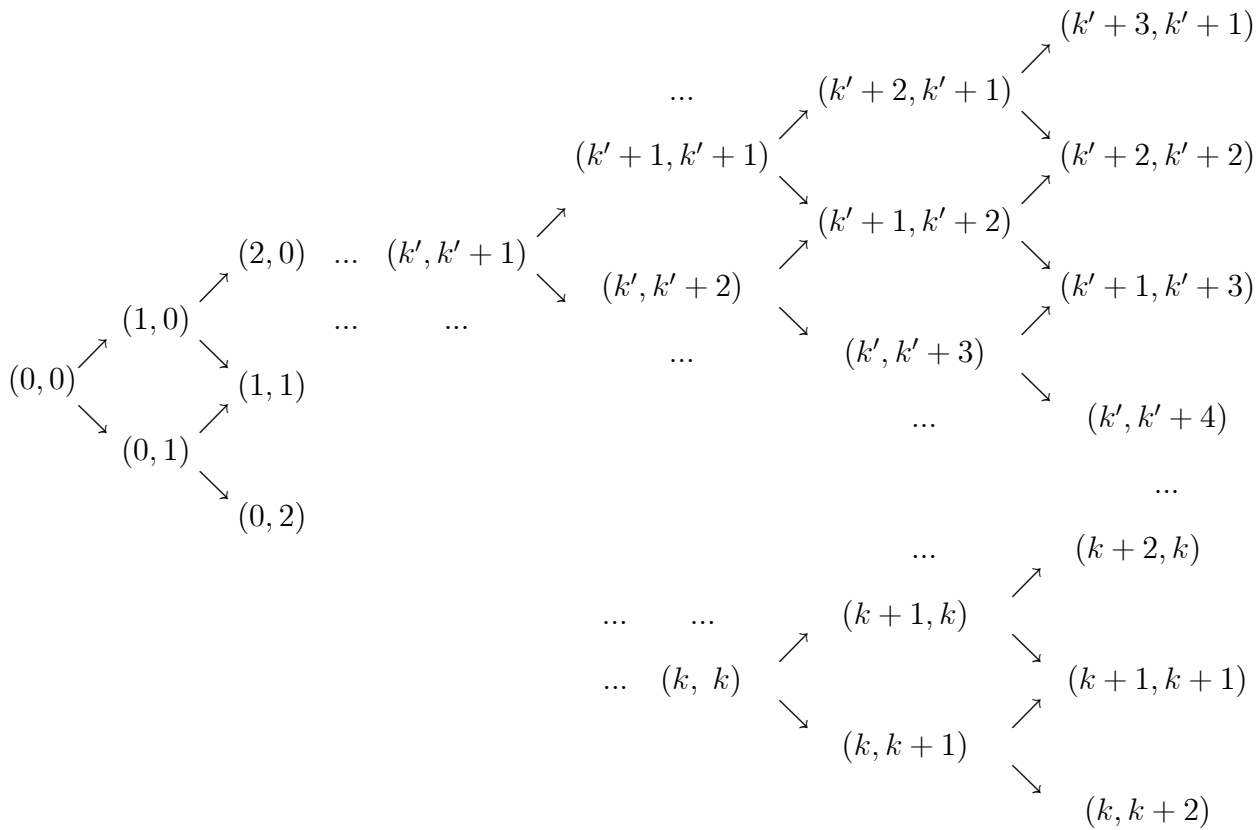
Figure 1: Industry capacity development tree

We characterize the capacity acquisition path and the competition intensity prevailing at various stages of market and industry development: first, in the early stage when firms hold no capacity (Case 1); second, at a later stage, when firms hold symmetric (Case 2) or asymmetric (Case 3) capacities due to the unraveling of their respective investment strategies. We first consider situations that are "near" the end of the game: from the nodes considered, a limited number of investments will lead to a situation where the investment game is over in the sense of Proposition 1. Once these investment developments are characterized, the previous relevant investment histories can be obtained by backward induction: a limited number of investments will lead to a situation or node from which the (not necessarily unique) unraveling of the investment game has been characterized till endgame conditions are met. Once we have characterized those situations that are "near" the end of the game, we generalize the analysis (in section 3.4) to arbitrary nodes in the industry development tree. Hence, we virtually consider all relevant cases as the path to such characterizations is clearly beaconed.

3.1 Case 1: No existing capacity

We start with a situation where initial capacities are zero; let us assume that the market is such that unconstrained firms would produce at most one unit each in Cournot duopoly, that is:

Assumption 2 $0 < x^c \leq 1$.

Although this assumption allows the monopoly output to exceed unity, so that the acquisition of more than one unit may be considered by any one firm, it also implies that, if both hold one unit or more, the game is over by Proposition 1. Assumption 2 also implies that, whatever the (strictly) positive number of capacity units held by its opponent, a firm obtains instantaneous profit $Y_t \pi_{11}$ once it invests in one unit or more; consequently it will typically not acquire more than one unit.

Therefore, the payoff from not investing immediately is (with $\pi_{1\nu} = \pi_{11}$ by Assumption 2)

$$F(0,0,y) = \left(\frac{y}{y_{0\nu}}\right)^{\beta} \left(\frac{\pi_{11}}{r-\alpha} y_{0\nu} - I\right),$$

where ν is the number of units acquired by the opponent before the firm acquires its first and single unit. The stopping problem faced by the firm is then:

$$F^*(0,0,y) = \sup_{y_{0\nu}} \left[\left(\frac{y}{y_{0\nu}}\right)^{\beta} \left(\frac{\pi_{11}}{r-\alpha} y_{0\nu} - I\right)\right] \tag{3}$$

with solution:

$$y_{0\nu}^* = y_{01}^* = \frac{r-\alpha}{\pi_{11}} I \frac{\beta}{\beta-1}, \quad \forall \nu \geq 1. \tag{4}$$

Knowing this, the value for the competitor of acquiring at least one unit immediately at $Y_t = y$, and any number of further units before Y reaches the threshold y_{01}^* can be computed explicitly. For example if it acquires one unit immediately and abstains from

any further investment its value is, according to Lemma 1:[17]

$$L\left(0,0,y\right) = \frac{\pi_{10}}{r-\alpha}y - I + \left(\frac{y}{y_{01}^*}\right)^\beta \frac{\pi_{11}-\pi_{10}}{r-\alpha}y_{01}^*, \quad y < y_{01}^*. \tag{5}$$

where $\frac{\pi_{11}}{r-\alpha}y_{01}^* = c\left(1,0,y\right)$, since no more investment is forthcoming beyond y_{01}^* by Proposition 1. Similarly, if the investment in the first unit is to take place in the future at $y_{00} > y$, then the value of the firm is:

$$L(0,0,y_{00}) = \left(\frac{y}{y_{00}}\right)^\beta \left(\frac{\pi_{10}}{r-\alpha}y_{00} - I\right) + \left(\frac{y}{y_{01}^*}\right)^\beta \left(\frac{\pi_{11}-\pi_{10}}{r-\alpha}y_{01}^*\right). \tag{6}$$

Its maximum $L^*\left(0,0,y\right)$ with respect to y_{00} is reached at:

$$y_{00}^L = \frac{r-\alpha}{\pi_{10}}I\frac{\beta}{\beta-1}. \tag{7}$$

Figure 2 illustrates the functions $L\left(0,0,y\right)$, $L^*\left(0,0,y\right)$ and $F^*(0,0,y)$.

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[17]We leave to the reader the straightforward task to adapt the formula and the rest of the argument for any number of units acquired by the first investor before the other one invests at y_{01}^*. For example if the firm plans to acquire a second new unit at some y', $y \leq y' < y_{01}^*$, the candidate value for $L\left(0,0,y\right)$ is $\frac{\pi_{10}}{r-\alpha}y - I + \left(\frac{y}{y'}\right)^\beta \left[\frac{\pi_{20}-\pi_{10}}{r-\alpha}y' - I\right] + \left(\frac{y}{y_{01}^*}\right)^\beta \frac{\pi_{21}-\pi_{20}}{r-\alpha}y_{01}^*$, $y \leq y' < y_{01}^*$ where $\pi_{21} = \pi_{11}$ by Assumption 2. If this value is higher than (5), then it gives the correct expression for $L\left(0,0,y\right)$; if it is lower, then (5) is the appropriate expression. Note that the number of candidates to try is low as it cannot exceed the monopoly capacity under Assumption 2.
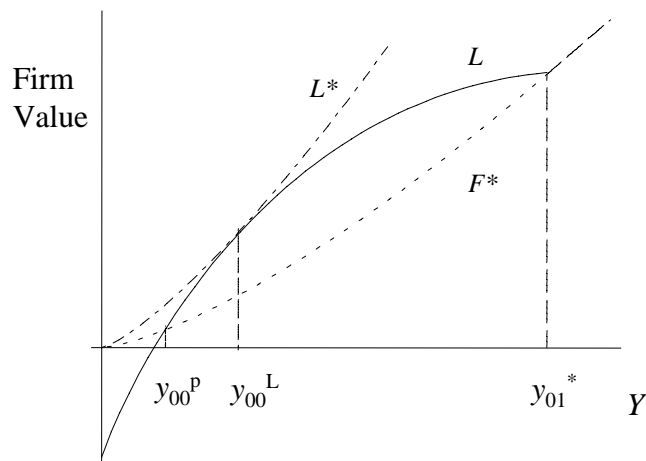
Figure 2: Firm values under alternative strategies

It is straightforward to check from (3)–(6) that, within the interval $(0, y_{01}^*)$, there exists a unique value y_{00}^p such that for $y < [>, =] y_{00}^p$, $L(0, 0, y) < [>, =] F(0, 0, y)$, with the corresponding stochastic stopping time being $\tau_{00}^p = \inf\{t \geq 0 \,|\, Y_t \geq y_{00}^p\}$.

We now determine the firms' equilibrium strategies before any firm has invested, that is, in states of the form $(0, 0, y)$. If $y < y_{00}^p$, investing is for both firms a strictly dominated strategy while for $y \geq y_{01}^*$, delaying investment any further is also a strictly dominated strategy. To determine the equilibrium outcome when $y_{00}^p \leq y < y_{01}^*$, it is helpful to consider what would happen if one of the firms were protected from preemption and could thus choose its optimal stand-alone investment date as a monopoly.[18] Given a current industry-wide shock y, the maximal expected payoff that this firm could then achieve by taking the lead is $L^*(0, 0, y)$. This is strictly higher than $F^*(0, 0, y)$.[19] In an *MPE*, however, such a value gap cannot be sustained. If a firm anticipates that its rival will first invest at y_{00}^L, then the former is better-off preempting the latter at $y_{00}^L - dy$. This is true for any y between y_{00}^p and y_{00}^L. When the industry-wide shock Y_t is equal to y_{00}^p, the value of both firms is the same, so each firm is indifferent between investing immediately and letting its rival invest while waiting to invest until Y_t reaches y_{01}^*, at the

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[18]Katz and Shapiro (1987).

[19]Everything happens as if the firm is myopic and takes no account of the future entry of the follower. This is is line with Leahy (1993): when computing its optimal stand-alone date, a myopic firm overstates by the same amount the value of the investment option and the marginal benefit from investing, leaving the investment rule unaffected.

stochastic time $\tau_{01}^* = \inf\{t \geq 0 \,|\, Y_t \geq y_{01}^*\}$. The following proposition is a transposition of Fudenberg and Tirole (1985, Proposition 2A) in a stochastic context.

Proposition 2 *(Preemption equilibrium) Under Assumptions 1 and 2, if $Y_0 \leq y_{00}^p$,*

1. *There exists only one* MPE *outcome of the investment game: one firm invests at τ_{00}^p, while the other firm waits until τ_{01}^* to invest; both times are stochastic.*

2. *Rents are equalized to the value of the second investor given by* (3).

The preemption *MPE* is characterized by intense competition. The first capacity unit is installed earlier than under protection from preemption since $y_{00}^p < y_{00}^L$, reflecting a partial dissipation of monopoly rents (Posner, 1975, Fudenberg and Tirole, 1987).

3.1.1 Socially optimal investment timings

It is more difficult to compare the *MPE* outcome with the social optimum. Specifically, let $k^0 = \lceil D(0) \rceil$ be the minimum capital stock required to produce $D(0)$. The social planner's problem is to find an increasing sequence of stopping times that solves:

$$\sup_{\tau_1 \leq \cdots \leq \tau_{k0}} \left\{ \mathrm{E}_y \left[\sum_{k=1}^{k^0} \int_{\tau_k}^{\tau_{k+1}} e^{-rt} Y_t \int_0^k D^{-1}(q) \, dq \right] \right\},$$

where by convention $\tau_{k^0+1} = \infty$. Standard computations imply that it is optimal for the social planner to invest in the first capacity unit when Y_t reaches the investment trigger y^O such that:

$$y^O \int_0^1 D^{-1}(q) \, dq = \frac{\beta}{\beta - 1} (r - \alpha) I.$$

Clearly, $y^O < y_{00}^L$. Since $y_{00}^p < y_{00}^L$ as well, there is no obvious way to compare y^O and y_{00}^p.

However, modifying slightly the model allows an unambiguous comparison between the *MPE* outcome and the social optimum, thus identifying the key factors involved in a general comparison. Indeed, suppose that the inverse demand curve is a step function, $D^{-1}(Q) = D^{-1}(\lceil Q \rceil)$; because of the assumption of unit capacity increments, the steps correspond to capacity levels, so that each capacity unit produces at full scale once

installed.[20] Then $\pi_{10} = D^{-1}(1) = \int_0^1 D^{-1}(q)\,dq$, so that $y_{00}^L = y^O$. It follows that $y_{00}^p < y^O$, so that the first capacity unit is introduced too early in an *MPE*, relative to the social optimum. This happens because rents accruing to each firm must be equalized in an *MPE*, while, from a social point of view, each successive capacity unit yields less value than the preceding one as the consumers' marginal willingness to pay decreases. In order for the first capacity unit to yield no more rent than the second unit in equilibrium, the first investor must therefore waste resources to compensate for interim earnings so that its value does not exceed that of the second investor.

The result that the first industry investment occurs earlier under duopoly than in the social optimum does not depend on the market size assumption $k^c = 1$. As $\pi_{11} = D^{-1}(2) = \int_1^2 D^{-1}(q)dq$, the second investor introduces the second capacity unit at the socially optimal date. Since it acts like a monopoly with respect to the market residual demand and since it does not hold any capacity, it will invest as soon as the market is able to support a second capacity unit. However, if the firm that makes the last industry investment already held some capacity, it would postpone its investment in order not to cannibalize its demand. We will show that this is indeed the case.

3.2 Case 2: Symmetric capacities

Let us now investigate the role of existing capacity, starting in this section with situations where firms have identical capacities, as illustrated by the subgame starting at node (k, k) in Figure 1. As in the previous subsection, we will assume that the firms hold a capacity lower than the unconstrained short-run Cournot output, which implies that both firms are initially capacity constrained and that a firm remains constrained if its opponent invests:

Assumption 3 $0 < x^c - k \leq 1$

Assumption 3 is compatible with an unconstrained monopoly output exceeding $k+1$, so that it does not rule out investments exceeding one unit, allowing a firm to get ahead by more than one unit. It does imply that the end of the game is not too far in the

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[20]The assumption $D^{-1}(Q) = D^{-1}(\lceil Q \rceil)$ reduces the consumer surplus by the triangles between the initial inverse demand curve and the steps of the new inverse demand curve, located entirely below the former. In an industry involving indivisible capacity units the same assumption of a stepwise demand would be necessary to ensure that perfect competition coincides with the social optimum.

sense that, by Proposition 1, the game is over once both firms have acquired at least one more unit. To simplify exposition, we take $k = 1$. Then Assumption 3 implies that $\pi_{21} > \pi_{11}$, $\pi_{22} > \pi_{12}$, and $\pi_{\nu 2} = \pi_{22} = \pi_{2\nu}$, $\forall \nu \geq 2$.

When considering a new investment, firms will now take into account the consequences on the profits they derive from their existing capacity. We will show that, as a result of the cannibalism effect, tacit collusion equilibria may exist besides the preemption equilibrium, provided that either late joint investment or no more investment dominates preemption over the whole relevant market development range.

3.2.1   The preemption MPE

The investment game with symmetric capacities always has a MPE. Assume that one of the firms has taken the lead by acquiring at least one new unit, bringing its total capacity to $\nu \geq 2$. For its rival, whatever the number of units held by the first investor, it is a dominant strategy by Assumption 3 to acquire one and only one unit at the market development threshold determined by the following optimal stopping problem: for $y < y_{1\nu}$,

$$F^* \left(1, \nu, y\right) = \sup_{y_{1\nu}} \left[\frac{\pi_{1\nu}}{r - \alpha} y + \left(\frac{y}{y_{1\nu}}\right)^{\beta} \left(\frac{\pi_{22} - \pi_{1\nu}}{r - \alpha} y_{1\nu} - I\right) \right], \tag{8}$$

that is, at:

$$y_{1\nu}^* = \frac{r - \alpha}{\pi_{22} - \pi_{1\nu}} I \frac{\beta}{\beta - 1}. \tag{9}$$

The situation is similar to the case with no initial capacity except that the trigger value, at which the second investor invests, depends on the number ν of units held by the initial investor. The higher ν, the earlier the second investor will invest because its profits $\pi_{1\nu}$ while waiting are lower the higher ν.

The firm that invests first, whether it acquires one single unit or more units, understands all implications of its investment(s) on the behavior of its competitor, so that $L\left(1, 1, y\right)$ can be computed explicitly. For example if the early investor acquires only

one unit, its payoff at the current level of $y \leq y_{1\nu}^*$ is, for $\nu = 1$:

$$L(1, 1, y) = \frac{\pi_{21}}{r - \alpha} y - I + \left(\frac{y}{y_{12}^*}\right)^\beta \left(\frac{\pi_{22} - \pi_{21}}{r - \alpha} y_{12}^*\right). \tag{10}$$

As before, if the firm were able to choose the investment threshold in the absence of any threat of preemption, the maximum $L^*(1, 1, y)$ with respect to y, for $\nu = 1$, would be reached at:[21]

$$y_{11}^L = \frac{r - \alpha}{\pi_{21} - \pi_{11}} I \frac{\beta}{\beta - 1}.$$

But under a preemption threat, the firm cannot wait until Y_t reaches $y_{1\nu}^L$ and invests at trigger level $y_{1\nu}^p$, at which rents are equalized. The following result then parallels Proposition 2.

Proposition 3 *(Preemption with equal capacities) Under Assumptions 1 and 3, the investment game has a preemption* MPE *such that any one firm invests when Y_t reaches $y_{1\nu}^p$ while the other firm invests when Y_t reaches $y_{1\nu}^*$.*

In this equilibrium, the threat of preemption leads to rent equalization and thus to the complete dissipation of any first-mover advantage. However, with positive capacities, the preemption equilibrium may not be the sole type of *MPE*, as we shall now see.

3.2.2 Tacit collusion *MPE*

The fact that the firms hold strictly positive capacities gives rise to the possibility of a different type of *MPE*. The strategies involved consist in coordinating on a random joint investment date or in abstaining from investing forever. We call these strategies *tacit collusion* strategies as they imply an increase of firms' values above the preemption equilibrium level. Note that short-run output decisions are still determined according to Cournot competition. Collusion is achieved only through firms' investment strategies and not through production decisions. This implies that the only way firms can sustain a

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[21] Again the reader can adapt the candidate expressions for $L(1, 1, y)$, with $y_{1\nu}^*$ given by (9), for any new capacity purchase exceeding one unit ($\nu > 2$). The highest such candidate gives $L(1, 1, y)$. It is certain to exist because, as shown in the proofs, the candidate for L corresponding to $\nu = 1$ exceeds $F^*(1, 1, y)$ for some range of y values lower than y_{12}^*.

tacit collusion outcome is by investing simultaneously, rather than at different times, and by doing so at a threshold y_{12}^s exceeding y_{12}^*. Indeed if one of the firms were to invest in a second capacity unit at some $y < y_{12}^*$, the latter's unique optimal continuation strategy would be to invest at y_{12}^*. This can be a *MPE* only if $y = y_{12}^p$ as shown in the analysis of the preemption *MPE* characterized above. Since simultaneous investments of one unit imply by Assumption 3 that both firms then hold more capacity than the unconstrained Cournot output, they will not acquire more than one unit. Furthermore the game is then over by Proposition 1.

Suppose that the firms could commit to invest simultaneously at some random future date or to abstain from investing forever. Given a current industry-wide shock y, the expected payoff that they could achieve in this way is, according to Lemma 1:

$$S(1, 1, y) = \frac{\pi_{11}}{r - \alpha} y + \left(\frac{y}{y_{11}^s} \right)^\beta \left(\frac{\pi_{22} - \pi_{11}}{r - \alpha} y_{11}^s - I \right). \tag{11}$$

If $\pi_{22} > \pi_{11}$, $S(1, 1, y)$ has a maximum with respect to y_{11}^s, denoted $S^*(1, 1, y)$, at y_{11}^{s*}:

$$y_{11}^{s*} = \frac{r - \alpha}{\pi_{22} - \pi_{11}} I \frac{\beta}{\beta - 1} > y_{12}^* = \frac{r - \alpha}{\pi_{22} - \pi_{12}} I \frac{\beta}{\beta - 1}, \tag{12}$$

with $\tau_{11}^{s*} = \inf\{t \geq 0 \,|\, Y_t \geq y_{11}^{s*}\}$ as the corresponding investment (stochastic) timing. If $\pi_{22} \leq \pi_{11}$, $S(1, 1, y)$ attains a maximum of $\frac{\pi_{11}}{r - \alpha} y$ by letting $y_{11}^{s*} = \infty$ (tacit collusion by inaction), in which case $\tau_{11}^{s*} = \infty$. Clearly if $L(1, 1, y)$ exceeds $S^*(1, 1, y)$ at any $y \leq y_{11}^{s*}$, tacit collusion is not an equilibrium since each firm then has an incentive to deviate and invest earlier. Hence,

Proposition 4 *(Tacit collusion with equal capacities) Under Assumptions 1 and 3, if $Y_0 \leq y_{11}^p$,*

1. *A necessary and sufficient condition for the existence of a tacit collusion MPE is $L(1, 1, y) \leq S^*(1, 1, y) \ \forall y < y_{12}^*$. If this inequality is strict for all such y, there exists a continuum of tacit collusion MPE, indexed by their joint investment triggers y_{11}^s in a range $[\underline{y}^s, y_{11}^{s*}]$, where $y_{12}^* \leq \underline{y}^s \leq y_{11}^{s*}$.*

2. *Rents are equalized in each tacit collusion MPE and exceed the preemption MPE rents; the Pareto optimal tacit collusion MPE corresponds to the joint profits max-*

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imizing investment rule under the constraint that firms invest simultaneously if they do.[22] *In this joint-profit maximization tacit collusion* MPE, *each firm invests in one capacity unit with intensity:*

$$s_1^f(1,1,y) = s_1^{-f}(1,1,y) = \begin{cases} 0 & \text{if } y \in [0, y_{11}^{s*}), \\ \\ 1 & \text{if } y \in [y_{11}^{s*}, \infty). \end{cases}$$

3. *If* $\pi_{22} > \pi_{11}$, *the Pareto optimal tacit collusion* MPE *has both firms investing when* Y_t *reaches* y_{11}^{s*}; *otherwise it is such that neither firm ever invests.*

Propositions 2 and 4 highlight the role of existing capacity in the exercise of market power. A firm that holds no capacity has no incentive to restrain output and thus tacit collusion cannot exist if one firm has zero capacity (Proposition 2).[23] Moreover, the mere existence of an incentive to tacitly collude is not enough to guarantee that tacit collusion is sustainable: firms must also follow investment strategies such that a deviation from the tacit collusion outcome would trigger a reaction leading to a new equilibrium with a lower value for the deviating firm. This "punishment" is made difficult because our assumption of a Cournot production equilibrium in any period implies that restraining output can only be achieved by postponing capacity investments in the industry. It follows in particular that the joint investment trigger in any tacit collusion equilibrium must be higher than both triggers in the *MPE* characterized in Proposition 3. Moreover, a firm becomes more vulnerable to a deviation by its competitor once the trigger value for the first investment in the preemption equilibrium has been crossed: once $y > y_{11}^p$ and until y reaches the threshold for the second investment, a deviation yields the defector a higher rent $L(\cdot)$ than the rent $F^*(\cdot)$ obtained by its competitor who would then invest optimally at y_{12}^*. Therefore, the rents $S^*(\cdot)$ under tacit collusion *MPE* must be attractive enough (Proposition 4(b)) to beat such defection at any level of y preceding y_{12}^*.

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[22]Absent that constraint, joint profits maximization would involve sequential investments. As mentioned above, such an investment sequence cannot be sustained as an MPE outcome of our duopoly model, as it would generate a strictly higher expected payoff for the first investor and would therefore be subject to preemption.

[23]In the language of contestability, this says that the level of contestability is stronger when the contesting firm is not yet active.

Proposition 4 provides a necessary and sufficient condition for tacit collusion *MPE*s to exist. This condition implies restrictions on the components of $L(1,1,y)$ and $S^*(1,1,y)$: first, the four profit values π_{ij} determined by the non-stochastic component of demand $D(\cdot)$ under Cournot competition; second, the parameters underlying real option values, that is, the value of β as determined by the discount rate r as well as the drift α and the volatility σ of the stochastic demand shock process.

Let $\widetilde{\Lambda}(\beta, I) = \{(\pi_{11}, \pi_{12}, \pi_{22}, \pi_{21}) \mid E(y; I, \beta) = S^*(1,1,y) - L(1,1,y) \geq 0 \ \forall y < y_{12}^*\}$ ($\widetilde{\Lambda}(\beta, I)$ is the set of π_{ij} quadruples for which tacit collusion equilibria exist given β and I); the following proposition states that this set is non empty, is independent of I (that is, $\widetilde{\Lambda}(\beta, I) = \Lambda(\beta)$), and is larger in industries with higher volatility, faster growth and lower cost of capital (that is, $\Lambda(\beta') \subset \Lambda(\beta)$ iff $\beta < \beta'$).

Proposition 5 *(Tacit collusion: existence) Under Assumptions 1 and 3:*

1. *There exists a set of market parameters guaranteeing the existence of tacit collusion MPE.*

2. *This set is independent of the investment cost I of a capacity unit.*

3. *It is larger, the higher demand volatility, the faster market growth, and/or the smaller the discount rate.*

As we know from the real option literature, increased volatility raises the option value of an irreversible investment under no preemption threat: the firm increases its investment threshold to reduce the probability that the stochastic process reverts to undesirable levels after the firm has invested. The flexibility to do so increases the value of the firm; the more so, the higher the volatility. Such an effect is also present here. But there is another effect of volatility: an increase in volatility raises firm values more in a tacit collusion equilibrium than in the preemptive equilibrium, thus favoring the emergence of the former. The reason comes from both timing and discounting. Tacit collusion equilibria involve higher investment thresholds (longer delays), while an increase in volatility amounts to a lower discount rate (recall that β decreases with volatility σ) because it raises the probability that a given threshold value of y be reached in any given amount of time. Although instantaneous profits are always independent of β, the discounted value of the profit flows corresponding to each equilibrium does depend on

β: the (state space) discount factors used in (11) and (10) are respectively $\left(\frac{y}{y_{11}^s}\right)^{\beta}$ and $\left(\frac{y}{y_{12}^*}\right)^{\beta}$ and since $y_{11}^s > y_{12}^*$, the former increases more than the latter when β decreases, that is, when volatility increases. To put it differently, the benefits of restraining supply through delaying investments occur in a distant future, that is, in a higher state of market development, while the benefits from deviating occur in the immediate future. Other things equal, more volatility gives relatively more weight to the former than to the latter, contrary to conventional wisdom whereby increased volatility, because it warrants a risk premium, amounts to an increase in the discount rate.

The intuition for the role of the (time) discount rate and the market growth rate is similar: a lower discount rate favors future payoffs and a larger expected growth rate raises future prospects relative to immediate ones. Hence, both favor the emergence of tacit collusion equilibria through a lower β.

3.3 Case 3: Different capacities

While we have shown that existing capacity is a necessary condition for tacit collusion between identical firms, capacity is also often said to play a role as a barrier to entry and thus can be used as a way to acquire and maintain a dominant position or a first-mover advantage. We assume now that firms differ in their initial sizes. Referring to Figure 1, we now investigate investment subgames such as the game starting at node $(k', k' + 1)$ and contrast them with sub-games such as the game starting at node (k, k) analyzed in the previous section. We showed that, with symmetric capacities, there are two possible types of MPE : the preemption equilibrium and the tacit collusion equilibrium. The former always exists, is highly competitive and involves rent equalization. The latter exists under some conditions, provides higher rents to both firms, and also involves rent equalization. We will show that some of these characteristics are modified under asymmetric capacities: initial capacity asymmetry prevents rent equalization in equilibrium and makes collusion more difficult in the sense that joint profits maximization is not compatible with a MPE.

Without loss of generality, suppose that one firm holds k', $k' \geq 1$, capacity units while the other holds $k' + 1$ units, that $k' = 1$ to facilitate exposition, and that

Assumption 4 $0 < x^c - k' \leq 2$.

The unconstrained Cournot output is then $x^c \leq 3$, either $1 < x^c \leq 2$ or $2 < x^c \leq 3$, with $\pi_{31} > \pi_{21}$, $\pi_{2\nu} > \pi_{1\nu}$, $\forall \nu$.

Consider first the case where $1 < x^c \leq 2$. The larger firm holding two units may be capacity constrained when the smaller firm holds only one unit but it will become unconstrained if the smaller firm invests in a second capacity unit. Thus, by Proposition 1, the investment game cannot be over at node $(1, 2)$. If the smaller firm invests, both firms then hold enough capacity to produce x^c and the game is over by Proposition 1. Moreover, the smaller firm benefits more from acquiring one new unit than the bigger firm does and this benefit from investing is positive at high enough levels of Y_t. Therefore, the smaller firm is the sole investor in equilibrium and the game ends when both firms hold 2 units of capacity.

Consider now the case where $2 < x^c \leq 3$. Both firms hold a lower capacity than the unconstrained short-run Cournot output so that both are initially constrained and each firm remains constrained if its opponent invests. By Proposition 1 this may be the end of the game, although not necessarily so; this possibility will be considered further below. Two alternative candidate preemption equilibria may be considered: one,where the bigger firm invests first and the smaller firm acts accordingly; another, where the roles are reversed. The corresponding values of the bigger and the smaller firm, acting as first or second investor are respectively $L(2, 1, y)$ and $F^*(2, 1, y)$ for the bigger firm, and $L(1, 2, y)$ and $F^*(1, 2, y)$ for the smaller firm.[24] When the smaller firm invests first, node $(2, 2)$ is reached and both firms remain capacity constrained, which is the situation we analyzed in subsection 4.2: both firms then hold 2 units of capacity and assumption 3 holds with $k = 2$, so that Propositions 3, 4, and 5 apply. The continuation of the game is then known and $L(2, 1, y)$ and $F^*(1, 2, y)$ can be computed.[25] If the bigger firm invests first, then it is a dominant strategy for the smaller firm to do invest at some finite future level of Y_t, since $\pi_{13} < \pi_{23} < \pi_{33}$ as the larger firm must accommodate (Cournot equilibrium). It is then straightforward to obtain $F^*(2, 1, y)$ and $L(1, 2, y)$.

We will show that, unlike the case with symmetric initial capacities, the next invest-

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[24]Explicit expressions are given in the proof of Lemma 2. As in previous cases, it is tedious but conceptually easy to check whether the first mover acquires only one, or more, new capacity units before its rival invests. We treat the case where the first mover acquires only one extra unit here.

[25]If tacit collusion MPE exist besides the preemption equilibrium, we assume that the firms reach the tacit collusion MPE that maximizes joint firm value under the constraint of simultaneous investment.

ment is undertaken by the smaller firm in any preemption equilibrium. In order to prove that result, we need the following lemma.

Lemma 2 *If $L(2,1,y) > F^*(2,1,y)$ for some $y < y_{13}^*$, then there is exactly one value $y_{12}^p \in (0, y_{13}^*)$ such that $L(2,1,y_{12}^p) = F^*(2,1,y_{12}^p)$ and $L(2,1,y) < F^*(2,1,y)$ for $y < y_{12}^p$.*

The lemma indicates that, by investing at $y = y_{12}^p$, the smaller firm leaves the bigger firm indifferent between investing immediately or waiting. Furthermore we show in the proof of the next proposition that, at $y = y_{12}^p$, the smaller firm strictly prefers investing. Also, at any other relevant level of y, the gain for the bigger firm from investing first is smaller than the gain for the smaller firm to do so. These results imply that the sole preemption equilibrium is one where the smaller firm catches up. Trivially, if the bigger firm finds unprofitable to invest, then the smaller firm can invest at its stand-alone date y_{12}^* without worrying about preemption.

Proposition 6 *(Preemption with different capacities) Under Assumptions 1 and 4,*

1. *There exists a unique preemption equilibrium, where the smaller firm invests when Y_t first reaches $\min\{y_{12}^p, y_{12}^*\}$.*

2. *In this preemption equilibrium, the smaller firm enjoys a strictly positive rent from investing first as $L(1,2,y_{12}^p) - F^*(1,2,y_{12}^p) > 0$, while the bigger firm is either indifferent between investing immediately and waiting, or prefers waiting as $L(2,1,y_{12}^p) - F^*(2,1,y_{12}^p) \leq 0$.*

3. *Once node (2,2) is reached, Proposition 3 applies, mutatis mutandis.*

In the preemption equilibrium the laggard not only catches up but also enjoys an advantage in terms of value. The reason is not because the laggard is in a better position to avoid immediate cannibalism: although the drop in revenues from existing capacity is indeed smaller for the smaller firm when industry output increases, the drop in price is the same, whichever firm invests. Thus the source of the first-mover advantage must be found in future decisions rather than current effects. If the bigger firm were investing first, the other firm could plan its own investment at its stand-alone date. Having less to loose from the cannibalism effect, it would invest earlier in the future than a bigger

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firm would. This reduces the advantage enjoyed by its bigger opponent from taking the lead.

The preemption equilibrium of Proposition 6 always exists and it is unique in the class of equilibria involving investment by both firms at different dates or investment by one firm only. As with equal capacities, there may exist another class of equilibria, tacit collusion equilibria, involving simultaneous investment or inaction by both firms. The next proposition shows that, as with equal capacities, higher volatility and faster growth make tacit collusion *MPE* more likely.

Proposition 7 *(Tacit collusion with different capacities) Under Assumptions 1 and 4,*

1. *If $\pi_{32} - \pi_{22} = 0$: no tacit collusion equilibrium exists.*

2. *If $\pi_{32} - \pi_{22} > 0$: the set of market parameters ensuring the existence of tacit collusion* MPE *becomes larger, the larger demand volatility is, the faster market growth is, and/or the smaller the discount rate is.*

3. *Joint-profits maximization is not compatible with equilibrium.*

As discussed in the case of equal initial capacities, tacit collusion involves postponing capacity investments in order to restrain output. Benefits from tacit collusion arise in a more distant future than benefits from taking the lead. Consequently the existence of a tacit collusion equilibrium rests on conditions under which the future weights relatively more, either because of significant market growth, or because of high volatility, or because of a low discount rate, as previously. However tacit collusion is less attractive when firms hold different capacities since joint profit maximization is not compatible with equilibrium: being different, firms prefer different thresholds for simultaneous investment and the smaller firm would deviate (invest earlier) from a strategy of joint investment at the joint profit maximizing threshold.
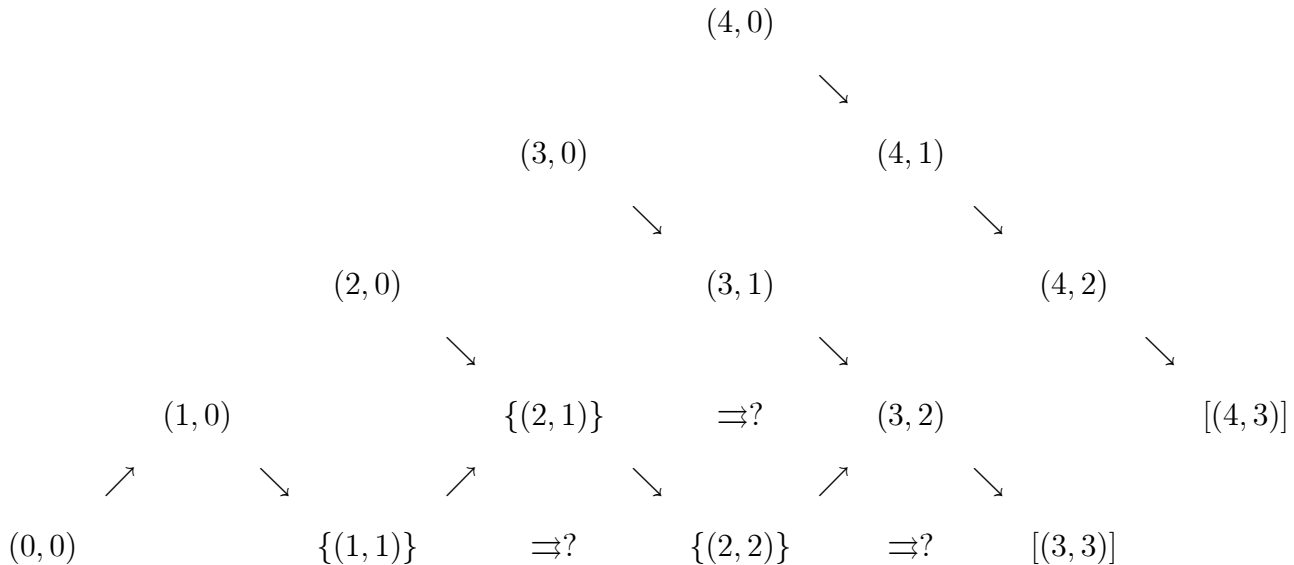
3.4 Generalization

We have considered explicitly three cases in this section: no initial capacity $(0,0)$, equal initial capacities (k,k) and different initial capacities $(k',k'+1)$, each with an assumption on the maximum market size limiting the number of possible remaining investments (in

the sense of Proposition 1): $0 < x^c \leq 1$ for the $(0,0)$ case; $0 < x^c \leq 2$ for the (k,k) case (taking $k = 1$); and $0 < x^c \leq 3$ for the $(k', k'+1)$ case (taking $k' = 1$). The three cases cannot be viewed as particular subgames of a wider game because these assumptions differ from one another. Let us now relax assumptions 2 and 3 keeping only Assumption 4 with $k' = 1$. The entire game can be solved using the above results.

This is illustrated in Figure 3 giving all possible capacity combinations if the market is such that $x^c \leq 3$ and $k^m \leq 4$. Since the game is symmetric we represent only combinations where Firm 1 is at least as big as Firm 2. Any node may be considered as initial node for a subgame. However we are interested in industry development, that is the game that starts at $(0,0)$ with y low and we will focus on equilibrium paths for that game. This eliminates all capacities in excess of the monopoly capacity.[26] Nodes that are necessarily endgame nodes, according to Proposition 1(A), and at which no firm holds more than the monopoly capacity, are represented with square brackets in the figure. Other possible endgame nodes, in the sense of Proposition 1(B), are denoted with curly brackets; they correspond to tacit collusion situations, in the sense of propositions 4, 5, and 7. Equilibrium steps are indicated by single arrows in case of single moves (preemption or stand alone) or double arrows in case of simultaneous moves (collusion). A question mark next to an equilibrium step indicates the corresponding step is not necessarily an equilibrium.

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[26]For a linear inverse demand curve $P = \left(1 - \frac{X_1 + X_2}{8}\right) Y_t$, $x^c = k^c = 3$ and the maximum monopoly capacity is $k^m = 4$.

$$(4,0)$$

$$\searrow$$

$$(3,0) \qquad\qquad (4,1)$$

$$\searrow \qquad\qquad\qquad \searrow$$

$$(2,0) \qquad\qquad (3,1) \qquad\qquad (4,2)$$

$$\searrow \qquad\qquad \searrow \qquad\qquad \searrow$$

$$(1,0) \qquad\quad \{(2,1)\} \;\; \Rightarrow? \quad (3,2) \qquad\qquad\quad [(4,3)]$$

$$\nearrow \quad \searrow \qquad \nearrow \quad \searrow \qquad \nearrow \quad \searrow$$

$$(0,0) \qquad\quad \{(1,1)\} \;\; \Rightarrow? \quad \{(2,2)\} \;\; \Rightarrow? \quad [(3,3)]$$

$\{(.,.)\}$: potential endgame; $\Rightarrow?$: potential tacit collusion *MPE* branch;

$[(.,.)]$: endgame (if reached); $\nearrow$: stand alone or preemption *MPE* branch.

Figure 3: Complete industry development game when $x^c = 3$

The subgame starting at node $(2,2)$ satisfies Assumption 3 for $k = 2$. It admits a preemption *MPE* described in Proposition 3 leading to an end at $[3,3]$. As described in Propositions 4 and 5 and denoted by the double arrow with a question mark, the subgame starting at $(2,2)$ may also have tacit-collusion *MPE*'s. In that case the end is either at $(2,2)$ or at $(3,3)$ and the corresponding firm values are equalized, but higher than in the preemption *MPE*.

Trivially subgames starting at $(3,0)$, $(3,1)$, or $(3,2)$ end up at $[3,3]$ as the bigger firm is then either passive in which instance the smaller firm invests at its stand-alone thresholds, or is preempted by the smaller firm in *MPE*.[27] Similarly subgames starting at $(4,0)$ or $(4,1)$ end up at $[4,3]$.

---

[27]For example, consider the possible alternatives from $(3,0)$: either the bigger firm invests first, leading to $(4,0)$ and a continuation with the small firm investing at its stand-alone thresholds until $[4,3]$ is reached; or the small firm invest first, leading to $(3,1)$, $(3,2)$, and $(3,3)$, or to $(3,1)$, $(4,1)$, $(4,2)$ and $(4,3)$. Two conditions are necessary for the first alternative to be a *MPE*: first the monopoly tenure of the big firm on its fourth unit must be sufficiently long to earn back the investment cost I on the unit. Second the small firm must not invest before the bigger one. Adapting the proof of Proposition 6 where it is shown that the smaller firm invests first in preemption *MPE*, it can be shown that the first condition is violated if the second one is satisfied.

The Subgame starting at node $(2, 1)$ is studied in propositions 6 and 7. The preemption path leads to $(2, 2)$ for a possible end of game at $(2, 2)$ or continuation to $(3, 3)$, whether directly in collusion *MPE*, or via $(3, 2)$ in preemption *MPE*. There may also exist a collusion path to $(3, 2)$ and $(3, 3)$.

The subgame starting at $(1, 1)$ has not been studied for $x^c \leq 3$ but only for $x^c \leq 2$. However, now that its two possible continuations, via $(2, 1)$ or via $(2, 2)$, are known, Propositions 3, 4, and 5 may be adapted accordingly. Precisely suppose that the continuation of the game follows the preemption path $(1, 1) - (2, 1)$. When applying Lemma 1 to evaluate $L(1, 1, y)$, one must substitute for the continuation value $c(2, 1, y)$. If the next equilibrium segment is the preemption segment $(2, 1) - (2, 2)$, this is $F^*(2, 1, y)$ as given in the proof of Lemma 2; similarly the expression for $F^*(1, \nu, y)$, $\nu = 1$, given by (8) for $x^c \leq 2$ must be replaced by the expression applying when $x^c \leq 3$, as provided in the proof of Lemma 2. Alternatively, if the next equilibrium segment is the tacit collusion segment $(1, 2) - (2, 3)$, then $c(2, 1, y)$ is equal to $S(2, 1, y)$ given in the proof of Proposition 7. The qualitative results are unchanged. That is the subgame starting at $(1, 1)$ always admits a preemption *MPE* via $(2, 1)$ and $(2, 2)$; a collusion equilibrium via $(2, 2)$ may exist depending on considerations discussed in Proposition 5. In both cases the continuation is known and the game ends at $(2, 2)$ or $(3, 3)$. If a collusion *MPE* exists for the subgame starting at $(2, 1)$, Proposition 7 indicates that it does not exhibit rent equalization. Then by the standard preemption argument used repeatedly in this paper, a preemption MPE exists at node $(1, 1)$ where the first investor invests at such a threshold that firms values are equalized: $L(1, 1, y) = F^*(1, 1, y)$.

Comparing the subgames starting at $(1, 1)$ and at $(2, 2)$ we note that a preemption equilibrium always exists and collusion equilibria may exist. However the game starting at $(1, 1)$ may also involve collusion from $(2, 1)$ to $(3, 2)$ unlike the game starting at $(2, 2)$ where collusion at $(3, 2)$ is not possible. This raises the issue of multiple equilibria. We have shown however that firm values are higher under collusion than under preemption in the game starting at $(2, 2)$. Although we do not provide a formal proof, this is also likely to be the case in the subgame starting at node $(1, 1)$; Equilibria can probably be Pareto ranked. In any case, if tacit collusion from node $(2, 1)$ is a possible *MPE*, it leads to $(3, 2)$. By Proposition 1, this cannot be the end of the game; it is a dominant strategy for the smaller firm to acquire one further unit and for the bigger one to abstain so node

$(3, 3)$ is reached. By Proposition 1, this is the end of the game, with both firms holding equal capacities.

Turning to the subgames starting at $(2, 1)$ and $(1, 1)$, it can be shown that the small firm invests first from $(1, 0)$ or from $(2, 0)$.[28] Finally, considering the initial node $(0, 0)$ it is now trivial to adapt Proposition 2 replacing the continuation values corresponding to the initial Assumption 2 with the equilibrium values for the game starting at $(1, 1)$ under the new assumption $x^c \leq 3$. A unique preemption *MPE* leading to $(1, 1)$ via $(1, 0)$ exists for each possible continuation at $(1, 1)$; several *MPE* continuation may exist from that node on, all leading to equal size firms at the end of the game, as indicated in Figure 3 and further discussed in the conclusion below.

Thus the complete game starting at $(0, 0)$ with y small and under assumption $x^c \leq 3$ can be solved entirely following the procedure just described. Further generalization to higher maximum industry size would not affect the qualitative properties of the model, which we summarize in the next section.

4. CONCLUSION

We characterized the development of a stochastically growing industry where duopolists make irreversible lumpy investments in capacity units without commitments regarding their future actions and make optimal use of their flexibility to adapt to the stochastic evolution of the market. The capacity unit never becomes small relative to the market despite unbounded market development, so that there is an end to the investment game.

We found that the early phase of development is characterized by intense competition: while only one firm is active, competition is fierce as the unique equilibrium is the preemption equilibrium. This competition intensity causes the first industry investment to occur earlier than would be socially optimal.[29] This deadweight loss is inevitable as the rents of both firms must be made equal, no matter what market volatility and growth are. The empirical implication of this result is that the first entrant ends up

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[28]The proof is similar to that sketched in Footnote 27.

[29]In a setup anologous to natural monopoly, that is, if the cost of acquiring the next unit is decreasing in the number of units already held by a firm, this effect would be galvanized as the entrant would have to enter sufficiently early and waste enough resources to dissipate a monopoly rent that would be enjoyed forever. Only then would its competitor be indifferent between entering or abstaining from producing forever.

facing riskier returns and a higher probability of bankruptcy than socially optimal. This happens even if it is known that the market will develop over the long run. Intense competition destroys value in the early phase of market development as the preemption motive overwhelms the option value.

The smaller firm eventually catches up to the larger one as a firm cannot durably keep its opponent at bay by holding as many capacity units as the market can bear.[30] Moreover, tacit collusion equilibria may exist when both firms hold positive capacity; they take the form of postponed simultaneous investment by both firms. Such equilibria are more likely to emerge in highly volatile and/or faster growing industries. This effect of volatility is new to the literature. The conventional real option result that high volatility delays investment is reinforced by the fact that higher volatility may allow a switch from the preemption equilibrium to a tacit collusion equilibrium involving further delayed investment and higher firm values. Tacit collusion requires simultaneous investment by both firms. When firms are of equal size, this is compatible with joint profit maximization; but when firms differ in size, the simultaneous investment threshold that maximizes joint profits is beyond the level that maximizes the expected value of the smaller firm. Hence tacit collusion is more attractive for firms of equal size.[31] Traditional measures of competition may be deceiving: competition is more intense when one single firm is active, as preemption is then the sole equilibrium, while tacit collusion is more likely when firms are both active, of equal size, and the market develops quickly, with much volatility, under low interest rates or cost of capital.

---

[30]Possible sources of first-mover advantage or rationale for a dominant position have been considered repeatedly in the literature. In Stiglitz and Dasgupta (1988), the fact that contesting a dominant firm is costly secures the latter's dominant position. Although investment is costly in our model, this argument does not apply because the market develops, so that competition is not only over current sales but also over the next capacity investment. No firm enjoys any cost advantage over that investment. In dynamic situations - patent races or investment games - the issue has often been whether an exogenous advantage in terms of timing could generate rents. In Gilbert and Harris (1984) this does not prevent rent dissipation. In Mills (1988), the exogenous ability to move first can be used to make a costly preliminary investment which works as a threat that keeps the rival at bay and thus generates rents for the first mover. Similarly, in patent race games, Fudenberg *et al.* (1983) and Harris and Vickers (1985) have established that when a firm exogenously gets an arbitrarily small head start, there is a unique perfect Nash equilibrium in which the firm with the head start surely wins. In the present paper, differences between firms can only result from past capacity investments. It is rather remarkable that being big is not like enjoying a head start in a race; quite the contrary, being big makes the threat of early subsequent investment less credible, which implies that, in a preemption equilibrium with firms of different sizes, it is the smaller firm that moves and invests first.

[31]This further suggests that explicit coordination, such as alliances, acquisitions and mergers, may be more valuable (more attractive), the more unequal the firm sizes are.

APPENDIX: PROOFS

Proof of Lemma 1. Let $Y_t = y$. The value of a firm at date t is the expected present value of its profits over the periods between investments by either firms, minus the present value cost of the investments made by the firm. In the case of a firm of capacity i that invests immediately, at t, while its opponent holds j units and does not make any investment at t,

$$
\begin{aligned}
L\left(i, j, y\right) &= E^y\left\{\int_t^{\tau_{kj}} e^{-rs}\pi_{kj}Y_s ds + e^{-r\tau_{kj}}\left[c\left(k, j, Y_{\tau_{kj}}\right)\right]\right\} - I \\
&= E^y\left\{\int_t^\infty e^{-rs}\pi_{kj}Y_s ds - \int_{\tau_{kj}}^\infty e^{-rs}\pi_{kj}Y_s ds + e^{-r\tau_{kj}}\left[c\left(k, j, Y_{\tau_{kj}}\right)\right]\right\} - I,
\end{aligned}
$$

where τ_{kj} is the random time, possibly infinite, at which some further investment occurs. The profit flow $\pi_{kj}Y_s$ replaces $\pi_{ij}Y_s$ at t. If it is altered by some new investment by either firm later on, at τ_{kj}, the continuation function $c\left(k, j, Y_{\tau_{kj}}\right)$ accounts for the new state.

The time homogeneity of $(Y_t)_{t\geq 0}$ and the strong Markov property for diffusions imply that, for all $y \geq 0$,

$$
L\left(i, j, y\right) = \frac{\pi_{kj}}{r - \alpha}y - I + E^y\left\{e^{-r\tau_{kj}}\left[c\left(k, j, Y_{\tau_{kj}}\right) - \frac{\pi_{kj}}{r - \alpha}Y_{\tau_{kj}}\right]\right\}.
$$

We are interested in stopping regions of the form $[y_{kj}, \infty)$. For any $y_{kj} > 0$, let $\tau\left(y_{kj}\right) = \inf\{t > 0 \mid Y_t \geq y_{kj}\}$, so that $Y_{\tau\left(y_{kj}\right)} = y_{kj}$ $P - a.s.$; then $L\left(i, j, y\right)$ may be rewritten as:

$$
L\left(i, j, y\right) = \frac{\pi_{kj}}{r - \alpha}y - I + E^y\left\{e^{-r\tau(y_{kj})}\right\}\left[c\left(k, j, y_{kj}\right) - \frac{\pi_{kj}}{r - \alpha}y_{kj}\right]. \tag{A.1}
$$

Following Harrison (1985, chapter 3), the Laplace transform $E^y\left\{e^{-r\tau(y_{kj})}\right\}$ is $\left(\frac{y}{y_{kj}}\right)^\beta$ for any $y \in [0, y_{kj})$. Substituting into (A.1) yields the formula for $L\left(i, j, y\right)$ given in the Proposition. The other expressions are obtained in a similar way. ∎

Proof of Proposition 1.

A strictly positive capacity is necessary. Suppose one firm has zero capacity. Then its profit is zero. If it buys one unit, the lowest instantaneous profit it can make at any time after making that investment is $Y_t\pi_{1k}$, where k is the capacity at which its opponent is unconstrained in the short run in response to an output of one: this corresponds to the worst-case scenario where its opponent holds the capacity which leaves the firm the lowest instantaneous profit and the firm does not acquire any further units even if it is profitable for it to do so. The maximized expected discounted present value from buying one capacity unit at some future time τ is, in that worst-case scenario,

31

$V\left(0,k,y\right)=\sup_{\tau}E^{y}\left\{\int_{\tau}^{\infty}e^{-rt}Y_{t}\pi_{1k}dt-e^{-r\tau}I\right\}$. Using the approach of Lemma 1 to evaluate V leads to $V(0,k,y)=\sup_{y_{0k}}\left(\frac{y}{y_{0k}}\right)^{\beta}\left(\frac{\pi_{1k}}{r-\alpha}y_{0k}-I\right)$. The value of y_{0k} that solves the maximization is $y_{0k}^{*}=\frac{I(r-\alpha)}{\pi_{1k}}\frac{\beta}{\beta-1}$ so that $V(0,k,y)>0$. Thus the strategy of never buying in the future is strictly dominated for the firm whose capacity is zero. In consequence both firms will eventually hold strictly positive capacity.

Either A or B is necessary. Assume that neither A nor B holds, that is: let l and k be the respective capacities; let l be such that the corresponding firm is capacity constrained and let k be such that the firm that holds k units is not constrained if the other firm has a capacity of $l+1$ or more units. If the first firm increases its capacity to $l+1=n$ its current instantaneous profit increases to $Y_{t}\pi_{nk}>Y_{t}\pi_{lk}$ and stays at that level forever since the opponent, not being capacity constrained, has no alternative but to accommodate by reducing output. The maximized gain in expected discounted present value from bringing capacity to n at some future time τ is $V(l,k,y)=\sup_{y_{lk}}\left(\frac{y}{y_{lk}}\right)^{\beta}\left(\frac{\pi_{nk}-\pi_{lk}}{r-\alpha}y_{lk}-I\right)$. This is positive, implying that a strategy of never investing in a situation where one firm is constrained, while the other is unconstrained or would become unconstrained after a unit investment by its opponent, is strictly dominated.

Condition A is sufficient. If neither capacity constraint is binding, no firm can increase profit by further investing so that the game is necessarily over. ∎

Proof of Proposition 2. #1 and #2. As shown in the main text, if a firm invests the first time Y_{t} reaches y from below while the other firm waits, its payoff is $L\left(0,0,y\right)$ as given by (5) and the payoff of its opponent is $F^{*}(0,0,y)$ given by (3). If both firms invest simultaneously at $Y_{t}=y$, taking $y_{00}^{s}=y$ in Lemma 1, their payoff is, $\widetilde{S}\left(0,0,y\right)=\frac{\pi_{11}}{r-\alpha}y-I$. Let:[32]

$$s_{1}^{f}\left(0,0,y\right)=s_{1}^{-f}\left(0,0,y\right)=\begin{cases}0,\text{ if }y\in[0,y_{00}^{p})\\[4pt]\frac{L(0,0,y)-F^{*}(0,0,y)}{L(0,0,y)-\widetilde{S}(0,0,y)},\text{ if }y\in[y_{00}^{p},y_{01}^{*})\\[4pt]1,\text{ if }y\in[y_{01}^{*},\infty)\end{cases}.$$

$$s_{1}^{f}\left(0,\nu,y\right)=s_{1}^{-f}\left(0,\nu,y\right)=\begin{cases}0,\text{ if }y\in[0,y_{01}^{*}),\ \nu\geq1\\[4pt]1,\text{ if }y\in[y_{01}^{*},\infty),\ \nu\geq1\end{cases}$$

$$s_{1}^{f}\left(\nu,\nu',y\right)=s_{1}^{-f}\left(\nu,\nu',y\right)=0\ \forall y;\ \nu,\nu'\geq1$$

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[32]If the first investor can increase its rent by investing in a second unit, that is if $\Delta L\left(1,0,y\right)=\frac{\pi_{20}-\pi_{10}}{r-\alpha}y-I-\left(\frac{y}{y_{01}^{*}}\right)^{\beta}\frac{\pi_{20}-\pi_{10}}{r-\alpha}y$ is positive on some interval $\left[\underline{y_{10}},\overline{y_{10}}\right]$ then the MPE strategy profile must also specify $s_{1}^{f}\left(1,0,y\right)=\begin{cases}1,\ \underline{y_{10}}\leq y<\overline{y_{10}}\\0,\text{ otherwise}\end{cases}$ where $\underline{y_{10}}\leq\overline{y_{10}}<y_{01}^{*}$. It is tedious, but not difficult, to also work out the corresponding value of y_{00}^{p}, which is lower since the rent of the first investor would otherwise exceed that of its opponent. We leave it to interested readers to adapt the foregoing proof to such cases where it might be profitable for the first investor to invest more than once before its opponent does.

where $s_1^f(i, j, y)$ is a probability distribution satisfying the detailed definition given in Boyer *et al.* (2004, Appendix A). It can be interpreted as the intensity with which firm f invests in one unit of capacity in state (i, j, y), *i.e.* when it holds i capacity units, its opponent holds j units, and $Y_t = y$. We have shown already that, on $[0, y_{00}^p)$, it is a dominant strategy not to invest, and on (y_{01}^*, ∞), it is dominant for a firm with zero capacity to invest if the other holds one unit. The above strategy combination implies that an investment is sure to occur the instant Y_t reaches y_{01}^* because then $s_1^f(0, 0, y)$ and $s_1^{-f}(0, 0, y)$ start increasing, while no simultaneous investment can occur at y_{00}^p because $s_1^f(0, 0, y_{00}^p)$ and $s_1^{-f}(0, 0, y_{00}^p)$ are still zero. Once one firm has invested, the other one abstains from investing $\left(s_1^{-f}(0, \nu, y) = 0\right)$ until Y_t reaches y_{01}^*.

We now show that the above strategy profile is an *MPE* strategy profile in any subgame starting at $y \in [y_{00}^p, y_{01}^*)$. For $y \in [y_{00}^p, y_{01}^*)$, if firm f deviates by choosing $s'(0, 0, y) = 0$, the other firm invests at y so that firm f's dominant strategy in the continuation is to invest at y_{01}^* for a continuation payoff of $F^*(0, 0, y)$. If it chooses to deviate with intensity $s'(0, 0, y) = \lambda \in (0, 1]$, its continuation payoff is:

$$\frac{\lambda\left[1 - s_1^{-f}(0, 0, y)\right] L(0, 0, y) + (1 - \lambda) s_1^{-f}(0, 0, y) F^*(0, 0, y) + \lambda s_1^{-f}(0, 0, y) \widetilde{S}(0, 0, y)}{\lambda - \lambda s_1^{-f}(0, 0, y) + s_1^{-f}(0, 0, y)}.$$

Substituting for $s_1^{-f}(0, 0, y)$, this is equal to $F^*(0, 0, y)$. Thus, for any subgame starting at $y \in (y_{00}^p, y_{01}^*)$, both firms are indifferent between all possible choices. At $y = y_{00}^p$ the continuation payoff from the candidate *MPE* strategies is $F^*(0, 0, y_{00}^p) = L(0, 0, y_{00}^p)$ as for all possible alternatives. Last, the right partial derivative $\partial_y^+ s_1^f(0, 0, y'')$ is strictly positive as required by regularity condition (R_2) in Boyer *et al.*. For the proof that there is no other equilibrium outcome, we refer the reader to Fudenberg and Tirole (1985, Appendix 1). ■

Proof of Proposition 3. For each $y \in (0, y_{12}^*]$, $F^*(1, 1, y)$, $L(1, 1, y)$, and $\widetilde{S}(1, 1, y) = \frac{\pi_{22}}{r - \alpha} y - I$ are respectively the expected payoffs of becoming the first investor, the second investor, and of investing immediately, simultaneously with the other firm. As in the proof of Proposition 1, it can be shown that the strategy profile defined below is an *MPE* strategy profile:

$$s_1^f(1, 1, y) = s_1^{-f}(1, 1, y) = \begin{cases} 0, & y \in [0, y_{11}^p) \\ \frac{L(1,1,y) - F^*(1,1,y)}{L(1,1,y) - \widetilde{S}(1,1,y)}, & y \in [y_{11}^p, y_{12}^*) \\ 1, & y \in [y_{12}^*, \infty) \end{cases}.$$

$$s_1^f(1, 2, y) = s_1^{-f}(1, 2, y) = \begin{cases} 0, & y \in [0, y_{12}^*) \\ 1, & y \in [y_{12}^*, \infty) \end{cases}$$

$$s_1^f(2, 2, y) = s_1^{-f}(2, 2, y) = 0 \,\forall y$$

■

Proof of Proposition 4. #1 Let $L(1,1,y) \le S^*(1,1,y) \ \forall y \in (0, y_{12}^*]$. By the definition of $S^*(1,1,y)$, one has $L(1,1,y) \le S^*(1,1,y) \ \forall y \in (0, y_{11}^{s*}]$. We will show that the following (tacit collusion) strategies, whose equilibrium payoff is $S^*(1,1,y)$ for both firms, yield a *MPE*:

$$s_1^f(1,1,y) = s_1^{-f}(1,1,y) = \begin{cases} 0, & y \in [0, y_{11}^{s*}) \\ 1, & y \in [y_{11}^{s*}, \infty) \end{cases},$$

$$s_1^f(1,2,y) = s_1^{-f}(1,2,y) = \begin{cases} 0, & y \in [0, y_{12}^*) \\ 1, & y \in [y_{12}^*, \infty) \end{cases}.$$

For either firm, say f, a deviation from $s_1^f(1,1,y)$ either results in an investment after y_{11}^{s*} is reached, or in an investment before y_{11}^{s*} is reached. In the former instance, since $-f$ has already invested when f invests, the payoff is $F(1,2,y) < F^*(1,2,y) \le S^*(1,1,y)$ where the last inequality follows from the fact that $y_{11}^s = y_{12}^*$ is admissible in the maximization that defines $S^*(1,1,y)$. If the deviation results in an investment by f before y_{11}^{s*} is reached, then $-f$ applies $s_1^{-f}(1,2,y)$. The payoff to f is $L(1,1,y)$ if the deviation occurs before y_{12}^* is reached and $S(1,1,y)$ if it occurs at or after y_{12}^* (since in that case $-f$ invests immediately). Since $S(1,1,y) \le S^*(1,1,y)$, the above strategies yield a *MPE* with joint investment at y_{11}^{s*}.

Now we show necessity, *i.e.* that no equilibrium exists if $L(1,1,y) \le S^*(1,1,y)$ is violated. First, consider joint investment at y_{11}^s with payoff $\tilde{S}(1,1,y) \le S^*(1,1,y)$. Clearly the above strategy adjusted for joint investment at y_{11}^s rather than y_{11}^{s*} yields a *MPE* if $L(1,1,y) \le \tilde{S}(1,1,y) \ \forall y \in (0, y_{11}^s]$; but $L(1,1,y) \le \tilde{S}(1,1,y) \ \forall y \in (0, y_{11}^s]$ implies $L(1,1,y) \le S^*(1,1,y) \ \forall y \in (0, y_{12}^*]$, a contradiction. Second consider any situation with $L(1,1,y) > S^*(1,1,y)$ for some $y \in (0, y_{12}^*]$; this implies $L(1,1,y) > \tilde{S}(1,1,y)$ for any joint investment threshold other than y_{11}^{s*}; then deviation at y is preferable for any candidate joint investment threshold. This completes the proof of existence.

With respect to the existence of a continuum of tacit collusion *MPE*s, suppose now that $S(1,1,y) > L(2,1,y)$ for each $y < y_{12}^*$, and define $\underline{y}^s$ to be smallest value of $y_{11}^s \in [y_{12}^*, y_{11}^{s*})$ such that:

$$\frac{\pi_{11}}{r - \alpha} y + \left(\frac{y}{y_{11}^s} \right)^\beta \left\{ \frac{\pi_{22} - \pi_{11}}{r - \alpha} y_{11}^s - I \right\} \ge L(2,1,y)$$

for all $y \in [0, y_{12}^*]$. Then, for any $y_{11}^s \in [\underline{y}^s, y_{11}^{s*}]$, one can as above construct an *MPE* such that firms invest jointly at $\tau_{11}^s = \inf\{t \ge 0 \mid Y_t \ge y_{11}^s\}$. By definition of y_{11}^{s*}, the expected payoff from jointly investing at τ_{11}^s is an increasing function of the investment trigger y_{11}^s over the range $[\underline{y}^s, y_{11}^{s*}]$. It follows that these *MPE*s are Pareto ranked, and that the Pareto optimal *MPE* corresponds to joint investment at τ_{11}^{s*}.

#2. Rents are equal and exceed $F(1,2,y)$ by the definition of S. Since the firms act simultaneously, joint profits equal $2S^*(1,1,y)$ under joint investment at τ_{11}^{s*}. Joint investment is a contraint in the definition of S.

#3. As explained in the text, when $\pi_{22} < \pi_{11}$, $y_{11}^{s*} \to \infty$; thus firms never invest. Otherwise the above strategy profile implies joint investment at y_{11}^{s*}. ∎

Proof of Proposition 5. We first prove #3. Assume that $\pi_{22} - \pi_{11} > 0$. By Proposition 4.1, a tacit collusion equilibrium exists if and only if $E(y; I, \beta)$ is positive for all $y < y_{12}^*$.[33] Thus we study the sign of:

$$E(y; I, \beta) = -\frac{\pi_{21} - \pi_{11}}{r - \alpha} y + I + K(\beta) y^\beta$$

for $y \in [0, y_{12}^*]$ where, after substitution of the expressions for y_{12}^* and y_{11}^{s*},

$$K(\beta) = \left(\frac{\beta - 1}{\beta I}\right)^{\beta - 1} \left(\beta^{-1} \left(\frac{\pi_{22} - \pi_{11}}{(r - \alpha)}\right)^\beta + \left(\frac{\pi_{12} - \pi_{22}}{r - \alpha}\right)^\beta \left(\frac{\pi_{22} - \pi_{21}}{\pi_{22} - \pi_{12}}\right)\right)$$

The function E is strictly convex, strictly decreasing in a right neighborhood of zero, and $\lim_{y \to \infty} E(y; I, \beta) = \infty$. It follows that E attains its minimum at a unique point $y_E > 0$ that is characterized by the first-order condition:

$$\beta K(\beta) y_E^{\beta - 1} = \frac{\pi_{21} - \pi_{11}}{r - \alpha}. \tag{A.2}$$

Substituting in the expression for $E(y_E)$, it follows that the minimized value of E is:

$$E^*(\beta) \equiv \min_{y \geq 0} E(y; I, \beta) = (1 - \beta) K(\beta) y_E^\beta + I = I - \frac{\beta - 1}{\beta} \frac{\pi_{21} - \pi_{11}}{r - \alpha} y_E. \tag{A.3}$$

Changes in σ affect the function E only through β; β is a function that is strictly decreasing in σ and α (increasing in r) and that goes to 1 as $\sigma \to \infty$ and as $\alpha \uparrow r$ (as $r \downarrow \alpha$), with $\sigma \geq 0$ and $r > \alpha$. By the envelope theorem, $E^{*\prime}(\beta) < 0$. It follows that if $\beta < \beta'$ and $E^*(\beta') = 0$, then $E^*(\beta) > 0$, so that $E(y; I, \beta) > 0 \; \forall y$. Consequently $\Lambda(\beta') \subset \Lambda(\beta)$. This proves #3.

We now establish conditions under which $\Lambda(\beta)$ is non empty and prove its independence on I. Using (A.2) and (A.3), the condition for $E^*(\beta) \geq 0$ can be written

---

[33]For $y_{12}^* < y \leq y_{11}^s \leq y_{11}^{s*}$, $S^*(1,1,y)$ is higher than the continuation of $L(1,1,y)$.

as

$$\frac{\pi_{21} - \pi_{11}}{\pi_{22} - \pi_{11}} \leq \left[1 + (\beta - 1) \frac{\pi_{22} - \pi_{11}}{\pi_{21} - \pi_{11}} \frac{\pi_{21} - \pi_{22}}{\pi_{22} - \pi_{12}} \left(\frac{\pi_{22} - \pi_{12}}{\pi_{22} - \pi_{11}} \right)^{\beta} \right]^{\frac{1}{\beta - 1}}$$

This is independent of I. Now we take $\beta = 2$ because this allows to define the admissible set of π_{ij} sufficiently explicitely to prove that the set is not empty. The condition $E^* (2) \geq 0$ can be written, after some manipulations, as:

$$Q(x) = -x^2 + bx + c \geq 0 . \tag{A.4}$$

where $b = \pi_{22} - \pi_{11}$, $c = (\pi_{21} - \pi_{22})(\pi_{22} - \pi_{12})$, and $x = \pi_{21} - \pi_{11}$. This quadratic expression is subject to features implied by the output competition model, first, under Assumption 1 on demand; and second, under Assumption 3 for the equal capacity Case 2 under scrutiny. These features are: $\pi_{21} > \pi_{22} > \pi_{11} > \pi_{12} > 0$ and $\pi_{21} - \pi_{11} > \pi_{22} - \pi_{12}$ where the last inequality means that the rise in profit from increasing capacity from 1 to 2 units is higher when the opponent holds 1 unit than when it holds 2 units. Taking $\pi_{21} - \pi_{22} = 1$ as normalization, the conditions of the Cournot model are equivalent to:

$$x > 1 ; \quad x > c ; \quad c > b ; \quad b > 0. \tag{A.5}$$

For values of x, b, and c satisfying conditions (A.5), $Q(x) \geq 0$ if and only x is smaller than or equal to the positive root of $Q(x)$, which is equal to $\frac{1}{2}\left(b + \sqrt{b^2 + 4c}\right)$. This is possible if and only if the positive root is greater than both c and 1, or:

$$b \geq \max\left\{1 - c,\ c - 1\right\}$$

Existence of the tacit collusion MPE when $\beta = 2$ is therefore ensured when, in addition to the regular features arising from the Cournot model and under the normalization $\pi_{21} - \pi_{22} = 1$,

$$\pi_{22} - \pi_{11} \geq \max\left\{1 - (\pi_{22} - \pi_{12}),\ (\pi_{22} - \pi_{12}) - 1\right\}$$

For example, if $\pi_{21} = 1$; $\pi_{22} = \frac{5}{6}$; $\pi_{12} = \frac{3}{6}$; $\pi_{11} = \frac{3.5}{6}$, then $\pi_{21} - \pi_{22} = \frac{1}{6}$; normalizing requires multiplying all those values by 6; then $\pi_{22} - \pi_{11} = \frac{9}{6} > \pi_{22} - \pi_{12} - 1 = 1$. ∎

Proof of Lemma 2. The values of the bigger firm and the smaller firm when their opponent invests immediately are respectively $F^*(2, 1, y)$ and $F^*(1, 2, y)$. For the bigger

firm,

$$F^* (2, 1, y) = \max \left\{ S^* (2, 2, y), \frac{\pi_{22}}{r - \alpha} y + \left(\frac{y}{y_{22}^p} \right)^\beta \left(L (2, 2, y_{22}^p) - \frac{\pi_{22}}{r - \alpha} y_{22}^p \right) \right\}$$

where $S^* (2, 2, y)$ and $L (2, 2, y)$ correspond to the tacit collusion and the preemption equilibria analyzed in Section 3.2 (for $k' = 2$), respectively given by (11) taken at the joint-profit maximizing trigger (12), and by (10). Thus, in case of tacit collusion, it is assumed that the firms reach the highest payoff equilibrium; the proof can be adapted for any other tacit-collusion equilibrium. If a tacit-collusion equilibrium does not exist at node $(2, 2)$, the maximum is trivially taken to be given by the second term.

The smaller firm remains capacity constrained until it holds three units; if it allows the bigger firm to invest first, the latter will then have to accomodate whenever the smaller firm introduces a new unit. Consequently, the smaller firm's dominant policy in that case is to acquire two units successively at its stand-alone trigger values:

$$F^* (1, 2, y) = \sup_{y_{13},\, y_{23}} \left[\frac{\pi_{13}}{r - \alpha} y + \left(\frac{y}{y_{13}} \right)^\beta \left(\frac{\pi_{23} - \pi_{13}}{r - \alpha} y_{13} - I \right) + \left(\frac{y}{y_{23}} \right)^\beta \left(\frac{\pi_{33} - \pi_{23}}{r - \alpha} y_{23} - I \right) \right].$$

Let $y_{13}^* = \frac{1}{\pi_{23} - \pi_{13}} (r - \alpha) I \frac{\beta}{\beta - 1}$, and $y_{23}^* = \frac{1}{\pi_{33} - \pi_{23}} (r - \alpha) I \frac{\beta}{\beta - 1}$ be the corresponding investment triggers.

Given the dominant policy of the smaller firm when the bigger firm invests first, the value of the latter, if it purchases its third unit at $Y_t = y$ when the small firm holds one unit, is:

$$L (2, 1, y) = \frac{\pi_{31}}{r - \alpha} y - I + \left(\frac{y}{y_{13}^*} \right)^\beta \frac{\pi_{32} - \pi_{31}}{r - \alpha} y_{13}^* + \left(\frac{y}{y_{23}^*} \right)^\beta \frac{\pi_{33} - \pi_{32}}{r - \alpha} y_{23}^*$$

If the smaller firm invests first, at $Y_t = y$,

$$L (1, 2, y) = \max \left\{ S^* (2, 2, y), \frac{\pi_{22}}{r - \alpha} y + \left(\frac{y}{y_{22}^p} \right)^\beta L (2, 2, y_{22}^p) \right\} - I.$$

For $y \leq y_{13}^*$, let $G (2, 1, y) \equiv L (2, 1, y) - F^* (2, 1, y)$ denote the gain for the bigger firm from investing at the current value y of Y_t, as opposed to allowing its opponent to

take the lead:

$$
G\left(2,1,y\right) = \frac{\pi_{31}}{r-\alpha}y - I + \left(\frac{y}{y_{13}^{*}}\right)^{\beta}\left(\frac{\pi_{32}-\pi_{31}}{r-\alpha}y_{13}^{*}\right) + \left(\frac{y}{y_{23}^{*}}\right)^{\beta}\frac{\pi_{33}-\pi_{32}}{r-\alpha}y_{23}^{*}
$$

$$
- \max\left\{S^{*}\left(2,2,y\right),\frac{\pi_{22}}{r-\alpha}y + \left(\frac{y}{y_{22}^{p}}\right)^{\beta}\left(L\left(2,2,y_{22}^{p}\right) - \frac{\pi_{22}}{r-\alpha}y_{22}^{p}\right)\right\} \quad \text{(A.6)}
$$

Considering (11), (12), and (10), $G\left(2,1,y\right)$ is concave whatever the maximum in the last term and it is increasing in a right-neighbourhood of zero; also, $G\left(2,1,0\right) = -I$. Consequently, if $G\left(2,1,0\right)$ reaches a strictly positive value for some $y < y_{13}^{*}$, then there exists at least one value of y in the interval $[0,y_{13}^{*}]$ such that $G\left(2,1,y\right) = 0$. We define y_{12}^{p} as the smallest root. ∎

Proof of Proposition 6. The case $0 < x^{c} - k' \leq 1$ $(k' = 1)$ is discussed in the main text; we focus on $1 < x^{c} - k' \leq 2$ $(k' = 1)$ in this proof.

Strategies and outcomes. By Assumption 4, if the bigger firm invests at $y \leq y_{13}^{*}$, the sole possible continuation is one where the smaller firm invests at y_{13}^{*} and then again at y_{23}^{*}. The dominant strategy for the smaller firm is then:

$$
s_{1}\left(1,3,y\right) = \begin{cases} 0, & \text{if } y \in [0,y_{13}^{*}) \\ 1, & \text{if } y \in [y_{13}^{*},\infty) \end{cases} \quad ; \quad s_{1}\left(2,3,y\right) = \begin{cases} 0, & \text{if } y \in [0,y_{23}^{*}) \\ 1, & \text{if } y \in [y_{23}^{*},\infty) \end{cases}.
$$

Alternatively, if the smaller firm invests at some $y \leq y_{13}^{*}$, the outcome is equal capacity, which is the situation analyzed in Section 3.2. Propositions 3 and 4 give the alternative equilibria for the continuation. The preemption *MPE* always exist; tacit-collusion *MPE*s exist if and only if $L\left(2,2,y\right) \leq S^{*}\left(2,2,y\right)$ $\forall y \in (0,y_{23}^{*}]$. If there is more than one continuation *MPE*, we assume that the equilibrium ensuring the highest continuation payoff is selected.

Payoffs. The gain for the bigger firm to invest immediately, if the alternative is the smaller firm taking the lead, is $G\left(2,1,y\right)$ (see above). As far as the smaller firm is concerned, two alternatives may arise. Trivially, if investing when its opponent holds one unit is a dominated strategy for the bigger firm $(G\left(2,1,y\right) \leq 0 \ \forall y \leq y_{13}^{*})$, then Result #1 holds, with the smaller firm investing at its stand-alone date, *i.e.* when y reaches y_{12}^{*} for the first time. Alternatively, if $G\left(2,1,y\right) > 0$ for some values of $y \leq y_{13}^{*}$ and if the strategy of the bigger firm is to take the lead if the smaller firm does not do so first, then the gain for the smaller firm to invest immediately is $G\left(1,2,y\right) \equiv$

$L\left(1,2,y\right)-F^{*}\left(1,2,y\right),\ y\leq y_{13}^{*}$:

$$G\left(1,2,y\right)\ =\ -I+\max\left\{S^{*}\left(2,2,y\right),\frac{\pi_{22}}{r-\alpha}y+\left(\frac{y}{y_{22}^{p}}\right)^{\beta}\left(L\left(2,2,y_{22}^{p}\right)-\frac{\pi_{22}}{r-\alpha}y_{22}^{p}\right)\right\}$$

$$-\sup_{y_{13},\,y_{23}}\left[\frac{\pi_{13}}{r-\alpha}y+\left(\frac{y}{y_{13}}\right)^{\beta}\left(\frac{\pi_{23}-\pi_{13}}{r-\alpha}y_{13}-I\right)+\left(\frac{y}{y_{23}}\right)^{\beta}\left(\frac{\pi_{33}-\pi_{23}}{r-\alpha}y_{23}-I\right)\right]\text{(A.7)}$$

Gain from investing immediately rather than waiting. Result #1 has just been established when the gain $G\left(2,1,y\right)$ for the bigger firm to invest immediately is non positive $\forall y\leq y_{13}^{*}$. We now assume $G\left(2,1,y\right)>0$ for some $y\leq y_{13}^{*}$ and compare it, as given by $\left(\text{A.6}\right)$, with $G\left(1,2,y\right)$, the gain for the smaller firm to invest immediately, as given by $\left(\text{A.7}\right)$. Thus, for $y\leq y_{13}^{*}$,

$$G\left(1,2,y\right)-G\left(2,1,y\right)=-I+\max\left\{S^{*}\left(2,2,y\right),\frac{\pi_{22}}{r-\alpha}y+\left(\frac{y}{y_{22}^{p}}\right)^{\beta}\left(L\left(2,2,y_{22}^{p}\right)-\frac{\pi_{22}}{r-\alpha}y_{22}^{p}\right)\right\}$$

$$-\frac{\pi_{13}}{r-\alpha}y-\left(\frac{y}{y_{13}^{*}}\right)^{\beta}\left(\frac{\pi_{23}-\pi_{13}}{r-\alpha}y_{13}^{*}-I\right)-\left(\frac{y}{y_{23}^{*}}\right)^{\beta}\left(\frac{\pi_{33}-\pi_{23}}{r-\alpha}y_{23}^{*}-I\right)$$

$$-\frac{\pi_{31}}{r-\alpha}y+I-\left(\frac{y}{y_{13}^{*}}\right)^{\beta}\left(\frac{\pi_{32}-\pi_{31}}{r-\alpha}y_{13}^{*}\right)-\left(\frac{y}{y_{23}^{*}}\right)^{\beta}\frac{\pi_{33}-\pi_{32}}{r-\alpha}y_{23}^{*}$$

$$+\max\left\{S^{*}\left(2,2,y\right),\frac{\pi_{22}}{r-\alpha}y+\left(\frac{y}{y_{22}^{p}}\right)^{\beta}\left(L\left(2,2,y_{22}^{p}\right)-\frac{\pi_{22}}{r-\alpha}y_{22}^{p}\right)\right\}$$

Simplifying and using the fact that $S^{*}\geq L$,

$$G\left(1,2,y\right)-G\left(2,1,y\right)\geq-\frac{\pi_{13}+\pi_{31}}{r-\alpha}y-\left(\frac{y}{y_{13}^{*}}\right)^{\beta}\left(\frac{\pi_{23}-\pi_{13}+\pi_{32}-\pi_{31}}{r-\alpha}y_{13}^{*}-I\right)$$

$$-\left(\frac{y}{y_{23}^{*}}\right)^{\beta}\left(\frac{2\pi_{33}-\pi_{23}-\pi_{32}}{r-\alpha}y_{23}^{*}-I\right)+2\left(\frac{\pi_{22}}{r-\alpha}y+\left(\frac{y}{y_{22}^{p}}\right)^{\beta}\left(L\left(2,2,y_{22}^{p}\right)-\frac{\pi_{22}}{r-\alpha}y_{22}^{p}\right)\right)$$

By definition of the preemption equilibrium at capacities $\left(2,2\right)$ the rents of both firms are equalized if one of the firms invests at $Y_{t}=y_{22}^{p}$. That is: $L\left(2,2,y_{22}^{p}\right)-\frac{\pi_{22}}{r-\alpha}y_{22}^{p}=$
$F^{*}\left(2,3,y_{22}^{p}\right)-\frac{\pi_{22}}{r-\alpha}y_{22}^{p}=\frac{\pi_{23}-\pi_{22}}{r-\alpha}y_{22}^{p}+\left(\frac{y_{22}^{p}}{y_{23}^{*}}\right)^{\beta}\left(\frac{\pi_{33}-\pi_{23}}{r-\alpha}y_{23}^{*}-I\right)$; substituting into the above

inequality and rearranging,

$$G\left(1,2,y\right) - G\left(2,1,y\right) \geq \frac{2\pi_{22} - \pi_{13} - \pi_{31}}{r - \alpha}y + \left(\frac{y}{y_{13}^*}\right)^\beta y_{13}^* \frac{\pi_{13} - \pi_{23} + \pi_{31} - \pi_{32}}{r - \alpha}$$

$$+ 2\left(\frac{y}{y_{22}^p}\right)^\beta \frac{\pi_{23} - \pi_{22}}{r - \alpha}y_{22}^p + \left(\frac{y}{y_{23}^*}\right)^\beta y_{23}^* \left(\frac{\pi_{32} - \pi_{23}}{r - \alpha}\right) + \left(\left(\frac{y}{y_{13}^*}\right)^\beta - \left(\frac{y}{y_{23}^*}\right)^\beta\right) I$$

Evaluating the π_{ij} as price times quantity, and defining p_l as the industry price when there are $l = i + j$ capacity unit in the industry (without any ambiguity as long as $i, j \leq 3$ so that firms operate at full capacity), this can be written as:

$$G\left(1,2,y\right) - G\left(2,1,y\right) \geq \left(\frac{y}{y_{13}^*}\right)^\beta y_{13}^* \frac{4p_4 - 5p_5}{r - \alpha} + \left(\frac{y}{y_{22}^p}\right)^\beta \frac{4\left(p_5 - p_4\right)}{r - \alpha}y_{22}^p$$

$$+ \left(\frac{y}{y_{23}^*}\right)^\beta y_{23}^* \frac{p_5}{r - \alpha} + \left(\left(\frac{y}{y_{13}^*}\right)^\beta - \left(\frac{y}{y_{23}^*}\right)^\beta\right) I$$

We know that $y_{23}^* > y_{13}^*$ and $y_{23}^* > y_{22}^p$ and we note that the right-hand-side in the above expression would be zero if it was true that $y_{22}^p = y_{13}^* = y_{23}^*$. In order to show that $G\left(1,2,y\right) - G\left(2,1,y\right) > 0$ $\forall y \leq y_{13}^*$, we first show that the result holds if $y_{13}^* = y_{22}^p < y_{23}^*$. Then we note that $G\left(1,2,y\right) - G\left(2,1,y\right)$ is increasing in y_{22}^p and we show that $y_{22}^p \geq y_{13}^*$. Thus let $\widetilde{G}\left(\varepsilon\right) = G\left(1,2,y\right) - G\left(2,1,y\right)$ for $y_{13}^* = y_{22}^p = \varepsilon y_{23}^*$. Then $\widetilde{G}\left(1\right) = 0$ and setting $0 < \varepsilon < 1$ amounts to setting $y_{13}^* = y_{22}^p < y_{23}^*$. Since $\frac{d\widetilde{G}(\varepsilon)}{d\varepsilon} < 0$ $\forall \varepsilon$, $0 < \varepsilon \leq 1$, it follows that $G\left(1,2,y\right) - G\left(2,1,y\right) > 0$ for $y_{13}^* = y_{22}^p < y_{23}^*$. Since $p_5 - p_4 < 0$, $G\left(1,2,y\right) - G\left(2,1,y\right)$ is increasing in y_{22}^p and thus remains strictly positive for $y_{13}^* < y_{22}^p < y_{23}^*$. In order to show that $y_{13}^* < y_{22}^p$, we assume to the contrary that $y_{22}^p = y_{13}^* = \frac{r - \alpha}{\pi_{23} - \pi_{13}}I\frac{\beta}{\beta - 1}$. Evaluating $L\left(2,2,y\right)$ and $F^*\left(2,3,y\right)$ at that value of y, it can be shown that $L\left(2,2,y_{13}^*\right) < F^*\left(2,3,y_{13}^*\right)$ while, by definition, $L\left(2,2,y_{22}^p\right) = F^*\left(2,3,y_{22}^p\right)$. Since $L - F^*$ is increasing in y over the relevant range (see Figure 2), it follows that $y_{13}^* < y_{22}^p$, which concludes the proof that $G\left(1,2,y\right) - G\left(2,1,y\right) > 0$ $\forall y < y_{13}^*$.

Thus the gain from investing immediately while its opponent waits is higher for the small firm than it is for the bigger firm at any $y < y_{13}^*$. For any y such that $G\left(2,1,y\right) \geq 0$, $G\left(1,2,y\right) > 0$ so the best response for the small firm to a strategy by the bigger firm of investing at such level of Y_t is to preempt at $y - \varepsilon$. Consequently a preemption equilibrium with the bigger firm as first investor does not exist.

Consider preemption by the smaller firm. If $G\left(2,1,y\right) > 0$ for some $y < y_{13}^*$ so that the bigger firm may invest first if the smaller one does not preempt, then, by Lemma 2, $G\left(2,1,y_{12}^p\right) = 0$. Since $G\left(1,2,y\right) - G\left(2,1,y\right) > 0$, it follows that $G\left(1,2,y_{12}^p\right) > 0$. Then the smaller firm should invest at y_{12}^p which is achieved in equilibrium for the following

strategies (note that the smaller firm invests first with probability one):

$$s_1(1,2,y) = \begin{cases} 0, & \text{if } y \in [0, y_{12}^p) \\ 1, & \text{if } y \in [y_{12}^p, \infty) \end{cases} \; ; \; s_1(2,1,y) = \begin{cases} 0, & \text{if } y \in [0, y_{12}^p) \\ \frac{L(2,1,y)-F^*(2,1,y)}{L(2,1,y)-S(2,1,y)}, & \text{if } y \in [y_{12}^p, y_{12}^*) \\ 1, & \text{if } y \in [y_{12}^*, \infty) \end{cases} .$$

The rest of the proof of #1 and #2 is a mere adaptation of the proof of Proposition 2. For the proof of uniqueness, we refer the reader to Fudenberg and Tirole (1985, Appendix 1).

#3 can be readily verified. ∎

Proof of Proposition 7. Under Assumption 4 with $k' = 1$:

#1. If $\pi_{32} - \pi_{22} = 0$, there exists no value of Y_t at which it is profitable for the bigger firm to invest if the smaller does so; thus there exists no tacit-collusion MPE with simultaneous investment. Since $\pi_{22} > \pi_{12}$, abstaining from investing is a dominated strategy for the smaller firm; thus there exists no tacit-collusion MPE by inaction.

#2. $\pi_{32} - \pi_{22} > 0$. The sole alternative to the tacit-collusion MPE, if it exists, is the preemption MPE. The proof is similar to that of Proposition 4 so we only introduce the main elements. By Proposition 6, for the bigger firm, the alternative to tacit collusion is to be passive in the preemption MPE; for the smaller firm, the alternative to tacit collusion is to be first investor in the preemption MPE. Consequently, adapting Proposition 4, collusion is an MPE if and only if $S(2,1,y) - F^*(2,1,y) \geq 0$ and $S(1,2,y) - L(1,2,y) \geq 0$ for all $y \leq y_{21}^s$ where y_{21}^s is the threshold at which both firms invest simultaneously. We compute these gains from tacit collusion.

First we evaluate $S(2,1,y)$ and $S(1,2,y)$. Since $\pi_{32} - \pi_{22} > 0$, $2 < x^c \leq 3$, so that a capacity of three units is necessary to produce the unconstrained Cournot output. In case of simultaneous investment both firms acquire one unit at some common trigger y_{21}^s to be defined. Then the bigger firm holds three units and must accomodate any increase in production up to x^c by the smaller firm. Thus it is a dominant strategy for the latter to acquire a third unit at its stand-alone threshold y_{23}^*, if $y_{21}^s \leq y_{23}^*$, or at $y_{23} = y_{21}^s$ if $y_{21}^s > y_{23}^*$. Once both firms hold three units each, the game is over by Proposition 1(A). Thus the tacit-collusion equilibrium, if it exists, involves simultaneous investment at y_{21}^s, followed, possibly immediately, by an investment by the smaller firm. The corresponding values for the bigger and the smaller firms are respectively:

$$S(2,1,y) = \frac{\pi_{21}}{r-\alpha}y + \left(\frac{y}{y_{21}^s}\right)^\beta \left(\frac{\pi_{32}-\pi_{21}}{r-\alpha}y_{21}^s - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \frac{\pi_{33}-\pi_{32}}{r-\alpha}y_{23}$$

$$S(1,2,y) = \frac{\pi_{12}}{r-\alpha}y + \left(\frac{y}{y_{21}^s}\right)^\beta \left(\frac{\pi_{23}-\pi_{12}}{r-\alpha}y_{21}^s - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \left(\frac{\pi_{33}-\pi_{23}}{r-\alpha}y_{23} - I\right)$$

where either $y_{23} = y_{23}^* > y_{21}^s$ or $y_{21}^s = y_{23} \geq y_{23}^*$.[34] Now we evaluate the gain from colluding for the bigger firm, over its alternative of letting the smaller firm invest first, using the expression established in the proof of Lemma 2 for $F^*(2,1,y)$:

$$
\begin{aligned}
GS(2,1,y) &= S(2,1,y) - F^*(2,1,y) \\
&= \frac{\pi_{21}}{r-\alpha}y + \left(\frac{y}{y_{21}^s}\right)^\beta \left(\frac{\pi_{32} - \pi_{21}}{r-\alpha}y_{21}^s - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \frac{\pi_{33} - \pi_{32}}{r-\alpha}y_{23} \\
&\quad - \max\left\{ S^*(2,2,y), \frac{\pi_{22}}{r-\alpha}y + \left(\frac{y}{y_{22}^p}\right)^\beta \left(L(2,2,y_{22}^p) - \frac{\pi_{22}}{r-\alpha}y_{22}^p\right) \right\} \quad (A.8)
\end{aligned}
$$

Similarly, for the smaller firm, the gain from colluding over the alternative of investing first is, using the expression in the proof of Lemma 2 for $L(1,2,y)$:

$$
\begin{aligned}
GS(1,2,y) &= S(1,2,y) - L(1,2,y) \\
&= \frac{\pi_{12}}{r-\alpha}y + \left(\frac{y}{y_{21}^s}\right)^\beta \left(\frac{\pi_{23} - \pi_{12}}{r-\alpha}y_{21}^s - I\right) + \left(\frac{y}{y_{23}}\right)^\beta \left(\frac{\pi_{33} - \pi_{23}}{r-\alpha}y_{23} - I\right) \\
&\quad + I - \max\left\{ S^*(2,2,y), \frac{\pi_{22}}{r-\alpha}y + \left(\frac{y}{y_{22}^p}\right)^\beta \left(L(2,2,y_{22}^p) - \frac{\pi_{22}}{r-\alpha}y_{22}^p\right) \right\} \quad (A.9)
\end{aligned}
$$

Let y_{12}^{s*} and y_{21}^{s*} be the values of y that maximize $S(1,2,y)$ and $S(2,1,y)$ respectively with respect to y_{21}^s. That is, $y_{12}^{s*} = \frac{1}{\pi_{23} - \pi_{12}}(r-\alpha)I\frac{\beta}{\beta-1}$; $y_{21}^{s*} = \frac{1}{\pi_{32} - \pi_{21}}(r-\alpha)I\frac{\beta}{\beta-1}$. Note that $y_{12}^{s*} < y_{21}^{s*}$. Consider y_{12}^{s*} and y_{21}^{s*} as possible triggers in a tacit-collusion equilibrium; since $S(1,2,y)$ is decreasing in y beyond its maximum, it is a dominant strategy for the smaller firm to invest when $y \geq y_{12}^{s*}$. Thus in MPE, $y_{12}^s \leq y_{12}^{s*}$ and simultaneous investment at y_{12}^{s*} yields a higher payoff to both firms than at $y_{12}^s < y_{12}^{s*}$. This equilibrium exists if and only if both $S(1,2,y) - L(1,2,y)$ and $S(2,1,y) - F^*(2,1,y)$ are non negative for any $y \leq y_{12}^{s*} = y_{23}$. The rest of the proof of #2, about parameter conditions, is otherwise similar to that of Proposition 5. ∎

#3. It can be verified that the value that maximizes $S(1,2,y) + S(2,1,y)$ is higher than $\min(y_{12}^{s*}, y_{21}^{s*})$. ∎

---

[34]We take the case $y_{23} = y_{21}^s$ corresponding to situations where $y_{23}^* < y_{21}^s$: the second investment of the smaller firm occurs later under tacit-collusion than the stand-alone trigger y_{23}^* would imply because the smaller firm delays its first investment beyond y_{23}^* in order to collude. The approach is identical for the alternative case and leads to the same implications.

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