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Selection of the number of factors in presence of structural instability: a Monte Carlo study [*]

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Résumé/abstract

In this paper we study the selection of the number of primitive shocks in exact and approximate factor models in the presence of structural instability. The empirical analysis shows that the estimated number of factors varies substantially across several selection methods and over the last 30 years in standard large macroeconomic and financial panels. Using Monte Carlo simulations, we suggest that the structural instability, in terms of both timevarying factor loadings and nonlinear factor representations, can alter the estimation of the number of factors and therefore provides an explanation for the empirical findings.

Mots clés/Key words: Factor model, Number of factors, Large panels, Monte Carlo simulations.

Codes JEL *:* C12, C38, C55

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1 Introduction

Following the improvement of information technology, large panels of economic and financial time series are now available. Using a large data set in econometric analysis can lead to the curse-of-dimensionality problem. One such example is the rise in degrees-of-freedom when the number of variable increases. On the other hand, choosing among variables introduces an element of arbitrariness and can lead to misspecification and misleading results (see Hansen and Richard 1987; Ludvigson and Ng 2007). A solution to this problem is to use the factor analysis where the information in hundreds of economic and financial time series can be summarized by a relatively small number of (common and latent) factors (see, among others, Chamberlain and Rothschild 1983; Connor and Korajczyk 1986, 1988, 1993; Chen Roll and Ross 1986; Stock and Watson 2002, Bernanke, Boivin and Eliasz 2005; Ludvigson and Ng 2007, 2009, 2011).

The existence of the factor model is strictly related to the number of primitive shocks in a data set (Rao 1955). The choice of the number of factors is very important. In fact, researchers can face misspecification problems when the number of factors is underestimated, or problems related to power when the number of factors is overestimated. Many methods have been proposed to estimate the number of (static and dynamic) factors (see, among others, Bai and Ng 2002, 2007; Onatski 2009, 2010; Alessi, Barigozzi and Capasso 2010; Ahn and Horenstein 2013; Hallin and Liska 2007; Amengual and Watson 2007).

The aim of this paper is to study the selection of the number of factors of different tests and information criteria in presence of structural instability. First, we conduct an extensive comparison of all the procedures using several large macroeconomic and financial panels. The empirical results shows that: i) the estimated number of factors differs substantially across the selection methods; ii) it varies a lot over time across, and within, selection methods. Several explanations are possible. The factors (often perceived as states of economy) become more or less pervasive over time such that their dimension can be harder to estimate. This could be modelled by allowing for time-varying factor loadings. Another possibility is the presence of nonlinearity between observables and latent factors. For instance, the number of states is the same through the sample but during crisis periods their effects on the economy could be nonlinear. Such misspecification could lead the procedures to overestimate the number of factors because they interpret a squared or interaction term as a new state.

In the second part of the paper, we perform many Monte Carlo simulations to suggest that the structural instability, in terms of both time-varying factor loadings and nonlinear factor representations, can alter the estimation of the number of factors and therefore explain the empirical findings above. In particular, we consider two types of irregularities: time-varying factor loadings and nonlinear factor representations. Our work is related to Bates, Plagborg-Moller, Stock and Watson (2013), BPSW hereafter, who consider the estimation of the factor space in the presence of time variation in factor loadings. They also verify the performance of the Bai and Ng(2002) criterion to successfully predict the dimension of the factor space. The second related paper is Chen, Dolado, and Gonzalo (2014), who provide a framework to test for large breaks in factor loadings[1]. They also show that the Bai and Ng (2002) information criteria are likely to overestimate the true number of factors in the presence of large breaks. Finally, Guo-Fitoussi and Darné (2014) concentrate on comparing finite sample properties among many selection rules. Our contribution to this literature consists of: i) providing empirical evidence for the time varying factor structure, in terms of the number of factors, in macroeconomic and large financial data sets; ii) assessing the performance of several selection rules in the presence of irregularities discussed above. In addition, we study the robustness of selection methods in finite and large samples, and in exact and approximate factor structures.

The results from our extensive simulation exercise show that structural instabilities, taking several forms of time-variant factor loadings and nonlinear factor representations, together with cross-sectional and time dependence of the idiosyncratic component, do alter the estimation of the number of factors across all six most popular selection methods used in the literature.

In Section 2, we present the time-varying parameters factor model framework. The selection rules considered in our analysis are shown in Section 3. The empirical part of the paper is presented in Section 4. The Monte Carlo simulation experiments are detailed in Sections 5 and 6. Additional empirical results are presented in the Appendix.

2 The factor model

In this framework, the large number of observed time series are modelled as dependent on a small number of latent factors. The factor model can be written as follows:

$$X_{i,t} = \lambda_{i,t}^{'} F_t + e_{i,t}, \quad i = 1, ..., N \quad t = 1, ..., T \tag{1}$$

where $X_{i,t}$ is the observed data, $\lambda_{i,t} \in \mathbb{R}^{\mathbf{q}}$ is the possibly time varying factor loading, F_t is a $q \times 1$ vector of latent common factors and $e_{i,t}$ is an idiosyncratic error assumed to be uncorrelated with F_t at all leads and lags.

Define $\mathbf{X_t} = (\mathrm{X}_{\mathbf{1},\mathbf{t}},..., \mathrm{X}_{N,t})'$, $\wedge_{\mathbf{t}} = (\lambda_{\mathbf{1},\mathbf{t}},..., \lambda_{N,t})'$, $\mathbf{e_t} = (\mathrm{e}_{\mathbf{1},\mathbf{t}},..., \mathrm{e}_{N,t})'$, and $\mathbf{F_t} = (\mathrm{F}_{\mathbf{1},\mathbf{t}},..., \mathrm{F}_{q,t})'$ such that the model can be written in a more compact form:

[1]See also Breitung and Eickmeier (2011) who test for the presence of structural breaks in dynamic factor models.

$$\mathbf{X_t} = \wedge_{\mathbf{t}}\mathbf{F_t} + \mathbf{e_t} \tag{2}$$

Following BPSW the structural instability may be introduced by modelling the factor loadings as follows:

$$\wedge_t = \wedge_0 + h_{NT}\zeta_t \tag{3}$$

where h_{NT} is a deterministic scalar sequence that may depend on N and T. h_{NT} sets the scale of deviation. ζ_t is a possibly random process of dimension $N \times r$. ζ_t will be modelled depending on which type of instability we want to assess. For example, ζ_t may be white noise, in which case the factor loading $\wedge_t$ will be the initial loading matrix $\wedge_0$ plus uncorrelated noise. ζ_t may also be modelled as a random walk, which gives a standard continuous time-varying parameter model. Finally, ζ_t may be a single deterministic break. Of course, if $\wedge_t$ is constant, (1) becomes standard factor model with constant parameters.

Note that we only consider the time instability in factor loadings and do not specify a time-varying VAR process for factors, unlike in Korobilis (2013) and Eickmeier, Lemke and Marcellino (2014). Our goal is not to study how impulse responses of X_t are changing over time, but to verify if the estimation of the number of factors is affected by structural instabilities in the way the observable series are linked to latent states of the economy.

3 Tests and Criteria for selecting the number of factors

We consider several selection methods that have been recently developed in approximate static linear factor model framework. In this section, they are presented briefly, the details can be found in the original references. Information criteria procedures are represented by Bai and Ng (2002) and Amengual and Watson (2007). Onatski (2010) and Ahn and Horenstein (2013) are tests based on the theory of random matrices, while Bai and Ng (2007) exploit the rank of matrices. Finally, Hallin and Liska (2007) build on spectral density representation of factor models. Some of these procedures are suited for selecting the number of static factors and others seek to determine the number of dynamic factors. In our simulation designs, we only consider the case where the number of static and dynamic factors is the same, i.e. the two representations are equivalent.

3.1 Bai and Ng (2002)

Information criteria select the number of factors which minimizes the variance explained by the idiosyncratic component. The estimated number of factor is:

$$\hat{k} = \underset{0 \le k \le r_{max}}{argmin} \left(\left[\frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(X_{i,t} - \hat{\lambda}_i^{k\prime} \hat{F}_t^k \right)^2 \right] + kp(N,T) \right), \tag{4}$$

where $\hat{\lambda}_i^k$ and $\hat{F}_t^k$ are the principal components analysis estimators of the factor loadings and factors, when the number of static factors is k. $p(N,T)$ is a penalty function that is used to avoid over-parametrization. The authors provide 16 different specification of the objective function. The most useful one that we consider in the rest of the paper, is the IC_{p2}.

3.2 Amengual and Watson (2007)

To estimate the number of dynamic factors, the Bai and Ng (2002) estimator is applied to residuals obtained by projecting the observed data onto lagged values of principal components estimates of the static factors. Assume, in addition to the observational equation (1), that F_t follow a finite VAR process:

$$F_t = \sum_{i=1}^{p} \Phi_i F_{t-i} + \epsilon_t. \tag{5}$$

Let η_t represents the vector of q common *dynamic* shocks. The innovation ϵ_t can be written as $\epsilon_t = G\eta_t$, where G is $k \times q$ with full column rank. By substitution, we have:

$$\mathbf{e_{Xt}} = \mathbf{X_t} - \sum_{\mathbf{i=1}}^{\mathbf{p}} \wedge \mathbf{\Phi_i F_{t-i}} = \wedge \mathbf{G}\eta_\mathbf{t} + \mathbf{e_t} \tag{6}$$

Hence, e_{Xt} follows a static factor model with q factors that correspond to the common shocks η_t . In practice, e_{xt} is obtained by the following calculations:

$$\hat{e}_{Xt}^A = X_t - \sum_{i=1}^{p} \hat{\wedge} \hat{\Phi}_i \hat{F}_{t-i}, \tag{7}$$

$$\hat{e}_{Xt}^B = X_t - \sum_{i=1}^{p} \hat{\Pi}_i^{ols} \hat{F}_{t-i}, \tag{8}$$

where $\hat{F}$ and $\hat{\wedge}$ denote the principal components estimators of F and $\wedge$, using the consistent estimator of k, and $\left(\hat{\Phi}_1, \hat{\Phi}_2, ..., \hat{\Phi}_p\right)$ the ordinary least square estimator of $\hat{F}_t$ onto $\left(\hat{F}_{t-1}, \hat{F}_{t-2}, ..., \hat{F}_{t-p}\right)$. On the other hand, $\left(\hat{\Pi}_1^{ols}, \hat{\Pi}_2^{ols}, ..., \hat{\Pi}_p^{ols}\right)$ are the OLS estimators from projection of X_t onto $\left(\hat{F}_{t-1}, \hat{F}_{t-2}, ..., \hat{F}_{t-p}\right)$

Finally, the Bai and Ng (2002) IC_{p2} criteria are applied on an estimate of e_{Xt} to select the number of dynamic common shocks. In our exercises, we concentrate only on static factor models so the matrix G is identity, and we will use the estimator in (7).

3.3 Onatski 2010

Onatski (2010) develops an estimator of the number of factors - in the approximated factor models - that performs well even when the idiosyncratic terms are correlated. Assume that the idiosyncratic components of the data can been written as $e = A\epsilon B$, where A and B are two largely unrestricted matrices and ϵ is an $N \times T$ matrix with i.i.d Gaussian. Both (limited)cross-sectional and temporal correlations in e are allowed. Onatski (2010) observes that any finite number of the largest idiosyncratic eigenvalues of the sample covariance matrix clusters around a single point, while all the systematic eigenvalues - the number of which equals the number of factor - diverge to infinity. The estimator then separates the diverging eigenvalues from the cluster and counts the number of separated eigenvalues - this is the estimated number of factors. Bai and Ng(2002), Hallin and Liska(2007), and Watson and Amengual (2007) made the assumption that the factor's cumulative effect on the N cross-sectional units grows proportionally to N. According to this assumption, with r static factors, r eigenvalues of the data's covariance matrix grow proportionally to N while the rest of the eigenvalues stay bounded. Onatski (2010) estimates the number of factors without making any assumption on the rate of growth of the factor's cumulative effect.

Let k be the number of factors, and λ_j the j largest eigenvalues of XX'/T, Onatski (2010) shows that for j>k, the differences $\lambda_j - \lambda_{j+1}$ converge to zero while the differences $\lambda_k - \lambda_{k+1}$ diverge to infinity. Let $\{k_{max}^n, n \in \mathbb{N}\}$ be a slowly increasing sequence of real numbers such that $(k_{max}^n/n) \to 0$ as $n \to \infty$. The family of estimators is defined as:

$$\hat{k}(\delta) = max \left\{i \leq k_{max}^n : \lambda_i - \lambda_{i+1} \geq \delta\right\}$$

where k_{max}^n is the maximum possible number of factors having a sample of size n.

3.4 Ahn and Horenstein (2013)

The idea is based on the fact that the k largest eigenvalues of the variance matrix of N response variables grow unboundedly as N increases, while the other eigenvalues remain bounded. The estimators are obtained by maximizing the ratio of two adjacent eigenvalues. The two estimators are:

$$\hat{k}_{ER} = \underset{0 \le k \le k_{max}}{argmax} \frac{\tilde{\mu}_{NT,k}}{\tilde{\mu}_{NT,k+1}}$$

and

$$\hat{k}_{GR} = \underset{0 \le k \le k_{max}}{argmax} \frac{\log\left[V(k-1)/V(k)\right]}{\log\left[V(k)/V(k+1)\right]}$$

where $V(k) = \sum_{j=k+1}^{m} \tilde{\mu}_{NT,j}$ and $\tilde{\mu}_{NT,k} := \psi_k\left[XX'/(NT)\right]$ are the k^{th} largest eigenvalues of the positive semi definite matrix $XX'/(NT)$. ER refers to the eigenvalue ratio and GR to the growth ratio.

3.5 Bai and Ng (2007)

Bai and Ng (2007) exploit the fact that if a $r \times r$ matrix Σ_u has rank q, the $k-q$ smallest eigenvalues are zero. Let

$c_1 > c_2 > ... > c_N$ be the ordered eigenvalues of Σ_u, and

$D_{1k} = \left(\frac{c_{k+1}^2}{\sum_{j=1}^{r} c_j^2}\right)^{1/2}$ and $D_{2k} = \left(\frac{\Sigma_{j=k+1}^{r} c_{k+1}^2}{\sum_{j=1}^{r} c_j^2}\right)^{1/2}$

When the true eigenvalues $c_{q+1},...c_r$ are zero, D_{1k} and D_{2k} should be zero for any $k > q$. The covariance matrix Σ_u is estimated by $\hat{\Sigma}_u = \frac{1}{T-p}\sum_{t=1}^{T} \hat{u}_t \hat{u}_t'$, where $\hat{u}_t$ are the residuals from estimation of the VAR(p) process in $\hat{F}$. The cut-off point is used to account for estimation error.

3.6 Hallin and Liska (2007)

Let $\sum_n(\theta)$, $\theta \in [-\pi, \pi]$ represent the spectral density matrices and $\lambda_{n1}(\theta), ...\lambda_{nn}(\theta)$ its eigenvalues in decreasing order of magnitude.

- If the spectral density matrices $\sum_n(\theta)$ are known, Hallin and Liska (2007) propose selecting the number of factors as:

$$\hat{q}_n = \underset{0 \le k \le q_{max}}{argmin} \left[\frac{1}{n}\sum_{j=k+1}^{n}\int_{-\pi}^{\pi} \lambda_{n,j}(\theta) d\theta + kp(n)\right]$$

where $p(n)$ is a penalty function, and q_{max} is some predetermined upper bound. In this case, $\hat{q}_n$ is deterministic because the spectral density matrices $\sum_n(\theta)$ are assumed known. Under assumptions in their paper, if the penalty is such that $\lim_{n\to\infty} p(n) = 0$ and $\lim_{n\to\infty} np(n) = \infty$, $\lim_{n\to\infty} \hat{q}_n = q$

If the spectral density matrices $\sum_n(\theta)$ are unknown, they can be estimated by the lag window estimator $\sum_n^T(\theta)$:

$$\textstyle\sum_n^T(\theta) := \frac{1}{2\pi} \displaystyle\sum_{u=-M_T}^{M_T} w(M_T^{-1}u)\Gamma_{n,u}^T e^{-iu\theta},$$

where $x \to w(x)$ is a positive even-weight function and $M_T > 0$ is a truncation parameter, $\Gamma_{n,u}^T$ is the sample cross-covariance matrix of $X_{n,t}$ and $X_{n,t-u}$ based on T information.

The estimated factor number, for a given pair n and T, are:

$$\hat{q}_{1,n}^T = \underset{0 \leq k \leq q_{max}}{argmin} \left[\frac{1}{n} \sum_{i=k+1}^{n} \frac{1}{2M_T + 1} \sum_{l=M_T}^{M_T} \lambda_{ni}^T(\theta_l) + kp(n,T) \right]$$

or

$$\hat{q}_{2,n}^T = \underset{0 \leq k \leq q_{max}}{argmin} \left[\log \left(\frac{1}{n} \sum_{i=k+1}^{n} \frac{1}{2M_T + 1} \sum_{l=M_T}^{M_T} \lambda_{ni}^T(\theta_l) \right) + kp(n,T) \right]$$

where $p(n,T)$ is a penalty function, $\theta_l := \pi l/(M_T + 1/2)$ for $l = -M_T, ..., M_T$, q_{max} is the predetermined upper bound and the eigenvalues $\lambda_{ni}^T(\theta_l)$ are those of the lag window estimator $\sum_n^T(\theta)$. Under the assumptions in Hallin and Liska (2007), the estimators $\hat{q}_{1,n}^T$ and $\hat{q}_{2,n}^T$ are consistent.

4 Empirical evidence

Despite a good performance of all selection methods in simulation experiments under regular conditions, their application to large macroeconomic and financial data sets produces mitigated results. In particular, the estimated number of factors varies significantly across the selection procedures and even within a single one[2].

In this section we compare all the procedures using a variety of macroeconomic and financial panels. A number of conditions can affect the performance of selection methods. First, the macroeconomic panel must be constructed in a way that is representative of economy: time series for different sectors of economic real activities, prices, monetary and credit aggregates, interest rates, etc. The sectoral and disaggregate data are more and more readily available, but adding many series of the same type is not recommended because it may alter the estimation of common factors, as pointed by Boivin and Ng (2006). The most used

[2]A typical example is Bai and Ng (2002) information criteria that have 16 slightly different specifications where the suggested numbers of factors can diverge substantially.

US macroeconomic panel is the one from Stock and Watson (2002). While it has been updated by a number of researchers, the core of the data set - in terms of the relative importance of sectors - is always the same. Second, all of these time series must be stationary. In some cases the solution is easy, but in others the transformation to be applied is not obvious. For example, some researchers kept interest rates and inflation rates in levels (Bernanke, Boivin and Eliasz 2005), while others considered the first difference (Stock and Watson 2005). Since these series usually represent an important part of the sample, the stationary transformations may substantially modify the correlation structure and hence alter the estimation of the number of factors. Finally, the frequency in which the time series are observed and transformed can be important. Financial indicators are often available on a daily basis while real economic activity series are observed at best monthly. If, in addition, one requires quarterly series such GDP and government spending indicators, the construction of the data set involves several temporal aggregations that are known to change the properties time series (see Lutkepohl 1984).

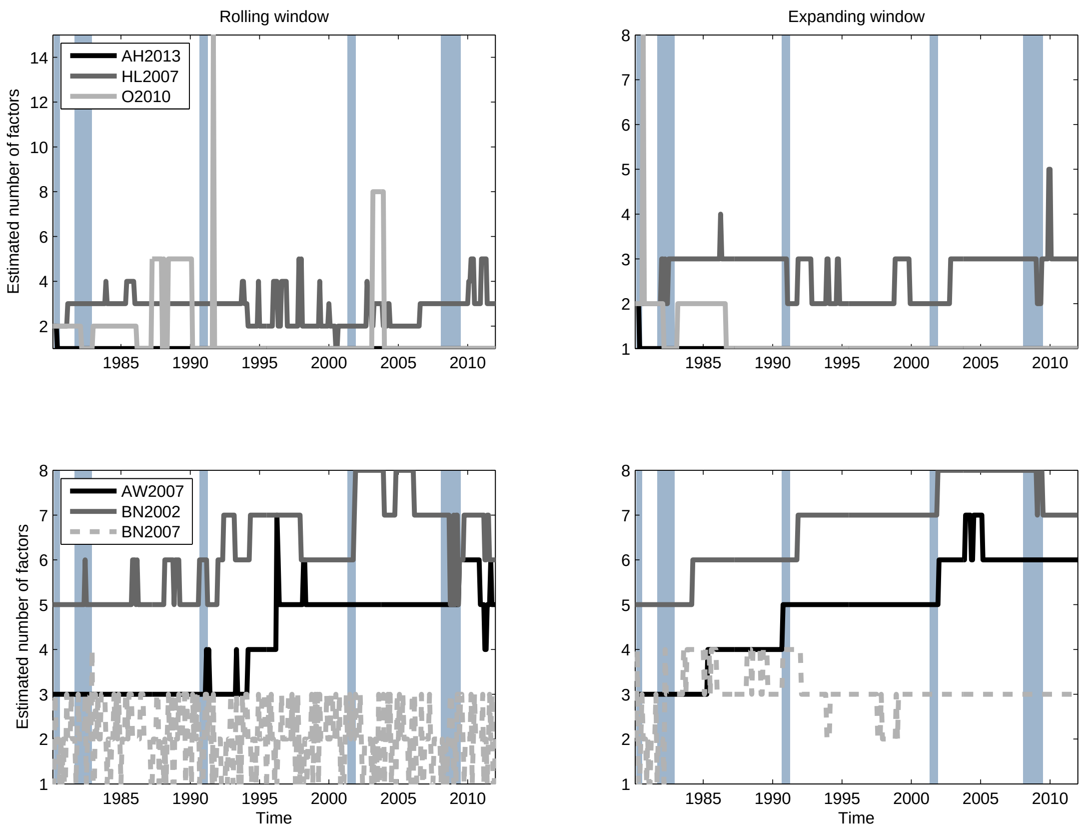
To investigate the empirical stability of results, we estimate the number of factors in several data sets and across time.

4.1 Number of factors in a large panel of macroeconomic variables

Figure 1 presents the selection of the number of factors in a large US macroeconomic panel used in Jurado, Ludvigson, and Ng (2013). Essentially, it is an updated version of the Stock-Watson data set, which consists of 132 monthly macroeconomic series spanning between 1964M01 - 2011M12. The data have been stationarized following Stock and Watson (2005): interest, unemployment rates, and inflation measures are in first-difference. We start selecting number of factors within the 1964M01 - 1979M12 sub-period and then continue until the end with rolling and expanding windows (first and second column panels, respectively). The first row panels present results for Ahn and Horenstein (2013), Hallin and Liska (2007), and Onatski (2010) procedures while the results for information criteria are presented in the second row. In the case of Bai and Ng (2002), we show the IC_{p2} criterion, which is also used in the second step of Amengual and Watson (2007).

We remark important instabilities over time and between methods. Firstly, the suggested number of factors varies significantly across the criteria - in the full sample case, at the end of the expanding window, it goes from 1 to 7. Typically, the estimates of the number of dynamic factors are smaller than those of static factors. Secondly, there exists a lot of instability over time. For example, consider the Amengual and Watson (2007) criteria in the rolling window panel. The suggested number of factors during the 80s was stable at 3, but then rose to 4 and 5 until the 2008-09 recession. A similar behavior is observed in the expanding window exercise.

Figure 1: Number of factors over time: Macroeconomic panel



This figure presents the selection of the number of factors during 1980-2012 period. The first column presents results computed for a rolling window with 192 months in size (the initial period is 1964M01 - 1979M12). The second column presents results for the expanding window where the time series size grows every period. AH2013 stands for Ahn and Horenstein (2013), HL2007 for Hallin and Liska (2007), O2010 for Onatski (2010), AW2007 for Amengual and Watson (2007), BN2002 and BN2007 for Bai and Ng (2002,2007), respectively.

Interpretation of factors

It is well known that the factors are identified up to a rotation. The estimation of F_t by principal components of X_t specifies a particular rotation matrix such that factors are orthonormal and $\Lambda'\Lambda$ is diagonal[3]. However, after the estimation, it is

[3] See Bai and Ng (2013) for more details on identification issues within principal components estimation of factor models

common practice to verify which type of variables loads on each factor. Since we have found that the number of factors is likely to change over time, it is interesting to see if their interpretation remains stable.

The interpretation of factors is formulated in terms of the marginal R^2 of each element in F_t for all series in X_t. To fix the ideas, we evaluate separately the part of the variance of each series explained by every factor . Then, we order the series by highest marginal R^2s for each factor. We start with the initial period 1964M01-1979M12 and expand the panel month by month. The results are presented in Figure 3. Consider, for example, the first north-west panel. The blue line corresponds to the highest marginal R^2 of the first factor. The five series in the text box are those that load the most on F_1, in descending order. Hence, the variation in the growth rate of the industrial production index of manufacturing industries (IP: mfg) are explained by more than 82% by the first factor during 1980, but its explanatory power decreases to 77% for the full sample. We note that the interpretation of the first factor did not change over time, it is highly related to the real sector, which includes the other series, employment and capital utilization. The explanatory power of the second factor did not change significantly: it represents the credit spread and the long term spread measures. Its determination coefficient goes from 65% during the 1980 decade to 55% at the end of the sample. The vertical lines correspond to periods where the ordering of five most-explained series by the factor has changed. In the case of F_2, these changes are minor: usually only the fifth variable is affected.

On the other hand, the interpretations of the third and the fourth factor have changed through the sample. Before 1986, F_3 was clearly related to the term structure of interest rate, but subsequently became an inflation factor. In addition, its explanatory power has risen significantly over time, especially from the year 1990. On the contrary, the fourth factor was related to prices before 1990, then became a term structure factor from 1999 onwards. The fifth factor's interpretation remained quite stable over time – it explains around 35% of the variation in short term spreads. The sixth factor exhibited an interesting behavior. It is clearly related to the stock markets, with a respectable R^2 of 30% for the S&P industrial returns until 1990. However, between 2001 and 2008, it explained almost 60% of the variation in total reserves growth - clearly making it a monetary factor. Finally, the interpretations of F_7 and F_8 have evolved a lot during the 1980-2011 period. In the case of F_7, it changes from being an exchange rates, inflation, and stock market factor to a housing market factor in 2001.

Now, let us see how the estimated number of factors relates to their interpretations. Consider, for example, the Bai-Ng (2002) criteria at the south-east panel in Figure 1. The estimated number of factors is five until 1984M02. Hence, the underlying states of the economy until 1984 were: real, credit spread, term structure of interest rates, inflation, and term spread. Then, from 1984 to 1991, the estimated K grows to six, implying the following decomposition of elements of F_t: real, credit spread, inflation and term structure, term spread, and stock market factors. The ordering is important since F_t is estimated by principal components: they are ordered by explanatory power of the total variance of X_t. Between 1992 and 2001, a seventh factor is suggested by the information criteria. The interpretation from the previous period did not change except that the seventh factor is also related to the stock market. Between 2002 and mid-2009, eight factors are needed. Now, the 6th, 7th, and 8th components correspond to monetary aggregates, housing market, and stock market, respectively. Finally, when we consider the full sample, seven factors are estimated.

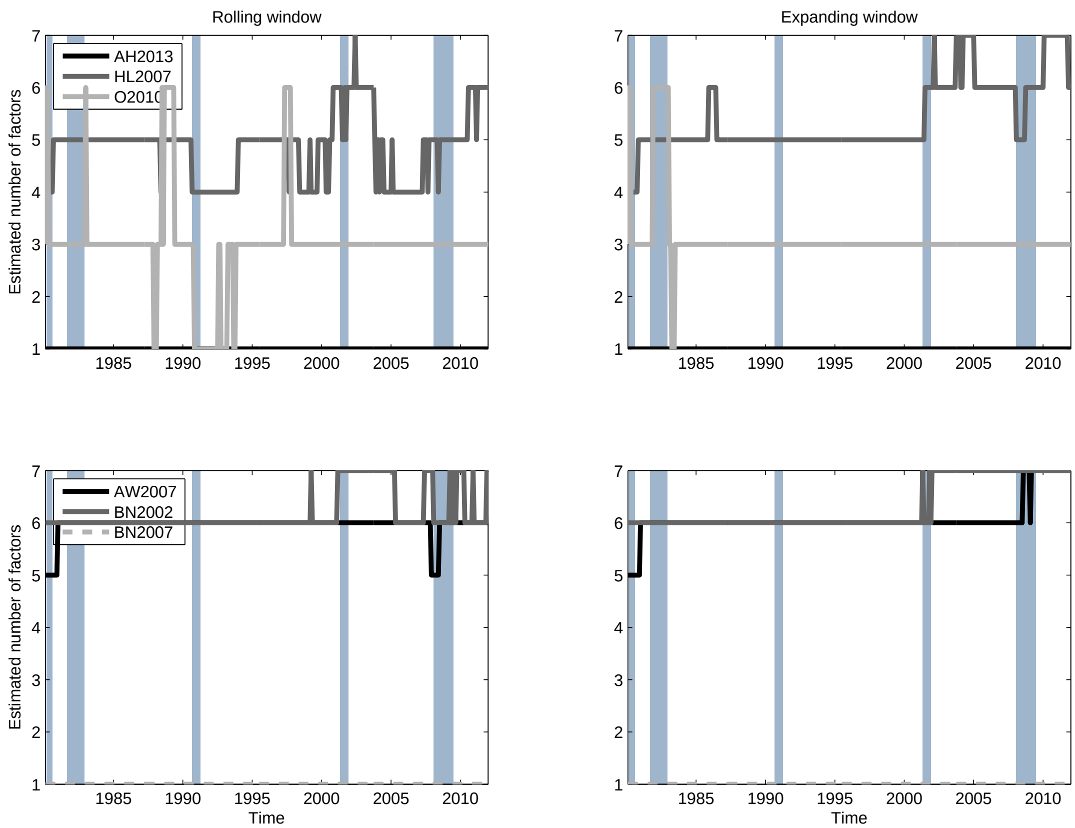
4.2 Number of factors in a large panel of financial variables

As noted by Onatski (2012), macroeconomic panels may suffer from a weak factor structure . In fact, macroeconomic aggregates and sectoral data are strongly correlated within groups but less across them. For example, inflation series are very similar amongst each other but much less correlated to employment indicators. The presence of correlation clusters may alter the strength of the *common* factor structure and hence the estimation of pervasive factors.

The factor analysis has been applied in finance to characterize the determinants of a large set of returns. In this section, we consider a large financial data set from Jurado, Ludvigson, and Ng (2013), which is an update from Ludvigson and Ng (2007). There are 147 financial market variables observed from 1960M01 to 2011M12. Figure 2 shows the selection of the number of factors over time. There is less instability in case of the information criteria on second row panels for both rolling and expanding windows, in comparison to results from the macroeconomic panel in Figure 1. Interestingly, Hallin and Liska (2007) and Onatski (2010) suggest that number of factors varies more, especially for the rolling window. The former seems to be highly unstable since the late 90s while the latter suggests between 1 and 6 factors during 1988-1998 period.

In the Appendix, we presented more examples with other US and Canadian macroeconomic panels.. Overall, using a battery of selection methods, we find robust evidence that the number of factors is changing over time. One can offer several explanations for this finding. Possibly, if the factors represent the latent states of economy, these could be more or less pervasive over time such that their number is harder to estimate. This hypothesis of structural instability can be represented by time-varying factor loadings. Hence, the number of factors is always the same but a subset of them may become more or less related to the series in X_t. Alternatively, we assume a linear factor structure. This assumption can be violated over time, especially during turbulent periods. For example, suppose the first element of F_t is a productivity factor and the second a financial factor representing the credit market conditions. There can be periods where the impact of productivity shocks on the macroeconomy depends on the level of credit market conditions: a negative productivity shock may produce even worse reactions if there is not enough liquidity in the financial system. This can be modelled by adding an interaction between $F_{1,t}$ and $F_{2,t}$ in (1) without a need for time-varying factor loadings.

Figure 2: Number of factors over time: Financial panel



This figure presents the selection of the number of factors during 1980-2012 period. The first column presents results computed for a rolling window with 240 months in size (the initial period is 1960M01 - 1979M12). The second column presents results for the expanding window where the time series size grows every period.

In the next section we will investigate if structural instability in factor loadings and the presence of nonlinearities in observational equation can alter the selection of the number of factors. Both exact and approximate factor models will be considered.

5 Monte Carlo simulation exercise I: time-varying factor loadings

The aim of this simulation is to assess the robustness of different tests and information criteria used when selecting the number of factors in a static factor model. Recall the model:

$$\mathbf{X_t} = \wedge_\mathbf{t}\mathbf{F_t} + \mathbf{e_t}$$

$$\wedge_t = \wedge_0 + h_{NT}\zeta_t$$

The focus is on instabilities of the factor loadings, $\wedge_t$; hence, one needs to impose a stochastic process for them. We consider several cases that can be summarized as follows (see BPSW 2012):

Case 1: the loading factor does not depend in time

- $q = 2$
- $(N, T) \in \{(50, 100), (100, 200)\}$
- $\wedge_t = \wedge_0, \forall_t$
- $\lambda_{i,j} \sim N(0, 1)$, $F_{j,t} \sim N(0, 1)$, $e_{i,t} \sim N(0, 1)$, $i = 1, \dots, N$, $j = 1, \dots, q$ and $t = 1, \dots, T$.

Case 2: the loading factors are random variables

- $q = 2$
- $(N, T) \in \{(50, 100), (100, 200)\}$
- $F_{j,t} \sim N(0, 1)$, $e_{i,t} \sim N(0, 1)$
- $\wedge_t = \zeta_t$
  For each $t \in \{1...T\}$ we draw $\lambda_{i,j}$, $1 \leq j \leq N$ and $1 \leq i \leq q$ from $N(0, 1)$ distribution.

Case 3: single large deterministic break with $h_{NT} = 10$

- $q = 2$
- $(N, T) \in \{(50, 100), (100, 200)\}$
- $\lambda_{i,j} \sim N(0, 1)$, $F_{j,t} \sim N(0, 1)$, $e_{i,t} \sim N(0, 1)$
- $\wedge_t = \begin{cases} \wedge_0 & for\ t = 1, ..., T/2 \\ \wedge_t = \wedge_0 + 10\wedge_0 & for\ t > T/2 \end{cases}$

Case 4: single small deterministic break with $h_{NT} = 1$

- $q = 2$
- $(N, T) \in \{(50, 100), (100, 200)\}$
- $\lambda_{i,j} \sim N(0, 1)$, $F_{j,t} \sim N(0, 1)$, $e_{i,t} \sim N(0, 1)$
- $\wedge_t = \begin{cases} \wedge_0 & for\ t = 1, ..., T/2 \\ \wedge_t = \wedge_0 + \wedge_0 & for\ t > T/2 \end{cases}$

Case 5: random walk

- $q = 2$
- $(N, T) \in \{(50, 100), (100, 200)\}$
- $F_{j,t} \sim N(0, 1)$, $e_{i,t} \sim N(0, 1)$
- $\wedge_t = \wedge_{t-1} + \zeta_t$ where $(\zeta_{i,j})_t \sim N(0, 1)$. The sequence $\wedge_t$ at $t = 0$ is initialized from $\lambda_{i,j} \sim N(0, 1)$,.

To complete the simulation exercise, we also consider several degrees of cross-sectional and time dependence among idiosyncratic components in the observational equation, e_t. In particular, we follow Boivin and Ng (2005), Onatski (2012), and Dufour and Stevanovic (2013). Assuming that

$$e_{i,t} = \rho_N e_{i-1,t} + \zeta_{i,t}$$

and

$$\zeta_{i,t} = \rho_T \zeta_{i,t-1} + \epsilon_{i,t}$$

$$\epsilon_{i,t} \sim N(0, 1).$$

Hence, the parameter ρ_N drives the degree of cross-sectional dependence while ρ_T is responsible for serial correlation among e_t. For each factor loadings cases above, following Dufour and Stevanovic (2013), we consider four correlation structures of e_t:

- *Exact factor structure*: $\rho_N = 0$ and $\rho_T = 0$.
- *Cross-sectional dependence:* $\rho_N = 0.5$ and $\rho_T = 0$.
- *Serial correlation*: $\rho_N = 0$ and $\rho_T = 0.9$
- *Cross-sectional and serial dependence* : $\rho_N = 0.5$ and $\rho_T = 0.9$

In addition, we consider two sets of panel dimensions: $N = 50,\ T = 100$ (small sample) and $N = 100,\ T = 200$ (large sample).

The Monte Carlo exercise consists of simulating 1000 times for each case, small and large samples, each correlation structure, and then applying all tests or criteria. For each selection procedure, we compute the percentage of underestimation, overestimation, and texact estimation. The mean and standard deviation of estimated number of factors are also computed.

5.1 Results and discussion

The results are summarized in four tables. Table 1 shows the simulation results in the case of exact factor structure. Table 2 presents the performance of selection methods in presence of cross-sectional dependence, while Table 3 presents results where only univariate serial correlation of e_t is considered. Lastly, Table 4 shows the behavior of selection methods in the case of the weakest factor structure implied by the presence of both cross-sectional and serial dependence.

Overall, it is most problematic when the factor loadings follow a random walk (case 5). In that case, each test and information criteria fails to capture the true number of factors in both small and large samples and in all four correlation structures of the idiosyncratic component (see Tables 1 - 4). In particular, Ahn and Horenstein (2013) and Bai and Ng (2007) systematically underestimate the number of factors, while the others largely overestimate.

In the case of classical factor structure, $\rho_N = 0$ and $\rho_T = 0$, results summarized in Table 1 show that, having a break on loading factors (cases 3 and 4) does not prevent the identification of the good number of factors in the large sample; however, this is not always the case in the small sample. For example, when there is a high break on loading factors (case 3), Hallin and Liska (2007) underestimate the number of factors at least 16% of the time, while Amengual and Watson (2007) information criteria overestimate approximately 15%. Hallin and Liska (2007) have the worst record on estimating the true number of factors in a small sample, this may be due to the fact that there is less accuracy when we try to estimate a spectral density matrix using a small sample.

However, as soon as we allow for time and/or cross-sectional dependence, the amplitude of the break increases the probability if failure to identify the true number of factors. For example, Bai and Ng (2002) have a perfect estimator when there were no dependence, but overestimate q when allowing for a break. Moreover, as the break becomes larger, the $\hat{q}$ also becomes larger (Table 2 shows that, in the large sample, when the break is 1 the estimated number of factors is three, versus eight when the amplitude of the break is 10). BPSW and Chen, Dolado, and Gonzalo (2014) find similar behavior of the Bai and Ng (2002) information criteria IC_{p2}. Another observation in Table 2 concerns Hallin and Liska (2007). In the large sample, it usually overestimates the number of factors when the magnitude of the break is 10 but performs perfectly when the break is smaller.

Strong time dependence leads many tests and information criteria to fail in identifying q even in the case of constant factor loadings. As expected, the situation is worse in small samples. However, even when the panel dimensions are larger, only Ahn and Horenstein (2013) and Bai and Ng (2007) perform well.

To summarize, the results from this extensive simulation exercise show that structural instabilities, taking several forms of time-variant factor loadings, together with cross-sectional and time dependence of the idiosyncratic component, do alter the estimation of the number of factors across all six most popular selection methods used in the literature.

Consequences

Here we discuss several consequences of the previous results for empirical analysis. Diffusion indices have been very popular in forecasting within the factor-augmented regressions. The typical framework consists of the forecasting equation for a series of interest y_t:

$$y_{t+h} = \alpha + \rho y_t + \beta F_t + \xi_{t+h}, \tag{9}$$

where a large number of potential predictors obey a factor model

$$X_t = \Lambda F_t + e_t.$$

Hence, the question is how the forecasting performance is affected in the presence of irregularities in the observational equation. Chen, Dolado, and Gonzalo (2014) show, using simulations, that imposing a priori number of factors that ignores the existence of a large break on Λ can worsen the forecasting power of the factor-augmented regressions. Overestimating the number of factors can help, but this entails more estimation uncertainty that ultimately increases the mean squared predicted errors. Barhoumi, Darné, and Ferrara (2013) compare several selection methods in the pseudo out-of-sample forecasting exercise and find that setting the number of factors with the Alessi, Barigozzi and Capasso (2010) information criterion (a modification of Bai-Ng (2002)) produces significantly lower squared prediction errors.

Our results contribute to these findings by showing that many selection methods typically overestimate the number of factors. Hence, if they are used to assess the dimension of F_t to include in (9), a similar forecasting behavior is expected to occur. More importantly, we showed that there are cases where Ahn-Horenstein (2013) and Bai-Ng (2007) tests underestimate the true number of latent common components. Obviously, this will misspecify the forecasting equation (9) as some important predictors would be omitted.

Another area of interest for factor models is the structural analysis. Since Bernanke, Boivin, and Eliasz (2005), the factor-augmented VAR (FAVAR) approach has been heavily used to identify and estimate the effects of structural shocks (monetary, news, productivity, credit, etc.) on real economy of many countries. The FAVAR model consists of the state-space representation

$$\begin{aligned} X_t &= \Lambda F_t + e_t, && (10)\\ F_t &= \Phi(L) F_{t-i} + u_t. && (11) \end{aligned}$$

where u_t are the reduced-form disturbances related to the structural shocks via $u_t = H\epsilon_t$. The objects of interest are impulse responses of X_t to the structural shocks ϵ_t

$$X_t = [I - \Phi(L)L]^{-1} H\epsilon_t. \tag{12}$$

Clearly, the misspecification, and particularly the underestimation, of the number of elements in F_t will alter both the identification of structural shocks and the estimation of the impulse responses. An extensive study on the consequences on forecasting and structural analysis goes beyond the scope of this paper, but is a part of our research agenda.

6 Monte Carlo simulation exercise II: nonlinear factor model

In this section we conduct several experiments where the data are simulated from a nonlinear factor model and then the same tests and criteria are used to detect the number of factors. The nonlinearity takes the form of different polynomials with power and/or interaction terms within elements of F_t.

Case 1: quadratic polynomial with 2 factors

- $q = 2$
- $(N, T) \in \{(100, 200), (300, 200)\}$
- The model is:
$$X_t = \Lambda_1 F_t + \Lambda_2 F_t^2 + e_t$$
- The elements of Λ_1 are drawn from $N(0, 1)$ and those of Λ_2 from $N(0, 0.25)$. Changing the variance of Λ_2 keeps the average R^2 of the model around 80%.
- $F_{j,t} \sim N(0, 1)$, $e_{i,t} \sim N(0, 1)$, $i = 1, \ldots, N$, $j = 1, \ldots, q$ and $t = 1, \ldots, T$.

Case 2: cubic polynomial with 1 factor

- $q = 1$
- $(N, T) \in \{(100, 200), (300, 200)\}$
- The model is:
$$X_t = \Lambda_1 F_t + \Lambda_2 F_t^2 + \Lambda_3 F_t^3 + e_t$$
- The elements of Λ_1 are drawn from $N(0, 1)$, those of Λ_2 from $N(0, 0.25)$ and of Λ_3 from $N(0, 0.09)$. Changing the variance of factor loadings keeps the average R^2 of the model around 80%.
- $F_t \sim N(0, 1)$, $e_{i,t} \sim N(0, 1)$.

Case 3: 3 factors plus an interaction term

- $q = 3$
- $(N, T) \in \{(100, 200), (300, 200)\}$
- The model is:
$$X_t = \Lambda_1 F_t + \Lambda_2 (F_{1,t} \times F_{2,t}) + e_t$$
- The elements of Λ_1 are drawn from $N(0, 1)$ and those of Λ_2 from $N(0, 0.25)$.
- $F_{j,t} \sim N(0, 1)$, $e_{i,t} \sim N(0, 1)$, $i = 1, \ldots, N$, $j = 1, \ldots, q$ and $t = 1, \ldots, T$.

Case 4: quadratic polynomial with 3 factors plus an interaction term

- $q = 3$
- $(N, T) \in \{(100, 200), (300, 200)\}$
- The model is:

$$X_t = \Lambda_1 F_t + \Lambda_2 F_{1,t}^2 + \Lambda_3 (F_{2,t} \times F_{3,t}) + e_t$$

- The elements of Λ_1 are drawn from $N(0, 1)$ and those of Λ_2 Λ_3 from $N(0, 0.25)$.
- $F_{j,t} \sim N(0, 1)$, $e_{i,t} \sim N(0, 1)$, $i = 1, \ldots, N$, $j = 1, \ldots, q$ and $t = 1, \ldots, T$.

This exercise is unfair for the estimation of linear factors given an observational equivalence. For example, consider Case 1. One can define $F_{3,t} \equiv F_{1,t}^2$ and $F_{4,t} \equiv F_{2,t}^2$ and rewrite the nonlinear 2-factor model as a linear 4-factor model. However, our goal is to verify if the methods used to estimate the number of primitive shocks are robust to this type of nonlinearity.

6.1 Results and discussion

The simulation results for all cases and selection methods are presented in Table 5. As expected, all the procedures overestimate the number of true primitive shocks by interpreting polynomial terms as new factors. This confirmation is useful to show that if nonlinearity becomes important over time in the true data-generating process of macroeconomic and financial panels, it can alter the determination of the true number of factors.

Note that we have changed the time and cross-section sizes compared to simulations with structural instabilities. In fact, we also did the pair $(N, T) = (50, 100)$ and found that some methods selected much smaller number of factors than when N is very large. The reason is that the signal of polynomial terms gets weaker in small N sample where some methods simply have the tendency to underestimate the number of linear factors.

7 Conclusion

The objective of this paper is to verify the robustness of most important selection methods to identify the number of factors in large data sets. Empirically, we show that, in both large macroeconomic and financial panels, the estimated number of factors varies significantly across time procedures. To provide an explanation of these findings we conduct an extensive Monte Carlo simulation exercise where the instabilities took two forms. First, we consider several time-varying processes for factor loadings in both exact and approximate factor model structures. Second, we study the implication of different nonlinear factor representations on the estimation of the number of factors.

The simulation results show that structural instabilities do alter the estimation of the number of factors across all six most popular selection methods used in the literature. Their performance is particularly affected when factor loadings behave as random walks and in the presence of cross-sectional and time dependencies across idiosyncratic components. More research is needed to explore the exact theoretical reasons for the systematic failure of these procedures. In addition, we hope this work will provide a basis for pursuing research on developing new estimators of the factor space rank in the presence of time instabilities and in the case of nonlinear factor structures.

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Table 1: MC simulations: factor loadings instabilities with exact factor structure

	(N=50 ; T=100)					(N=100 T=200)				
Ahn and Horenstein (2013)										
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 1	Case 2	Case 3	Case 4	Case 5
under	0.5	0.6	0	0	100	0	0	0	0	100
over	0	0	0	0	0	0	0	0	0	0
average	1.995	1.994	2	2	1	2	2	2	2	1
std	0.0706	0.0773	0	0	0	0	0	0	0	0
Hallin and Liska (2007)										
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 1	Case 2	Case 3	Case 4	Case 5
under	45.5	27.9	20.2	16.90	0	0	0	0	0	0
over	0	0	0.9	0	100	0	0	0	0	100
average	1.537	1.692	1.605	1.68	7.96	2	2	2	2	7.998
std	0.5147	0.521	1.81	0.71	0.2011	0	0	0	0	0.447
Onatski (2010)										
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 1	Case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	1	0.8	1.10	0.8	100	0.6	0.9	1.3	0.6	100
average	2.018	2.013	2.014	2.009	7.997	2.006	2.016	2.013	2.011	7.998
std	0.24	0.2071	0.1607	0.1045	0.0547	0.0773	0.2276	0.1133	0.2023	0.447
Bai and Ng (2007)										
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 1	Case 2	Case 3	Case 4	Case 5
under	0.3	0.3	1.3	2	100	0	0	0	0	100
over	16.5	13.1	0	0.1	0	0	0.1	0	0	0
average	2.166	2.132	1.987	1.98	1	2	2.001	2	2	1
std	0.3906	0.3588	0.11	0.14	0	0	0.0316	0	0	0
Bai and Ng (2002)										
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 1	Case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	0	0	0	0	100	0	0	0	0	100
average	2	2	2	2	8	2	2	2	2	8
std	0	0	0	0	0	0	0	0	0	0
Amengual and Watson (2007)										
	Case 1	Case 2	Case 3	Case 4	Case 5	Case 1	Case 2	Case 3	Case 4	Case 5
under	1.4	2	0	0	0	0	0	0	0	0
over	0	0	15.8	0	100	0	0	0	0	100
average	1.986	1.979	2.213	2	6	2	2	2	2	6
std	0.1175	0.1503	0.5639	0	0	0	0	0	0	0

This table presents the selection of the number of factors with factor loadings instabilities without any dependencies within idiosyncratic components. Case 1: constant factor loadings. Case 2: factor loadings are random variables. Case 3: single large deterministic break on loadings. Case 4: single large deterministic break on loadings. Case 5: each factor loading follows a random walk.

Table 2: MC simulations: factor loadings instabilities with cross-sectional dependance

	(N=50 ; T=100)					(N=100 T=200)				
	Ahn and Horenstein (2013)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0	0	100	0	0	0	0	100
over	0	0	0	0	0	0	0	0	0	0
average	2	2	2	2	1	2	2	2	2	1
std	0	0	0	0	0	0	0	0	0	0
	Hallin and Liska (2007)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	0	0	40.2	0.1	100	0	0	57.4	0	100
average	2	2	2.685	2.001	7.999	2	2	2.768	2	8
std	0	0	1.06	0.0316	0.0316	0	0	0.8372	0	0
	Onatski (2010)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	2.5	3.2	6.8	3.7	100	1.1	1.9	0.7	2	100
average	2.045	2.043	2.277	2.231	8	2.011	2.028	2.009	2.029	8
std	0.3565	0.2593	1.0571	0.7671	0	0.1044	0.2435	0.1222	0.2573	0
	Bai and Ng (2007)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	2.8	5.6	1	1.6	100	0	0	0	0	100
over	0	0	0	0	0	0	0	0	0	0
average	1.972	1.944	1.99	1.984	1	2	2	2	2	1
std	0.1651	0.23	0.0995	0.1255	0	0	0	0	0	0
	Bai and Ng (2002)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	100	100	100	100	100	0	0	100	100	100
average	3	3	8	8	8	2	2	8	3	8
std	0	0	0	0	0	0	0	0	0	0
	Amengual and Watson (2007)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	91.40	93.8	100	100	100	33.8	64.9	100	96.1	100
average	4.603	4.939	6	5.959	6	2.42	3.099	6	4.578	6
std	1.3263	1.2739	0	0.2267	0	0.6709	1.0697	0	1.1914	0

This table presents the selection of the number of factors with factor loadings instabilities with cross-sectional dependencies within idiosyncratic components. Case 1: constant factor loadings. Case 2: factor loadings are random variables. Case 3: single large deterministic break on loadings. Case 4: single large deterministic break on loadings. Case 5: each factor loading follows a random walk.

Table 3: MC simulations: factor loadings instabilities with time dependence

	(N=50 ; T=100)					(N=100 T=200)				
	Ahn and Horenstein (2013)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	98.8	85.6	0	0.1	100	1.7	1.8	0	0	100
over	1.2	13.9	100	99.80	0	98.3	98.2	100	100	0
average	1.024	1.283	3	2.999	1	2.966	2.964	3	3	1
std	0.2179	0.6938	0	0.0837	0	0.2587	0.266	0	0	0
	Hallin and Liska (2007)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0.6	0	0	0	0	0	0	0
over	100	100	99.4	100	100	100	100	100	100	100
average	3.066	3.051	3.436	3.172	7.789	3.051	3.062	3.639	3.157	7.912
std	0.3432	0.3106	0.9863	0.541	0.4782	0.3413	0.4151	1.0667	0.6187	0.3104
	Onatski (2010)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	6.6	5.3	0	0	0	0	0	0	0	0
over	90.8	92.8	100	100	100	100	100	100	100	100
average	3.262	3.259	3.472	3.373	7.906	3.0360	3.032	3.043	3.033	7.862
std	1.2661	1.1838	1.0965	1.0232	0.3021	0.3699	0.2917	0.3182	0.3162	0.348
	Bai and Ng (2007)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	15.9	12.3	5.2	15.5	100	0	0	0	0	100
over	0	0	0	0	0	0	0	0	0	0
average	1.841	1.877	1.948	1.845	1	2	2	2	2	1
std	0.3659	0.3286	0.2221	0.3621	0	0	0	0	0	0
	Bai and Ng (2002)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	100	100	100	100	100	100	100	100	100	100
average	8	8	8	8	8	8	8	8	8	8
std	0	0	0	0	0	0	0	0	0	0
	Amengual and Watson (2007)									
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	100	100	100	100	100	100	100	100	100	100
average	5.139	5.246	5.939	5.504	6	4.9710	6	6	5.383	6
std	1.0596	1.0414	0.3121	0.8711	0	1.1625	0	0	0.966	0

This table presents the selection of the number of factors with factor loadings instabilities with serial dependencies within idiosyncratic components. Case 1: constant factor loadings. Case 2: factor loadings are random variables. Case 3: single large deterministic break on loadings. Case 4: single large deterministic break on loadings. Case 5: each factor loading follows a random walk.

Table 4: MC simulations: factor loadings instabilities with time and cross-sectional dependences

	(N=50 ; T=100)					(N=100 T=200)				
Ahn and Horenstein (2013)										
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	100	100	0	99.7	100	100	100	0	27.6	100
over	0	0	100	0.3	0	0	0	100	72.4	0
average	1	1	3	1.006	1	1	1	3	2.448	1
std	0	0	0	0.1094	0	0	0	0	0.8945	0
Hallin and Liska (2007)										
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0.5	0.1	8.3	1.6	0	0	0	0	0	0
over	98.4	99.8	91.7	98.30	100	100	100	100	100	100
average	3.009	3.042	3.268	3.104	7.782	3.063	3.052	3.838	3.163	7.933
std	0.2811	0.323	1.3386	0.5844	0.4676	0.3862	0.3456	1.1752	0.5889	0.2731
Onatski (2010)										
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	24.8	15.5	0	0.2	0	0	0.2	0	0	0
over	67.5	78.9	100	99.6	100	100	99.8	100	100	100
average	2.934	3.206	3.702	3.543	7.877	3.052	3.07	3.124	3.075	7.937
std	1.5748	1.5277	1.2177	1.124	0.3376	0.3089	0.4138	0.5088	0.3867	0.2472
Bai and Ng (2007)										
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	19.3	16.4	10.7	28.7	100	0	0	0	0	100
over	0	0	0	0	0	0	0	0	0	0
average	1.807	1.836	1.893	1.713	1	2	2	2	2	1
std	0.3949	0.3705	0.3093	0.4526	0	0	0	0	0	0
Bai and Ng (2002)										
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	100	100	100	100	100	100	100	100	100	100
average	8	8	8	8	8	8	8	8	8	8
std	0	0	0	0	0	0	0	0	0	0
Amengual and Watson (2007)										
	Case 1	case 2	Case 3	Case 4	Case 5	Case 1	case 2	Case 3	Case 4	Case 5
under	0	0	0	0	0	0	0	0	0	0
over	100	100	100	100	100	100	100	100	100	100
average	5.777	5.869	5.999	5.914	6	5.405	5.983	6	5.698	6
std	0.5635	0.4336	0.0316	0.3616	0	0.9273	0.1369	0	0.6937	0

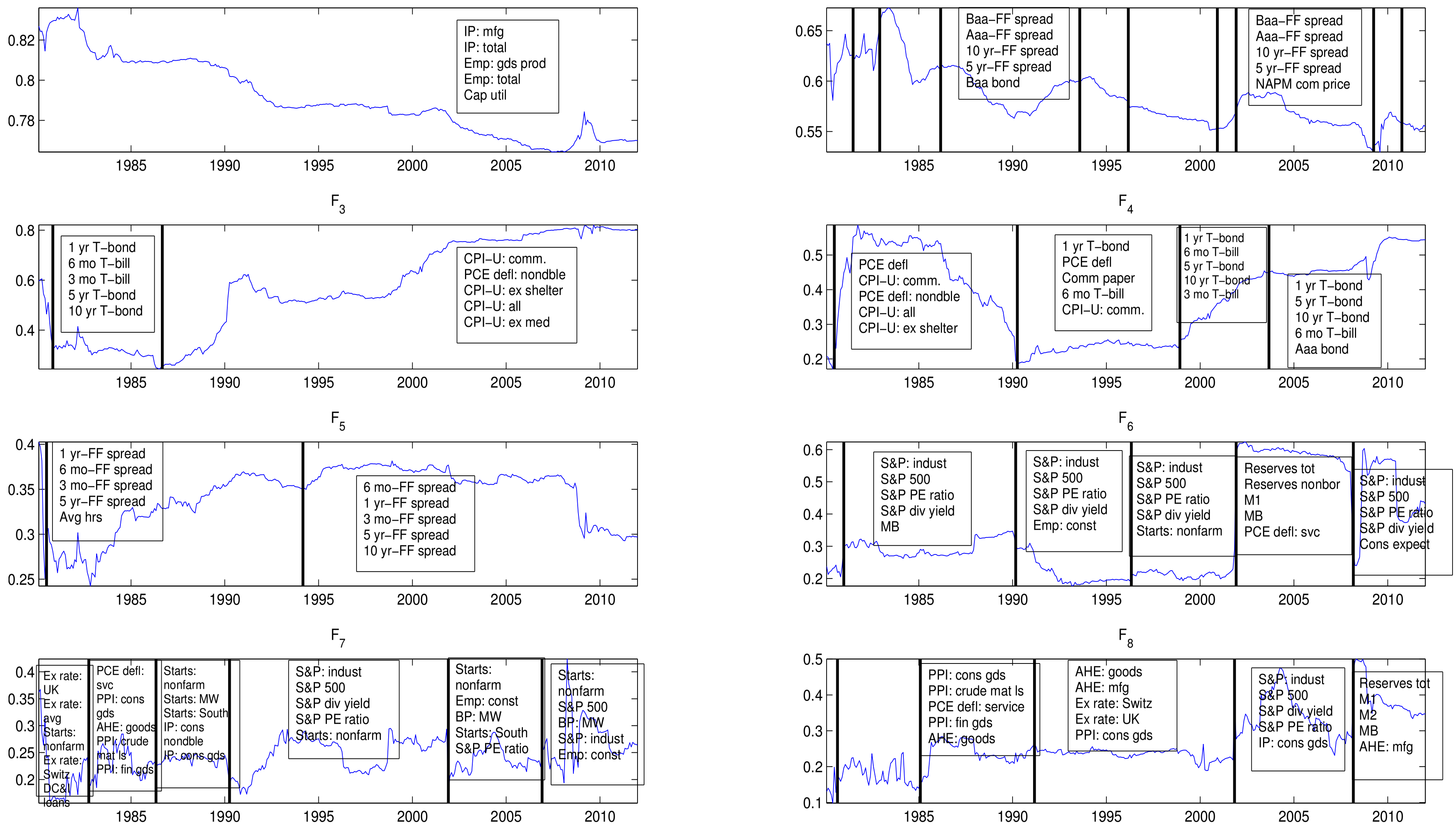
This table presents the selection of the number of factors with factor loadings instabilities with both cross-sectional and serial dependencies within idiosyncratic components. Case 1: constant factor loadings. Case 2: factor loadings are random variables. Case 3: single large deterministic break on loadings. Case 4: single large deterministic break on loadings. Case 5: each factor loading follows a random walk.

Table 5: MC simulations: nonlinear factor model

	N=100				N=300			
	Ahn and Horenstein (2013)							
	Case 1	Case 2	Case 3	Case 4	Case 1	Case 2	Case 3	Case 4
percentiles	[4 4 4]	[1 3 3]	[4 4 4]	[5 5 5]	[4 4 4]	[3 3 3]	[4 4 4]	[5 5 5]
average	4	2.88	3.98	4.99	4	2.99	4.00	5.00
std	0	0.45	0.15	0.09	0	0.03	0.00	0.00
	Hallin and Liska (2007)							
	Case 1	Case 2	Case 3	Case 4	Case 1	Case 2	Case 3	Case 4
percentiles	[4 4 4]	[3 3 3]	[4 4 4]	[5 5 5]	[4 4 4]	[3 3 3]	[4 4 4]	[5 5 5]
average	4	2.99	4.00	5.00	4	3.00	4.00	5.00
std	0	0.05	0.00	0.00	0	0.00	0.00	0.00
	Onatski (2010)							
	Case 1	Case 2	Case 3	Case 4	Case 1	Case 2	Case 3	Case 4
percentiles	[4 4 4]	[3 3 3]	[4 4 4]	[5 5 5]	[4 4 4]	[3 3 3]	[4 4 4]	[5 5 5]
average	4.02	3.01	4.02	5.03	4.01	3.01	4.01	5.02
std	0.20	0.17	0.17	0.21	0.05	0.05	0.07	0.19
	Bai and Ng (2007)							
	Case 1	Case 2	Case 3	Case 4	Case 1	Case 2	Case 3	Case 4
percentiles	[2 3 4]	[1 2 2]	[3 3 3]	[3 3 4]	[3 4 4]	[2 2 2]	[3 3 3]	[3 4 4]
average	2.98	1.62	3.00	3.20	3.83	2.00	3.03	3.81
std	0.55	0.49	0.03	0.42	0.37	0.11	0.16	0.40
	Bai and Ng (2002)							
	Case 1	Case 2	Case 3	Case 4	Case 1	Case 2	Case 3	Case 4
percentiles	[4 4 4]	[2 3 3]	[3 4 4]	[4 5 5]	[4 4 4]	[2 2 2]	[4 4 4]	[5 5 5]
average	4.00	2.69	3.94	4.93	4.00	3.00	4.00	5.00
std	0.00	0.46	0.25	0.25	0.00	0.14	0.00	0.00
	Amengual and Watson (2007)							
	Case 1	Case 2	Case 3	Case 4	Case 1	Case 2	Case 3	Case 4
percentiles	[4 4 4]	[2 3 3]	[3 4 4]	[4 5 5]	[4 4 4]	[2 2 2]	[4 4 4]	[5 5 5]
average	4.00	2.68	3.93	4.93	4.00	2.98	4.00	5.00
std	0.00	0.47	0.26	0.26	0.00	0.15	0.00	0.00

This table summarize main simulation results under nonlinear factor models as presented in Section 6. For each panel, percentile *line reports 5th, 50th and 95th percentile over 1000 simulation replications.*

Figure 3: Marginal R^2 of each factor over time

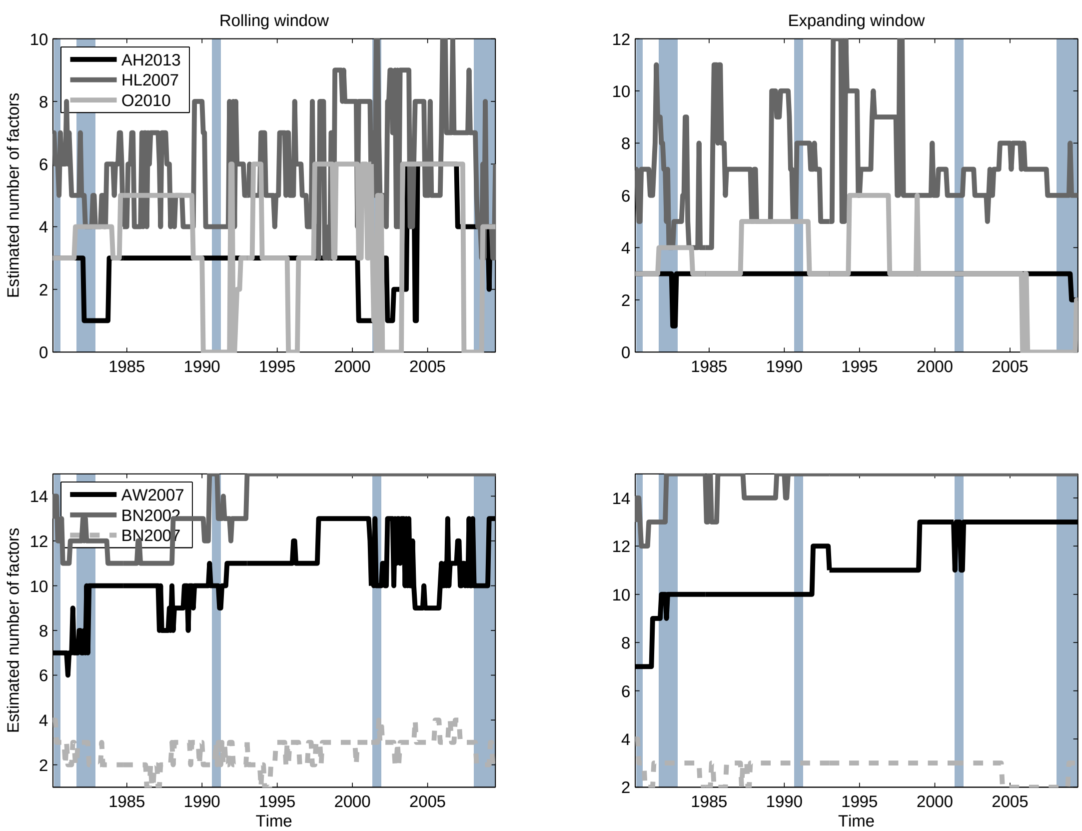


This figure presents the marginal contribution of each factor to the total R^2 from 1980 to 2012. For example, the first north-west panel shows the highest marginal R^2 of F_1. Through all sample, the first factor has the highest marginal contribution for variations in the growth rate of industrial production manufacturing (IP: mfg). The box contains five most explained series by the first factor, in terms of the marginal R^2.

A Appendix: additional empirical results

Figure 4 shows the estimated number of factors over time for the macroeconomic panel used in Boivin, Giannoni and Stevanovic (2013) which is an update of the data set in Bernanke, Boivin and Eliasz (2005). There are 124 variables observed from 1959M01 to 2009M06. This panel is very similar to the one used by Jurado, Ludvigson and Ng (2013) except for the stationarity assumptions on a subset of series. In this data set, interest, unemployment and inflation rates are supposed stationary, therefore they enter X_t in levels, contrary to Jurado, Ludvigson and Ng (2013) where the same series are in first difference of logs. Compared to Figure 1, these stationarity assumptions imply more factors on average over time.

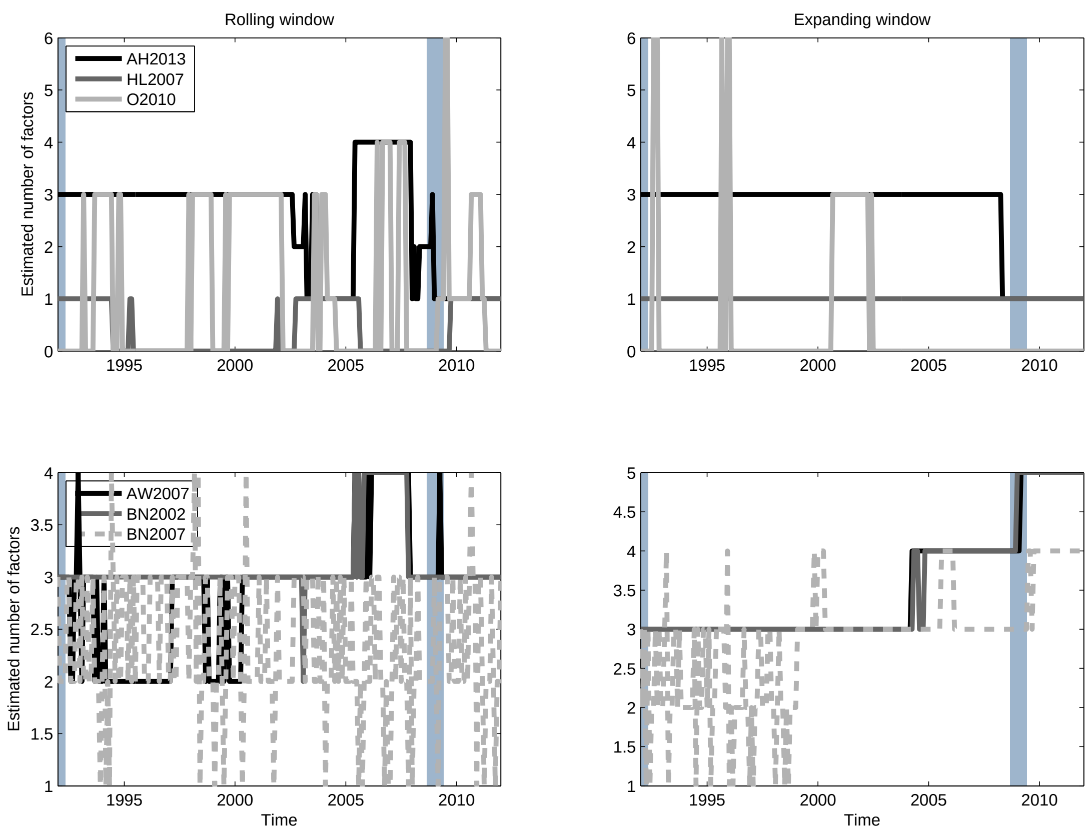
Figure 4: Number of factors over time: Macroeconomic panel from Boivin et al. (2013)



This figure presents the selection of the number of factors during 1980-2009 period. The first column results are computed for rolling window of size 251 months (the initial period is 1959M02 - 1979M12). The second column is for the expanding window where the time series size grows every period.

Figure 5 shows the estimated number of factors over time for a macroeconomic panel of Canadian series. The composition of the panel is very similar to the US data set used in Jurado, Ludvigson and Ng (2013). In addition, the same stationarity assumptions are imposed. There are 124 variables observed from 1981M01 to 2011M12. The Canadian macroeconomic data are typically less available and since the recent reform at Statistics Canada many series are constructed from 1981 only.

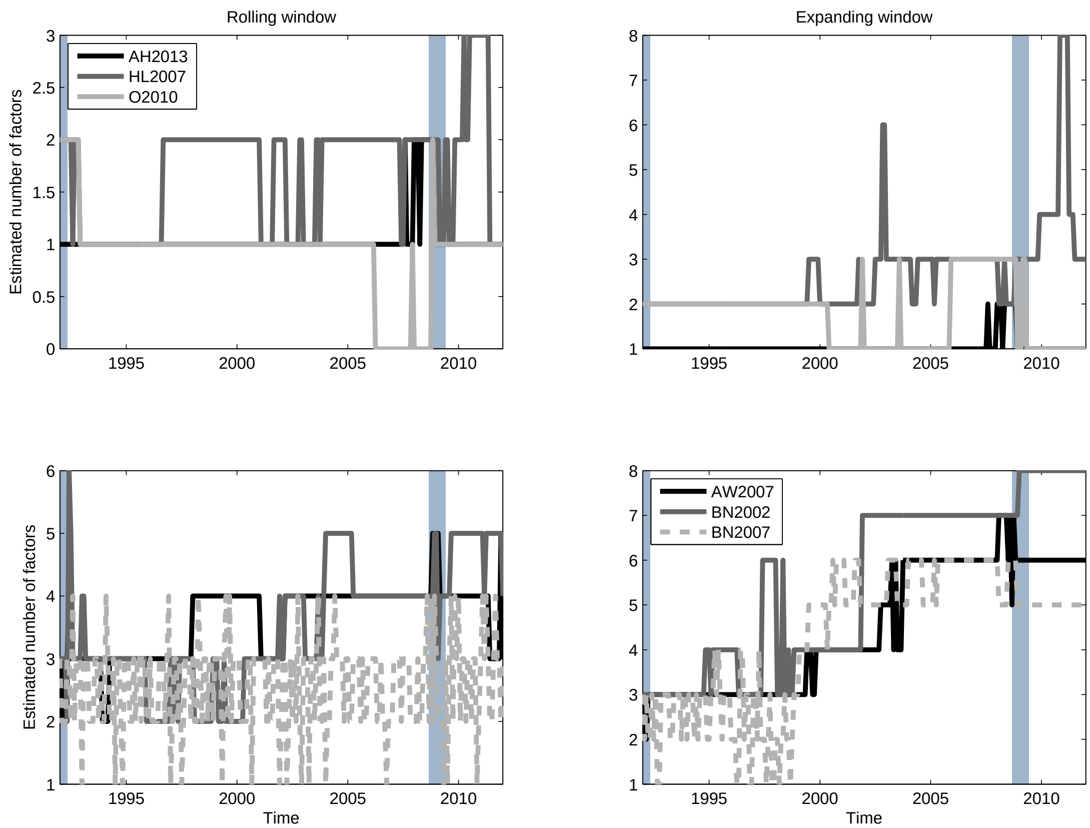
Figure 5: Number of factors over time: Canadian macroeconomic panel



This figure presents the selection of the number of factors during 1992-2012 period. The first column results are computed for rolling window of size 131 months (the initial period is 1981M01 - 1991M12). The second column is for the expanding window where the time series size grows every period.

Finally, we combine the previous Canadian data set with the US panel to construct a very large US-CAN macroeconomic panel containing 246 series during 1981 - 2012 period. The results are presented in Figure 6. Overall, the number of factors seems to grow over time.

Figure 6: Number of factors over time: US and Canadian macroeconomic panel



This figure presents the selection of the number of factors during 1992-2012 period. The first column results are computed for rolling window of size 131 months (the initial period is 1981M01 - 1991M12). The second column is for the expanding window where the time series size grows every period.