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2016s-56

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Série Scientifique/Scientific Series

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Montréal
Octobre/October 2016

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ISSN 2292-0838 (en ligne)

Resource Agency Relationship with Privately Known Exploration and Extraction Costs*

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Abstract

We analyze exploration and extraction under asymmetric information. The principal delegates the exploitation of a resource to an agent (a mining firm) who possesses private information about the cost of exploration and learns further extraction cost information once reserves have been established and constrain extraction. The principal can only commit to current period royalty contracts: one discovery-transfer menu; one extraction-royalty menu conditional on reserves discovered. Compared with the symmetric information first best, avoiding adverse selection in extraction requires the optimum mechanism to increase discoveries by the lowest cost type and possibly others. This is tempered by a countervailing effect stemming from information asymmetry in exploration which tends to reduce discoveries, especially by higher cost types. We further detail implications on the forms taken by the inefficiencies associated with asymmetric information: abandoned reserves, excessive use of low-cost exploration prospects, and inefficient levels of technological sophistication in exploration and extraction sectors.

Mot clés/Keywords: Nonrenewable resources; asymmetric information; endogenous stock of resource; incentive mechanisms

Codes JEL/JEL Codes: D82; H21; L72; Q38

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* We are grateful to FQRSC for financial support. We thank for useful comments participants at the SCSE, CREE and SURED conferences, as well as participants at the Montreal Environment and Resource Economics Workshop.

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1. Introduction

Nonrenewable natural resource management often implies an agency relationship between the owner of, for instance, the mineral rights to a land, and a specialized firm. In many countries, the government will possess the rights to search for and extract minerals even though the land on which these mining operations are taking place is privately owned. In such cases, the government often prefers, for various reasons, delegating the mining operations to a specialized firm in return for a preestablished royalty payment. Even if the rights are privately owned, as is in the case over land in the United States, the owner may want to entrust the exploration and the extraction of the resource to some specialized firm. Consequently, the exclusive right to search for and extract minerals usually is delegated to one or several private firms over the exploration period and the lifetime of the mine. This creates an agency relationship in which the owner is the principal and the firms are agents.

Adverse selection may occur when the agent, say a firm, has private information about some exogenous characteristics, extraction costs for instance. Moral hazard refers to situations where the private information held by the agent is about some endogenous variable, his effort for instance. In both cases, the uninformed party, say the government, moves first by offering the terms of a relationship to the agent. The agent accepts or refuses and acts accordingly. Another situation involves signalling, where the informed party moves first to communicate information to the principal before receiving a proposal from the latter. These different types of situations may arise in exhaustible resource exploitation (e.g. Segerson and Wu, 2006 on moral hazard; Gerlagh and Liski, 2014 on signalling) but, in this paper, we restrict our attention to a principal-agent problem with adverse selection.[1]

The general literature is huge. However this is not the sole reason for limiting our attention to this particular form of agency in nonrenewable resource exploitation. One fascinating issue in the fight against adverse selection is the trade-off between informational rents and efficiency. This paper will show that this trade-off takes novel forms in the dynamic exploitation of a nonrenewable resource that can be contrasted with standard results. The moral hazard problem has been extensively studied, but the rent efficiency trade-off is often blurred in that context. Moral hazard models yielding clear predictions are subject to drastic assumptions. According to Laffont and Martimort (2002, p.189), "there is no general lesson on the nature of the distortion entailed by moral hazard." Our results will be on the nature and the direction of distortions induced by adverse selection.

One aspect of resource exploitation is that it is a dynamic problem, which raises the issue of commitment. Early analyses of multi-period incentive mechanisms with full commitment are due to Baron and Besanko (1984), Roberts (1983), and Baron (1989). When the private information variable

[1] For thorough expositions of the general literature see Laffont and Tirole (1993), Salanié (1997), and Laffont and Martimort (2002)

is independent in the sense that a new independent value is drawn each period, the optimal mechanism yields the first best from the second period on; when it is perfectly correlated, the optimal mechanism is the optimal static mechanism. As a matter of fact, full commitment tends to trivialize dynamic aspects. This may explain why, as Pavan et al. (2014) put it, "the literature on dynamic contracting with adverse selection and limited commitment typically assumes constant types and generates dynamics through lack of commitment". See, e.g., Skreta (2006). In most of the paper we will assume that commitment is not possible beyond one period. In a resource exploitation context, this may be justified by the fact that a current government cannot bind future ones by taking decisions for them; or that no contracts exist that would bind an owner to future arrangements. However, we will also make the comparison with the optimal full commitment solution.

Gaudet et al. (1995) considered a dynamic agency relationship where the government entrusts the extraction of a deposit over several periods to some private firm. They characterized the optimum agency contract between the government and the firm, showing how the rate of extraction and the rent generated by the resource were affected by information asymmetry and departed from the first best optimum that would arise under perfect information. They did so while treating the mineral reserves available for extraction as given. However, the profits or benefits generated by the extraction of a resource are the main driver of exploration and discoveries. It is important to consider both exploration and extraction together. This is what we do in this paper: we have an exploration phase followed by extraction, and the stock of reserves is endogenous.

Natural resource exploitation is inherently dynamic; it can only be properly analyzed in a framework where current decisions physically constrain the future. However, in situations involving asymmetry of information, other dynamic issues may arise[2] because the information "state" may change over time as when the true value of the variable subject to asymmetric information changes over time. Complex dynamics may arise when the information variable is correlated over time because this adds one intertemporal link to the link arising from the physical connection between periods.

In this paper, we will intentionally simplify this former potential source of intertemporal link by assuming that the true value of the parameter which is subject to asymmetric information is temporally independent. In the two-period framework that we shall use (an exploration period followed by an extraction period), this seems like a natural assumption to make since the variable subject to asymmetric information represents an exogenous characteristic of the firm's exploration cost structure in the first period, and of the firm's extraction cost structure in the second period.

The resource literature is relatively scarce in adverse selection papers. Helm and Wirl (2014, 2016) consider climate contracts with multilateral externalities when information such as the willingness to

[2]See for instance Baron and Besanko (1984) or Baron (1989).

pay or costs is private. The contracts are static. Tatoutchoup (2015) studies optimal royalty contracts in forestry when the harvesting firm has private information on the cost of harvesting. The horizon is infinite and the costs may be correlated over time. In line with the early results mentioned above on commitment, the author finds that the rent received by the firm crucially depends on this correlation. Hung et al. (2006) introduce a private information parameter in a resource extraction model. They find that asymmetric information unambiguously increases the duration of the extraction program, which may be interpreted as meaning that the resource is used more sparingly even if that entails increased output toward the end of the program, relative to the symmetric information outcome. In that sense their results are in conformity with a pervasive static result: in order to fight adverse selection, the optimal contract reduces the activity of all types, except that of the lowest cost type ("no distortion at the bottom", or "no distortion for the most efficient type"). Our model will entail a different private cost at each period. As already shown by Gaudet et al. (1995), "no distortion at the bottom" (Laffont and Martimort, 2002, p.137) does not hold when such a dynamic element is introduced.[3]

Indeed the question whether asymmetric information reduces exploration is central to our investigation. While the theory of exploration as supplying a stock of reserves for later exploitation is well developed (Daubanes and Lasserre, 2014; Venables, 2014), we are not aware of any paper incorporating adverse selection into the process.[4] Osmundsen (1998) provides an early analysis of resource extraction involving reserves and asymmetric information. However reserves are not endogenous in that analysis but constitute the private information variable. Furthermore that private information variable is generated once and for all at the beginning of the principal agent relationship, so that the problem reduces to a static one. Indeed it is found that cumulative extraction is reduced and the "no distortion at the bottom" principle holds.

The level of generality of the beautiful paper by Pavan et al. (2014) is such that it covers the exploitation of nonrenewable resources. However it needs much specialization if any concrete results are to be generated on our special case. More importantly, the paper assumes commitment. Moreover, it is by no way clear that it can deal with the other question that we present now, because it includes a fixed, exogenous, set of types into the principal agent relationship.

The other question examined in this paper is the endogeneity of the set of types that are offered to participate in the relationship. Although our methodology, optimal control with endogenous boundary, is not new, this question has not raised interest in the literature. Yet it provides an interesting light

[3]For other exceptions to this principle, see Jullien (2000)

[4]Although the beautiful series of paper by Ken Hendricks, Robert Porter, and their coauthors (Hendricks et al., 2008; Hendricks and Porter, 2014) deal with oil auctions and the associated informational issues, they cannot be considered to address the supply of exploration and discoveries.

on the effects of asymmetric information. In a technological interpretation it shows how asymmetric information affects the technological efficiency level of the industry; in a geophysical interpretation it shows how adverse selection may cause excessive reliance on the best, low cost, deposits, while also allowing reserves to be wasted while they would be exploited under symmetric information.

In Section 2 we present the two period model used throughout the analysis. The two-period framework is the most parsimonious setup capable of highlighting the consequences of information asymmetry on exploration. It can be shown that all the qualitative results that are going to be established are robust to a generalization to any endogenous number of extraction periods,[5] although with additional complexity.

In Section 3 we solve for the full information first best solution that serves as a benchmark. As we seek a closed-loop solution, the extraction problem is first solved in Subsection 3.1, and the solution of the exploration problem follows in Subsection 3.2. We show in Section 4 that asymmetric information in general drives the optimal exploratory effort in opposite directions depending on firms' types. More efficient firms are directed to discover more reserves than under symmetric information, while less efficient ones are asked to produce a lower reserve stock. Typically one intermediate type sees its production of reserves unchanged, so that the principle calling for "no distortion for the most efficient type" is not observed. In Section 5 we provide additional light on the fundamental results by focusing on cases where adverse selection is a problem in one of the two periods only, and by focusing on the endogenous choice of the types (in exploration, in extraction) that are included in the principal agent relationship. This shows the effect of adverse selection on the level and composition of industry reserves, as well as the induced technological sophistication of resource sectors. We also consider full commitment.

We conclude in Section 6.

2. Model and objectives

To keep the analysis as simple as possible we make assumptions ensuring that exploration and extraction take place in a two-period framework. We assume that exploration lasts one period, called Period 1, and that it is never optimal for any firm to extract beyond Period 2, the first extraction period, possibly because the exogenous price of the resource collapses after that period. As mentioned in the introduction, it is possible to relax these assumptions at the cost of additional complexity without affecting the qualitative results. In fact, in Subsection 4.2 we briefly extend the model to

[5] For a generalization to many periods of the extraction model under asymmetry of information, see Gaudet et al. (1995).

allow for a second extraction period to show that the results are not only robust to, but reinforced, by such an extension.

The discovery of an amount x of reserves by a firm of type θ_1 costs

$$C_1(x,\theta_1) = \theta_1 x + \frac{c}{2}x^2, \;\; c \geq 0.$$

θ_1 is a cost parameter that may reflect technological abilities of the firm; it may also reflect knowledge by the firm about the difficulty of exploration of a particular site in which case it is a characteristic of the deposit known to the firm. In any interpretation, this knowledge is not directly accessible to the principal in configurations involving asymmetry of information.

Similarly in the next period, the firm incurs quadratic extraction costs

$$C_2(q,\theta_2) = \theta_2 q + \frac{b}{2}q^2, \;\; b \geq 0,$$

where q is the quantity of resource extracted and θ_2 is a cost parameter known only to the firm if there is asymmetry of information. This parameter can reflect various aspects of the firm's efficiency in extraction; it is learnt only after the reserves have been discovered because it reflects knowledge acquired by the firm during the exploration phase that does not transpire from the sheer size of the reserve stock, which is itself observable by both parties once exploration has taken place. Indeed, while it is hard to imagine the firm to ignore its general extraction abilities, it is very likely that the firm does not know the conditions under which it will exercise those abilities until the reserves are discovered. This is revealed to the firm at the beginning of Period 2 and remains hidden to the principal under information asymmetry. Given the different nature of the expenses in the two periods, it seems natural to restrict our attention to the case where the θ_ts are temporally independent.

The relationship between the principal and the firm starts at a time when the firm knows θ_1 but has not yet learnt about θ_2. The firm knows c, b, θ_1; it will learn θ_2 at the beginning of the extraction period. However, the principal only knows c, b, and the respective cumulative distribution functions of θ_1 and of θ_2, $G(\theta_1)$ defined on $[\theta_1^L, \theta_1^H]$, and $F(\theta_2)$ defined on $[\theta_2^L, \theta_2^H]$. To these distribution functions are respectively associated the density functions $g(\theta_1) > 0$, differentiable on $[\theta_1^L, \theta_1^H]$, and $f(\theta_2) > 0$, differentiable on $[\theta_2^L, \theta_2^H]$. Knowledge of these probability distributions is shared by both parties at the beginning of the relationship. Given any non-negative levels of x and q, θ_1^L and θ_2^L respectively correspond to the lowest possible values of the marginal cost of exploration and of the marginal cost of extraction, while θ_1^H and θ_2^H represent maxima. In order to deal with situations of economic interest, it is assumed that θ_1^L and θ_2^L are sufficiently low to warrant exploitation of the resource by some combination of types of firms under complete information. As far as θ_1^H and θ_2^H, they may be too high to warrant exploration or extraction. If so, firms of type θ_1^H do not carry out any exploration, so that they are not active in the second period either, even if they draw an efficient type θ_2 in Period

2, because they have no reserves to exploit. A type may also be active during the exploration period but fail to extract the reserves discovered, or extract only part of the stock, if it draws too high a cost characteristic in Period 2.

The exploration investment is to be recouped by the firm during the extraction period, when the extracted resource commands a unit price of p. The firm and the principal somehow must share the revenues and costs incurred over these two periods to generate cumulative profits and rents to their satisfaction. The profits of the firm are the revenues obtained from selling the extracted resource, minus exploration and extraction costs, and minus any royalties paid to the principal, net of any subsidies over the two periods. The rent that accrues to the principal is made of the royalties net of any subsidies the principal may give to the firm, plus any consumer or producer surplus that may enter the principal's objective function.

The problem faced by the principal is to establish the royalties (or subsidies) of periods 1 and 2, R_1 and R_2, in the way that best serve her objective. This problem will be examined under alternative information contexts. We assume that the price is exogenously established on the world market. Therefore, the resource good does not generate any consumer surplus. Thus, the two-period objective of the principal may be written as,

$$V = R_1 + \delta E(R_2) + \alpha \Pi_1, \tag{1}$$

where $E(R_2)$ is the expected royalty of the extraction period, discounted by δ, with $0 < \delta < 1$; and where Π_1 is the producers' total expected surplus. Parameter α is such that $0 \leq \alpha < 1$; it is zero if the principal is a private entity, indifferent to rents accruing to others; it is strictly positive if the principal attributes some value to producer surplus as a government typically does. Nonetheless, a dollar in the vaults of the government is valued more highly than a dollar left in the hands of the firm so that $\alpha < 1$. In the rest of the paper, we will refer to the principal or the government indifferently.

3. Full information

Under full information, the value of θ_t is revealed both to the firm and to the government at the beginning of each period t. Therefore, the firm has no private information and the government can achieve Pareto optimality while reaping the totality of the rent arising from the relationship.

3.1. The extraction period

In Period 2, the government wants to maximize

$$W = R_2 + \alpha \Pi_2, \tag{2}$$

where Π_2 is the surplus of the firm during the extraction period:

$$\Pi_2 = pq - C_2(q, \theta_2) - R_2(q). \tag{3}$$

The maximization is subject to the resource constraint

$$0 \leq q \leq x, \tag{4}$$

and to

$$\Pi_2(\theta_2) \geq 0 \ \forall \ \theta_2 \in [\theta_2^L, \theta_2^H]. \tag{5}$$

Since $0 \leq \alpha < 1$, the solution is to set R_2 at the maximum level compatible with $\Pi_2(\theta_2) \geq 0$ for all types, which requires:

$$q^s(\theta_2, x) = \begin{cases} x & \text{if } \frac{1}{b}\left[p - \theta_2\right] \geq x \\ \frac{1}{b}[p - \theta_2] & \text{if } \frac{1}{b}\left[p - \theta_2\right] < x \\ 0 & \text{if } \theta_2 \geq p. \end{cases} \tag{6}$$

The last line of (6) denotes a situation where the resource has no economic value; the second line denotes a situation where the resource is worth producing, but at a rate constrained by the cost of production and not by resource scarcity. The first line corresponds to a situation of definite natural resource scarcity, where the totality of developed reserves is to be used up. Although our results remain valid under weaker assumptions, we choose to focus on this case as more relevant to nonrenewable resources. This implies the following assumption:

$$p \geq \theta_2^H + bx^s(\theta_1^L), \tag{7}$$

where the right-hand side of (7) represents the highest possible marginal extraction cost, $x^s(\theta_1^L)$ being the largest possible amount of stock discovered; it is given further below by Equation (12).

3.2. The exploration period

In the exploration phase, the government wants to maximize the total social welfare, given by equation (1), subject to

$$x \geq 0, \tag{8}$$

and to

$$\Pi_1(\theta_1) \geq 0 \ \forall \theta_1 \in \ [\theta_1^L, \bar{\theta}_1^s]. \tag{9}$$

This last condition is the firms' participation constraint given that the contribution of the extraction phase to profits is zero ($\Pi_2(\theta_2) = 0 \ \forall \ \theta_2 \in [\theta_2^L, \theta_2^H]$). The value $\bar{\theta}_1^s$ separates active firms from inactive firms, for which x is zero, in the case where both parties share the same information about the firms' cost structures, if and only if $\bar{\theta}_1^s \leq \theta_1^H$; otherwise the maximum boundary is replaced by θ_1^H.

The only factors that influence producers' total expected surplus are its exploration cost and the first period royalty, R_1, which means: $\Pi_1(\theta_1) = -C_1(x, \theta_1) - R_1$. To maximize royalties, the principal sets $\Pi_1(\theta_1) = 0 \ \forall \ \theta_1 \in [\theta_1^L, \bar{\theta}_1^s]$ which implies that the royalty consists of a subsidy equal to $C_1(x, \theta_1)$. Using this, the problem faced by the principal in Period 1, given by (1), may now be rewritten as a point by point maximization

$$\begin{aligned} \underset{x}{\text{Max}} \quad & V = -\theta_1 x - \frac{c}{2}x^2 + \delta\Gamma^s(x) \\ \text{s.t.} \quad & \text{(8) and (9),} \end{aligned} \tag{10}$$

where $\Gamma^s(x) \equiv \int_{\theta_2^L}^{\theta_2^H} \left[pq^s(\theta_2, x) - \theta_2 q^s(\theta_2, x) - \frac{b}{2}q^s(\theta_2, x)^2\right] f(\theta_2)d\theta_2$ is the expected royalty of the extraction period and where $q^s(\theta_2, x) = x$ because of Assumption (7).

The first-order condition for an interior solution is

$$\theta_1 + cx = \delta\big[p - E\theta_2 - bx\big]. \tag{11}$$

The amount of reserves discovered must be such that the marginal discovery cost, the left-hand side of Equation (11), equals the expected discounted marginal rent of extracting exactly $q = x$. We will refer to this relation as the *"marginal discovery cost = expected marginal rent"* rule.

Isolating x yields

$$x^s(\theta_1) = \frac{-\theta_1 + \delta[p - E\theta_2]}{c + \delta b}, \tag{12}$$

where $x^s(\theta_1)$ denotes the firm's optimal discovery when both parties share the same information about the exploration and extraction cost structures.

If some firm draws a value of θ_1 such that $\theta_1 \geq \bar{\theta}_1^s$, where $\bar{\theta}_1^s = \delta[p - E\theta_2]$, then such a firm is offered an unprofitable royalty contract so that it is excluded from the relationship and does not produce any discovery. Thus $\bar{\theta}_1^s$ represents the critical type separating profitable from unprofitable firms. If $\delta[p - E\theta_2]$ is higher than θ_1^H then $\bar{\theta}_1^s$ is said not to exist and (12) applies to all types.

4. Asymmetric information

This section analyses the situation where there is asymmetric information in both periods. The cost parameter θ_t is revealed to the firm, and not to the government, at the beginning of each period $t = 1, 2$. The problem is modeled as a direct revelation game, which means that the government chooses an incentive mechanism in the form of a pair $(R_1(\tilde{\theta}_1), x(\tilde{\theta}_1))$ applying in the exploration period and in the form of a pair $(R_2(\tilde{\theta}_2, x), q(\tilde{\theta}_2, x))$ applying in the extraction period, where $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are the values willingly revealed by the firm for its cost parameters. This has two major implications. First we must seek a closed-loop solution to the problem, implying that the extraction period mechanism is not determined in Period 1 but only in Period 2, once reserves have been discovered; therefore R_2 and q are functions of x. Second, if the principal were able to commit, she would be able to deprive the firm of its second period informational advantage by committing to the second period contract conditions at the beginning of the relationship, before θ_2 were revealed to the firm; R_2 would then be specified as function of x and q in such a way that no profit were left to the firm in Period 2. We consider commitment in Subsection 5.3.

According to the revelation principle,[6] we may restrict our attention to mechanisms in response to which all firms will find it optimal to reveal the true value of its cost parameter; these mechanisms are such that $\tilde{\theta}_t = \theta_t$. Besides the usual incentive compatibility and participation constraints, the optimal mechanism is also constrained by the nonrenewability of the resource. Hence, if x is discovered in the exploration period, then extraction in the following period is such that $q \leq x$. Note that the royalty contract in Period 2 is conditional on the amount x discovered by the firm in Period 1, which is known at the beginning of Period 2 and is the amount $x(\theta_1)$ discovered under the first period royalty contract.

4.1. The extraction period

The properties of the incentive royalty contracts are derived as follows. Let $\Phi(\tilde{\theta}_2, \theta_2; x)$ be the firm's surplus for the extraction period if it declares $\tilde{\theta}_2$ while θ_2 is the true value of its cost parameter and it holds reserves x.

$$\Phi(\tilde{\theta}_2, \theta_2; x) = pq(\tilde{\theta}_2, x) - \theta_2 q(\tilde{\theta}_2, x) - \frac{b}{2} q(\tilde{\theta}_2, x)^2 - R_2(\tilde{\theta}_2, x). \tag{13}$$

To make sure the firm reveals truthfully its cost incurred in the extraction period, the incentive scheme must assure that the maximum surplus is reached when the firm chooses to declare $\tilde{\theta}_2 = \theta_2$, which is achieved if the following conditions are satisfied:

$$\Phi_1(\tilde{\theta}_2, \theta_2; x) = 0 \quad \text{for } \tilde{\theta}_2 = \theta_2, \tag{14}$$

or

$$\left[p - \theta_2 - bq(\tilde{\theta}_2, x)\right] \frac{dq(\tilde{\theta}_2, x)}{d\tilde{\theta}_2} - \frac{dR_2(\tilde{\theta}_2, x)}{d\tilde{\theta}_2} = 0 \quad \text{for } \tilde{\theta}_2 = \theta_2, \tag{15}$$

and

$$\Phi_{11}(\tilde{\theta}_2, \theta_2; x) \leq 0 \quad \text{for } \tilde{\theta}_2 = \theta_2. \tag{16}$$

Since these conditions must hold for all firm types and declarations, differentiating (15) totally with respect to $\tilde{\theta}_2$ and θ_2 while holding $\tilde{\theta}_2 = \theta_2$ (i.e. forcing $d\tilde{\theta}_2 = d\theta_2$) we obtain

$$\frac{dq}{d\theta_2} \leq 0 \quad \text{for } \tilde{\theta}_2 = \theta_2. \tag{17}$$

This condition states that, under the incentive scheme, a firm with a higher marginal extraction cost must not extract more than a firm with a lower marginal extraction cost.

[6]See Baron and Myerson (1982) or Baron (1989).

Facing a menu of royalty contracts that satisfies the above properties, a firm of type θ_2 reveals that type by choosing $\tilde{\theta}_2 = \theta_2$ so that its surplus may be written as

$$\Pi_2(\theta_2; x) \equiv \Phi(\theta_2, \theta_2; x). \tag{18}$$

By the envelope theorem,

$$\frac{d\Pi_2}{d\theta_2} = -q(\theta_2, x). \tag{19}$$

An additional consideration is that the principal does not need to offer a contract to all types. She may decide to exclude inefficient types from extraction activities if this is preferable. If so, the contract menu designed by the principal will not have to ensure that either the participation constraint nor the revelation constraint are satisfied for excluded types. As a result the participation constraints that apply to active firms are

$$\Pi_2(\theta_2, x) \geq 0 \;\; \forall \theta_2 \in \;\; [\theta_2^L, \bar{\theta}_2^a], \tag{20}$$

where $\bar{\theta}_2^a \leq \theta_2^H$ designates the critical value of θ_2 that delimits active firms from inactive firms in the extraction period.[7] Since $\Pi_2(\theta_2, x)$ is a decreasing function of θ_2 by (19), the set of firms' participation constraints may be rewritten as the single condition

$$\Pi_2(\bar{\theta}_2^a, x) \geq 0. \tag{21}$$

The objective of the principal is the maximization of

$$\begin{array}{ll} \underset{R_2(.,.),q(.,.),\bar{\theta}_2^a}{\text{Max}} & \displaystyle\int_{\theta_2^L}^{\bar{\theta}_2^a} \Big\{ R_2(\theta_2, x) + \alpha \Pi_2(\theta_2, x) \Big\} f(\theta_2) \mathrm{d}\theta_2 \\ \text{s.t.} & (4), (13), (17), (18), (19) \text{ and } (21). \end{array} \tag{22}$$

Using (13) and (18),

$$R_2(\theta_2, x) = pq(\theta_2, x) - \theta_2 q(\theta_2, x) - \frac{b}{2} q(\theta_2, x)^2 - \Pi_2(\theta_2, x), \tag{23}$$

so that the problem reduces to

$$\begin{array}{ll} \underset{q(.,.),\bar{\theta}_2^a}{\text{Max}} & \displaystyle\int_{\theta_2^L}^{\bar{\theta}_2^a} \Big\{ pq(\theta_2, x) - \theta_2 q(\theta_2, x) - \frac{b}{2} q(\theta_2, x)^2 - (1-\alpha)\Pi_2(\theta_2, x) \Big\} f(\theta_2) \mathrm{d}\theta_2 \\ \text{s.t.} & (4), (17), (19) \text{ and } (21). \end{array} \tag{24}$$

This may be solved as an optimal control problem in the θ_2 space with Π_2 the state variable and q the control variable. The boundary θ_2^L is given and $\bar{\theta}_2^a$ is free subject to $\theta_2^L \leq \bar{\theta}_2^a \leq \theta_2^H$. Temporarily leaving aside the monotonicity constraint (17), the Hamiltonian is

$$\mathcal{H} = \big[pq(\theta_2, x) - \theta_2 q(\theta_2, x) - \frac{b}{2} q(\theta_2, x)^2 \big] f(\theta_2) - \mu_2(\theta_2) q(\theta_2, x) - (1-\alpha)\Pi_2(\theta_2, x) f(\theta_2),$$

[7] Note that this critical value was not relevant in the full information context, i.e. $\bar{\theta}_2^s > \theta_2^H$, because we made parameters assumption, Equation (7) ensuring that every firm exhausted its discovery in the extraction period.

where μ_2 denotes the adjoint variable associated with Π_2.

An interior solution satisfying the maximum principle is

$$\frac{\partial \mathcal{H}}{\partial q} = 0 = [p - \theta_2] f(\theta_2) - \mu_2(\theta_2) - bq(\theta_2, x) f(\theta_2). \tag{25}$$

The trajectory of the adjoint variable must follow,

$$\dot{\mu}_2 = -\frac{\partial \mathcal{H}}{\partial \Pi_2} = (1 - \alpha) f(\theta_2). \tag{26}$$

Integrating this condition yields

$$\begin{aligned} \mu_2(\theta_2) &= \int_{\theta_2^L}^{\theta_2} (1-\alpha) f(\theta_2) d\theta_2 + \mu_2(\theta_2^L) \\ &= (1-\alpha) F(\theta_2), \end{aligned}$$

where the last line follows from the transversality condition $\mu_2(\theta_2^L) = 0$ that applies at θ_2^L since $\Pi_2\left(\theta_2^L\right)$ is free. Substituting in (25) and dividing by $f(\theta_2)$ yields

$$p = \theta_2 + bq(\theta_2, x) + (1 - \alpha) h(\theta_2), \tag{27}$$

where $h(\theta_2) = \frac{F(\theta_2)}{f(\theta_2)}$, the hazard rate, will be assumed positively monotonous, thus ensuring that (17) is satisfied.[8] As in other adverse selection models, the marginal cost is increased relative to the full information case by the *current marginal cost of incentive compatibility* $(1 - \alpha) h(\theta_2)$; this affects all firms types except the lowest-cost one, for which $h(\theta_2^L) = 0$.

Isolating $q(\theta_2, x)$ in equation (27) yields the optimal extraction at an interior solution, i.e. when the firm leaves part of its discovery in the ground at the end of the relationship,

$$q^{ai}(\theta_2) = \frac{1}{b}[p - \theta_2 - (1 - \alpha) h(\theta_2)].$$

More generally, the optimal extraction is

$$q^a(\theta_2, x) = \begin{cases} x & \text{if } \frac{1}{b}\left[p - \theta_2 - (1-\alpha)h(\theta_2)\right] \geq x \\ q^{ai}(\theta_2) & \text{if } \frac{1}{b}\left[p - \theta_2 - (1-\alpha)h(\theta_2)\right] < x \\ 0 & \text{if } \theta_2 \geq p - (1-\alpha)h(\theta_2). \end{cases} \tag{28}$$

Finally, the principal has to choose the range of firm types to which extraction is delegated. This is done by choice of the upper boundary $\bar{\theta}_2^a$. Given that the most efficient types will not be excluded, this choice is only subject to the constraint $\bar{\theta}_2^a \leq \theta_2^H$. The transversality condition is $\mathcal{H} = 0$ if $\bar{\theta}_2^a < \theta_2^H$

[8]See Laffont and Martimort (2002), especially Chapter 3, for the role of the monotone hazard rate condition and the related Spence-Mirlees condition.

and $\mathcal{H} \geq 0$ if $\bar{\theta}_2^a = \theta_2^H$. Given that (28) must also hold at $\theta_2 = \bar{\theta}_2^a$, it follows that $q^{ai}\left(\bar{\theta}_2^a\right)$ must equal zero if the cutoff type is strictly lower than θ_2^H which defines $\bar{\theta}_2^a$ by Condition

$$\bar{\theta}_2^a = \min\left\{\theta_2^H,\ [\theta_2|\ \theta_2 = p - (1-\alpha)h(\theta_2)]\right\}. \tag{29}$$

Thus asymmetry of information may cause some types of firms to be left completely inactive ($\bar{\theta}_2^a \leq \theta_2 \leq \theta_2^H$) despite having discovered reserves in Period 1; if so some other, more productive, types will abandon reserves after partial extraction (the second line of 28). This possibility arises despite Assumption (7) that guarantees all discovered reserves to be exhausted under symmetric information.[9] This is summarized in the following proposition:

Proposition 1. *Consider an extractive industry such that, under symmetric information, all firms exhaust the totality of the reserves that they previously discover. Asymmetry of information in extraction may require some types to extract only part of the reserves that they have previously discovered. When $\bar{\theta}_2^a < \theta_2^H$, the reserves of types θ_2 such that $\bar{\theta}_2^a \leq \theta_2 \leq \theta_2^H$ are left untouched while some types $\theta_2 \leq \bar{\theta}_2^a$ exploit them partially.*

4.2. The exploration period

The methodology used to solve the second period problem can be applied to solve the problem of the exploration period. Two major differences modify the analysis. First, unlike the extraction period where firms not only differ by their type θ_2 but also by the observable level of reserves x that they inherit from the first period, the firms analyzed in the exploration period differ only by their types. Therefore, the first period payoff function of a firm may be written $\phi(\tilde{\theta}_1; \theta_1)$. Second the payoff not only depends on current elements but also on the effect of discoveries on the expected value $\Psi(x)$ of the surplus that extraction activities will generate in Period 2.

Adapting the derivation outlined in Subsection 4.1, the total expected surplus of a firm under the incentive contract $R_1(\tilde{\theta}_1, x(\tilde{\theta}_1))$ is

$$\Pi_1(\theta_1) = -\theta_1 x(\tilde{\theta}_1) - \frac{c}{2} x(\tilde{\theta}_1)^2 + \delta\Psi(x(\tilde{\theta}_1)) - R_1(\tilde{\theta}_1), \tag{30}$$

where

$$\Psi(x) \equiv \int_{\theta_2^L}^{\bar{\theta}_2^a} \Pi_2(\theta_2, x) f(\theta_2) d\theta_2 \tag{31}$$

[9]The third line of (28) strictly does not apply since it corresponds to values of θ_2 above the cutoff type. However, since it prescribes the same extraction level $q = 0$ that applies to excluded firms, it is harmless to consider that (28) applies to the whole range of θ_2 types, whether they are active in the extraction period or not.

with $\bar{\theta}_2^a$ and Π_2 established in Subsection 4.1. The optimal contract must satisfy

$$\frac{dx}{d\theta_1} \leq 0 \quad \text{for } \tilde{\theta}_1 = \theta_1, \tag{32}$$

$$\frac{d\Pi_1}{d\theta_1} = -x(\theta_1). \tag{33}$$

One notes that R_1 may need to be a subsidy; combined with the expected surplus from the extraction period, it must be such that the participation constraint of all participating types $\theta_1 \leq \bar{\theta}_1^a$ is satisfied. This is the case if the constraint is met at the extensive margin, that is to say by the least efficient of the participating firms:

$$\Pi_1(\bar{\theta}_1^a) \geq 0. \tag{34}$$

Using (30) to eliminate R_1 when formulating the objective of the principal, the problem is

$$\begin{aligned} &\underset{x(\theta_1),\bar{\theta}_1^a}{\text{Max}} \quad \int_{\theta_1^L}^{\bar{\theta}_1^a} \Big\{ -\theta_1 x(\theta_1) - \frac{c}{2}x(\theta_1)^2 + \delta\big[\Psi(x) + \Gamma^a(x)\big] - (1-\alpha)\Pi_1(\theta_1)\Big\} g(\theta_1)\mathrm{d}\theta_1 \\ &\text{s.t.} \qquad (8), (32), (33) \text{ and } (34), \end{aligned} \tag{35}$$

where $\Gamma^a(x)$ represents the expected extraction royalty when there is asymmetric information:

$$\Gamma^a(x) = \int_{\theta_2^L}^{\bar{\theta}_2^a} \Big[pq^a(\theta_2,x) - \theta_2 q^a(\theta_2,x) - \frac{b}{2}q^a(\theta_2,x)^2\Big] f(\theta_2)d\theta_2 - \Psi(x), \tag{36}$$

with $q^a(\theta_2, x)$ and $\bar{\theta}_2^a$ respectively given by (28) and (29).

It is an optimal control problem with Π_1 the state variable, x the control variable, θ_1^L given and where $\bar{\theta}_1^a$ is free subject to $\theta_1^L \leq \bar{\theta}_1^a \leq \theta_1^H$. The optimal amount of reserves discovered by a firm of type θ_1 must be such that:[10]

$$\Big[-\theta_1 + \delta\big[\frac{d\Psi(x)}{dx} + \frac{d\Gamma^a(x)}{dx}\big]\Big] g(\theta_1) - (1-\alpha)G(\theta_1) - cx(\theta_1)g(\theta_1) = 0. \tag{37}$$

To simplify this expression, note that Ψ enters (36) additively so that the term between square brackets in (37) reduces to

$$\begin{aligned} \frac{d\Psi(x)}{dx} + \frac{d\Gamma^a(x)}{dx} = &\int_{\theta_2^L}^{\hat{\theta}^a(x)} \Big[p - \theta_2 - bx\Big] f(\theta_2)d\theta_2 \\ &+ \int_{\hat{\theta}^a(x)}^{\bar{\theta}_2^a} \Big[p - \theta_2 - bq\Big] f(\theta_2)d\theta_2 \frac{dq}{dx}, \end{aligned} \tag{38}$$

where the integral in (36) has been divided into two parts according to the value q^a implied by (28). This expresses the effect of a marginal change in x on the expected net of royalty surplus as the sum

[10] The resolution mimics the steps detailed in Subsection 4.1; in particular, the costate variable is replaced by its optimal value $(1-\alpha)\,G(\theta_1)$.

of two expected values. The first integral applies if the firm draws a relatively efficient type in the extraction period so that it will be able to extract the totality of its reserves ($q = x$). The second integral applies to less efficient types, defined on the interval $[\hat{\theta}^a, \bar{\theta}_2^a]$, that will abandon part of their discovery ($q < x$). Thus $\hat{\theta}^a(x)$ is defined as the value of θ_2 marking the transition between the first and the second lines of (28):

$$\frac{1}{b}\left[p - \hat{\theta}^a - (1-\alpha)h(\hat{\theta}^a)\right] = x. \tag{39}$$

At this stage, it will be assumed that a value of $\hat{\theta}^a(x)$ such that $\hat{\theta}^a(x) \le \theta_2^H$ exists; for short we will say that "$\hat{\theta}^a(x)$ exists". If it exists $\hat{\theta}^a$ is decreasing in x by the monotone hazard rate property of h.

Noticing that $\frac{dq}{dx} = 0$ by the second line of (28), and dividing by $g(\theta_1)$, equation (37) may thus be rewritten as

$$\theta_1 + cx + (1-\alpha)m(\theta_1) = \delta\left[p - E(\theta_2|\theta_2 \le \hat{\theta}^a(x)) - bx\right]F(\hat{\theta}^a(x)), \tag{40}$$

where $m(\theta_1) = \frac{G(\theta_1)}{g(\theta_1)}$.[11] This condition is a modified version of the "marginal discovery cost = expected marginal rent" rule (11) that must hold under symmetric information. Unlike more standard adverse selection problems, both sides of the relationship are affected by asymmetric information. On the left-hand side, the marginal discovery cost takes into account the current marginal cost of incentive compatibility, $(1-\alpha)m(\theta_1)$. On the right-hand side, the expected marginal rent is modified because discovered reserves will be left partially or totally unexploited if the firm draws a type $\theta_2 \ge \hat{\theta}^a(x)$ in the extraction period.

In order to compare the optimal amount of reserves $x^a(\theta_1)$ under asymmetric information with the amount $x^s(\theta_1)$ defined by (11) which is optimal under symmetric information, it will be convenient to rewrite (40) as

$$\theta_1 + cx + (1-\alpha)m(\theta_1) = \delta\left[p - E\theta_2 - bx\right] - \delta(1 - F(\hat{\theta}^a(x)))\left[p - E(\theta_2|\theta_2 > \hat{\theta}^a(x)) - bx\right],$$

rearranging one as

$$x^a = x^s - \frac{(1-\alpha)m(\theta_1)}{c+\delta b} + \frac{-\delta(1 - F(\hat{\theta}^a(x^a)))\left[p - E(\theta_2|\theta_2 > \hat{\theta}^a(x^a)) - bx^a\right]}{c+\delta b}, \tag{41}$$

where $\hat{\theta}^a$ and x^s are defined by (39) and (11) respectively.

By (39), the last term is positive so that it counteracts the second term. In particular, if $\hat{\theta}^a(x)$ exists for $x = x^a(\theta_1^L)$, it is certain that $x^a(\theta_1^L) > x^s(\theta_1^L)$ since m is then zero and the last term is strictly positive. Thus the most efficient type is requested to produce more reserves than it would under symmetric information. As θ_1 rises though, the marginal cost of incentive compatibility (second

[11] We make the monotone hazard rate assumption $\frac{dm(\theta_1)}{d\theta_1} \ge 0$, which is sufficient to insure that (32) is satisfied.

term) takes more weight, while the last term diminishes as $\hat{\theta}^a$ rises[12] reflecting the fact that reserves are less likely to remain unextracted if a lower stock is produced. It may then be that $x^a < x^s$ at higher values of θ_1 as would be expected if exploration was carried out in a conventional static agency context. If so, by continuity, there is an intermediate type whose production of reserves is the same under symmetric as under asymmetric information. We have established the following results.

Proposition 2. *If, for some level $x^a(\theta_1)$, an intermediate type $\theta_2 = \hat{\theta}^a$ exists above which firms do not exhaust their reserves during the extraction period, then the types that are most efficient in exploration are prescribed to discover more reserves under asymmetric information than is Pareto optimal under symmetric information. Precisely, $x^a(\theta_1) > x^s(\theta_1)$ if*

$$(1-\alpha)m(\theta_1) < -\delta(1-F(\hat{\theta}^a))\big[p - E(\theta_2|\theta_2 > \hat{\theta}^a) - bx\big], \tag{42}$$

which always holds for the lowest-cost firm type ($\theta_1 = \theta_1^L$); otherwise, $x^a(\theta_1) \leq x^s(\theta_1)$.

The existence of $\hat{\theta}^a$ is crucial to the theoretical significance of Proposition 2. If $\hat{\theta}^a$ does not exist (41) reduces to $x^a = x^s - \frac{(1-\alpha)m(\theta_1)}{c+\delta b}$, which means that asymmetry of information causes a reduction in exploration activity (except for the most efficient firm θ_1^L) reflecting the marginal cost of incentive compatibility $(1-\alpha)m(\theta_1)$. This is in line with usual results involving adverse selection in static contexts. The standard explanation given in the literature is that, by forcing the agents to produce below the efficient level, the principal reduces the rent that she has to concede in order to obtain agents not to mimic other types. As is well known, this necessity disappears in the case of the most efficient type as no other type wants to mimic it. Under the conditions of Proposition 2, this is not the end of the story because adverse selection during the extraction phase affects the revenues that the firm derives from its activity, as indicated on the right hand side of (40). The informational advantage enjoyed by the firm during extraction generates a rent in its favor. The firm can increase that rent by generating more reserves in the exploration phase. However this ability to increase the total second period rent is present only when the parameters of the problem justify extraction at a rate lower than $q = x$ for some high level of θ_2. This condition is realized when production is given by the second line of (28), which is possible if and only if $\hat{\theta}^a$ exists. The ability of types $\theta_2 \geq \hat{\theta}^a$ to abstain from producing at full capacity in Period 2 in that case increases the marginal revenue to be expected from a marginal increase in reserves in Period 1.

Parameter conditions for the existence of $\hat{\theta}^a$ are difficult to establish by brute force because $\hat{\theta}^a$ is conditional on x which itself depends on θ_1 in the solution: x and $\hat{\theta}^a$ must solve (39) and (40) jointly for each value of θ_1. However, consider $\hat{\theta}^a$ when $x = x^a\left(\theta_1^L\right)$. Since $x^a(\theta_1)$ is decreasing by (32)

[12] $\hat{\theta}^a$ is decreasing in x and x^a is decreasing in θ_1 by (32).

and considering (39), $\hat{\theta}^a$ does not exist at any value of $x^a(\theta_1)$ if it does not exist at $x^a\left(\theta_1^L\right)$. Clearly the existence of $\hat{\theta}^a$ at $x^a\left(\theta_1^L\right)$ is also sufficient to satisfy the condition of Proposition 2. Thus the existence of $\hat{\theta}^a\left(x^a\left(\theta_1^L\right)\right)$ is necessary and sufficient for Proposition 2. $\hat{\theta}^a\left(x^a\left(\theta_1^L\right)\right)$ exists if the value of $\hat{\theta}^a$ implied by(39) is not higher than θ_2^H, i.e. if $p-\theta_2^H-(1-\alpha)h(\theta_2^H)-bx^a\left(\theta_1^L\right)\leq 0$. Suppose that this condition holds with equality. Then it is violated for all other values of θ_1, which implies that $x^a\left(\theta_1^L\right)=x^s\left(\theta_1^L\right)$ by (41); on the contrary, if the condition holds strictly, then $\hat{\theta}^a$ must exist for some values of θ_1.[13] A necessary and sufficient condition for Proposition 2 to apply is thus[14]

$$p-\theta_2^H-(1-\alpha)h(\theta_2^H)-bx^s\left(\theta_1^L\right)\leq 0. \tag{43}$$

Before moving on to various implications and interpretations of the results, it is important to stress their generality and robustness. The fact that they have been established for a two-period model and that (43) further delimits the conditions of validity of Proposition 2 indeed casts a doubt on its relevance. On the other hand, the explanation given above for the novelty of our results - that information asymmetry in this dynamic setup gives firms incentives not only to disguise current costs but also to position themselves to reap higher information rents in the future - does not appear conditional on the number of periods. Indeed, the methodology just outlined can be extended to any finite number of periods, although at a considerable cost in terms of complexity. However an extension to three periods, one exploration period followed by two extraction periods, suffices to show that no restriction such as (43) survives the generalization.

As a matter of fact, (43) is an anecdotic restriction that applies only when extraction is limited to one period. It is easy to understand why it should be so. (43) limits the preponderance of corner solutions ($q=x$) during the extraction phase because firms then do not have the flexibility required to manipulate in the first period the informational advantage that they are to hold during the extraction period. On the contrary, when (43) holds, the firms have some flexibility because some types will choose output without being constrained by reserves ($q<x$).

When the number of extraction periods exceeds one, firms are able to adjust to changes in royalties by shifting extraction between periods, even if they are constrained by reserves over the total extraction duration. To illustrate let us modify the model by adding one period of sufficiently high price immediately following the first extraction period; adapt notation and assumptions accordingly in such a way that it is optimal under symmetric information for all types to extract the totality of discovered reserves within the two possible extraction periods. It can be established for the three period case

[13] This can be proven by contradiction: if $\hat{\theta}^a$ does not exist for any value of θ_1, then $x^a\left(\theta_1^L\right)=x^s\left(\theta_1^L\right)$, contradicting the maintained condition that $p-\theta_2^H-(1-\alpha)h(\theta_2^H)-bx^a\left(\theta_1^L\right)<0$.

[14] One can verify that Condition (43) is compatible with Assumption (7) which guarantees complete exhaustion of reserves under symmetric information.

that, when all types exhaust reserves,

$$x^a(\theta_1) = x^s(\theta_1) - \frac{(1-\alpha)m(\theta_1)}{c + \frac{2\delta^2}{(1+\delta)}b} + \frac{(1-\alpha)\frac{\delta(1-\delta)}{(1+\delta)}Eh(\theta_2)}{c + \frac{2\delta^2}{(1+\delta)}b}.$$

The right fraction has a strictly positive effect on $x^a(\theta_1)$ unlike what happens to the last term in (41) if all types exhaust reserves. We have sketched the proof of the following conjecture:

Conjecture 1. *In a multi-period extraction framework, information asymmetry during the extraction periods affects all discoveries in the first period positively. For the most efficient types in exploration, it follows that their discoveries are higher under asymmetry of information than would be efficient under symmetric information.*

5. Interpretations, implications, extensions

5.1. Adverse selection in exploration or in extraction only

When there is asymmetric information in exploration only, θ_1 is observed by the firm but not the principal at the beginning of Period 1; θ_2 is observed by both the firm and the principal at the beginning of Period 2. Then (40), the augmented "marginal discovery cost = expected marginal rent" rule reduces to

$$\theta_1 + cx(\theta_1) + (1-\alpha)m(\theta_1) = \delta\left[p - E\theta_2 - bx\right]. \tag{44}$$

The left-hand side is the marginal discovery cost, augmented by the current marginal cost of incentive compatibility, $(1-\alpha)m(\theta_1)$. The expected marginal rent, on the right-hand side, a constant independent of either θ_1 and x,is the same as given under symmetric information by (11). This is a standard result in the incentive literature: the marginal cost of the activity is increased by the cost of incentive compatibility; the marginal revenue is not affected. Hence the principal must reduce activity (exploration) by all firms except the most efficient one θ_1^L for which the cost of incentive compatibility is zero.

Consider the opposite case, when θ_1 becomes known to both the principal and the firm before the beginning of the relationship while θ_2 is learnt at the beginning of Period 2 by the sole firm. The augmented "marginal discovery cost = expected marginal rent" rule (40) reduces to

$$\theta_1 + cx(\theta_1) = \delta\left[p - E(\theta_2|\theta_2 \leq \hat{\theta}^{s,a}) - bx(\theta_1)\right]F(\hat{\theta}^{s,a}). \tag{45}$$

The left-hand side is as under symmetric information. Indeed the principal does not need to rely on the firm to know θ_1 so that there is no cost associated with the revelation of θ_1. However the firm will enjoy an informational advantage in Period 2 so that the revenue expected by the principal differs from what it is under symmetric information as per (11). Indeed both parties know in Period one that an information rent will be conceded to the agent in Period 2, not only according to the type θ_2 that the firm will draw, but also according to the level of reserves that it will hold at that time. Therefore, the principal must take account as soon as Period 1 of the effect of reserves on that rent. Higher reserves reduce the marginal value of that rent, so that it is in the interest of the principal to distort the efficient amount of discoveries positively.

These remarks are summarized in the following corollary of Proposition 2:

Corollary 1. *Private information in the exploration period causes discoveries to be reduced relative to the efficient symmetric information level; this affects all firm types except the lowest-cost one.*

Private information in the extraction period causes discoveries to be increased relative to the efficient symmetric information level; this affects all firm types including the lowest-cost one.

5.2. Technological level, economic reserves, extensive margin

Our results have obvious implications in terms of economic reserves and technology. Economic reserves, at least in the long run, are those physical reserves that are worth discovering given current and expected future economic conditions. In the model of this paper, economic conditions can be interpreted as the prevalence or absence of private information while economic reserves can be defined as the cumulative distribution of discovered reserves under symmetric information ($j = s$) or asymmetric information ($j = a$) alternatively:

$$S^j = \int_{\theta_1^L}^{\bar{\theta}_1^j} x^j(\theta_1)\, g(\theta_1) d\theta_1. \tag{46}$$

Resource economists also use the concepts of extensive and intensive margins. The extensive margin focuses on changes in the amount of economic reserves; it defines the limit between deposits (or firms) that are economic versus non-economic. In this paper, the extensive margin is given by $\bar{\theta}_1^j$. The intensive margin has to do with the speed of extraction and the recovery level. Although a multiperiod extraction version of the model can handle this concept, we will not pursue it here and shall focus on the extensive margin.

The economic reserves defined by (46) are homogenous in the sense that, until Period 2 reveals the variety of types that will exploit them, any part of the stock is expected to have the same extraction cost as any other part. However this homogeneity hides the variety of the costs incurred to constitute S^j. For example, for two identical hypothetical levels of S^a and S^s, if $x^a(\theta_1)$ exceeds $x^s(\theta_1)$ at low levels of θ_1 while the reverse is true at high costs, then it can be said that asymmetry of information tilts the composition of reserves toward lower cost reserves. If so, asymmetry of information causes excessive discoveries of the most valuable resources.

Consider the extensive margin $\bar{\theta}_1^j$. From (11)

$$\bar{\theta}_1^s = \delta[p - E\theta_2]. \tag{47}$$

Under asymmetric information, since $\bar{\theta}_1^a$ is free in Problem (35), implying that the Hamiltonian must be zero by the transversality condition at $\bar{\theta}_1^a$,

$$\bar{\theta}_1^a = -(1-\alpha)m(\bar{\theta}_1^a) + \delta[p - E(\theta_2|\theta_2 \leq \hat{\theta}^a)]F(\hat{\theta}^a),$$

where it was made use of the fact that (37) must also hold at $\bar{\theta}_1^a$. The last term on the right hand side is the expected second period surplus and is smaller than the expected second period surplus under symmetric information. As already discussed, it arises from asymmetry of information in the extraction period. It tends to reduce $\bar{\theta}_1^a$ relative to the value of $\bar{\theta}_1^s$ implied by (47). The first term is the familiar marginal cost of incentive compatibility associated with asymmetry of information in exploration. It increases $\bar{\theta}_1^a$ relative to the value of $\bar{\theta}_1^s$. This, together with Propositions 1 and 2 proves the following proposition:

Proposition 3. *Absent adverse selection in extraction, asymmetric information in exploration:*

1. *reduces the extensive margin $\bar{\theta}_1^a$ below its efficient level and causes the exploration sector to be more cost efficient than Pareto optimal;*
2. *reduces total reserves S^a relative to their efficient level and affects the composition of these reserves by increasing the share of relatively less costly discoveries; this increase is more pronounced the lower the cost of the discovery.*

Absent adverse selection in exploration, asymmetric information in extraction:

1. *increases the extensive margin $\bar{\theta}_1^a$ beyond it efficient level and causes the exploration sector to be less cost efficient than Pareto optimal;*
2. *increases total reserves S^a relative to their efficient level but has no effect on the composition of these reserves beyond the effect of the increase in $\bar{\theta}_1^a$ which is to increase the share of costly discoveries;*
3. *causes the extraction sector to be more cost efficient than Pareto optimal ($\bar{\theta}_2^a \leq \theta_2^H$).*

Cost efficiency in exploration may reflect the technology used by the firms or the exploration prospects that the firms turn into reserve deposits. In the first instance a departure from Pareto optimality reflects a waste of technological opportunities; in the second instance a waste of natural resources. In both cases, resources that are economic under symmetric information are wasted if the sector becomes too cost efficient.

5.3. Full intertemporal commitment

When considered at the exploration stage as in this corollary, private information in the extraction period is an instance of adverse selection involving an ex-post participation constraint. When risk aversion is not an issue (see Laffont and Rochet (1998); Lewis and Sappington (1995) for situations involving risk aversion on the part of the principal or the agent), the optimal incentive contract is known to implement the first best outcome (Proposition 2.4 in Laffont and Martimort (2002)). However, this result holds when the sole decision under scrutiny is occurring after the type becomes known to the agent. In the context of Corollary 1, extraction is indeed to be contracted upon ex-post, but exploration occurs before the realization of θ_2.

We have up until now made the assumption that the government could only commit to the present period's royalty schedule.[15] When the principal cannot fully commit to future arrangements, royalties must depend on the firm's cost reports $\tilde{\theta}_t$ made at the beginning of each period $t = 1, 2$. As we have seen, the firm anticipates the principal's response to its future cost report $\tilde{\theta}_2$, knows that it will be able to use to its advantage, and tries to position itself as soon as the first period in such a way as to maximize that advantage.

If full commitment is possible, the principal may commit to the same relationship that she would have offered were commitment not possible. It is therefore certain that she will not loose from the ability to commit. In fact the principal can gain by commiting as early as Period 1 to the response that she will give to the cost report $\tilde{\theta}_2$ given by the firm when it draws its second period type. By so doing she requires the mechanism to be accepted by the firm at a stage when both parties are ignorant of the true θ_2 and share the same information on its distribution. This eliminates the firm's informational advantage in Period 2. In fact, if the second period royalty eliminates any rent whatever the report, the firm has no incentive to misreport; if it enters the relationship at all it will truthfully report its extraction cost. Consequently, when the government can fully commit to future royalty rules, it faces adverse selection only in the first period.

How does commitment affect the amount of resource discovered and the selection of active types? The government commits at the beginning of the relationship to a pair of combinations $\{R_1(\tilde{\theta}_1), x^c(\tilde{\theta}_1)\}$ and $\{R_2(\tilde{\theta}_2, x), q^c(\tilde{\theta}_2, x)\}$, where the "^{c}" exponent means "commitment". In Period 2 the royalty $R_2(\theta_2, x)$ collects the maximized extraction rent, leaving the firm with just enough to cover its opportunity cost: $\Pi_2 = 0 \ \forall \ \theta_2 \in [\theta_2^L, \bar{\theta}_2^c]$. Consequently, the surplus expected by the firm for Period 2 at the beginning of Period 1, $\Psi(x) = 0$. Since it solves the same problem (2) as q^s, the extraction level is given by (6), the condition that applies under symmetric information. This equation is conditional on x, which is not the same under commitment as under symmetric information. However it requires $q = x$ when x is low enough, a condition ensured by (7) under symmetry. In the formulation of the

[15]This may reflect the fact that a present government cannot bind future ones by taking decisions for them; or that a contract that would take into account all future contingencies is either too costly or simply impossible to write. See Laffont and Tirole (1988) and Grossman and Hart (1986).

problem faced by the principal in Period 1, we are going to assume that (7) has the same implication in the commitment problem, i.e. that $q^c = x^c \ \forall \ \theta_2$, which further implies that $\hat{\theta}^c = \theta_2^H$ as written below. It turns out that this is indeed true once it is determined, further below, that $x^c \leq x^s$.

Accordingly, the two-period objective of the principal may be formulated as the following optimal control problem in the θ_1 space with Π_1 the state variable and x the control variable, and where θ_1^L is given and $\bar{\theta}_1^c$ is free subject to $\theta_1^L \leq \bar{\theta}_1^c \leq \theta_1^H$:

$$\begin{aligned} &\max_{x(\theta_1),\bar{\theta}_t} \quad \int_{\theta_1^L}^{\bar{\theta}_1^c} \left\{ -C_1(x,\theta_1) - (1-\alpha)\Pi_1(\theta_1) + \delta \int_{\theta_2^L}^{\theta_2^H} \{px - C_2(x,\theta_2)\} f(\theta_2) d\theta_2 \right\} g(\theta_1) \mathrm{d}\theta_1 \\ &\text{s.t.} \qquad (4), (32), (33), (34) \text{ and } (21).^{16} \end{aligned} \tag{48}$$

By the maximum principle,

$$\theta_1 + cx(\theta_1) + (1-\alpha)m(\theta_1) = \delta\big[p - E\theta_2 - bx\big], \tag{49}$$

so that

$$x^c(\theta_1) = \frac{-\theta_1 - (1-\alpha)m(\theta_1) + \delta[p - E\theta_2]}{c + \delta b}. \tag{50}$$

Using the transversality condition one also finds that:

$$\bar{\theta}_1^c = -(1-\alpha)m(\bar{\theta}_1^c) + \delta[p - E\theta_2].$$

Both discoveries and the critical value of θ_1 that delimits active types in exploration from inactive ones are the same as if information were symmetric during extraction. Perhaps not surprisingly, the ability to commit over the complete duration of the relationship allows the principal to eliminate the problem of adverse selection in extraction.

Proposition 4. *When the government can fully commit to future royalties:*

1. *the optimal amount of resource discovered is the same as if symmetric information prevailed in extraction, i.e. lower than under full information and also smaller than in the absence of commitment, strictly so if* (43) *holds;*
2. *the extensive margin* $\bar{\theta}_1^c$ *is the same as if symmetric information prevailed in extraction, i.e. lower than under full information and also smaller than in the absence of commitment.*

6. Conclusion

This analysis has combined exploration and extraction in a model of nonrenewable natural resource exploitation with adverse selection. Information asymmetry is represented a different parameter

[16] In constraints (34) and (21), $\bar{\theta}_t^a$ must be replaced with $\bar{\theta}_t^c$.

brought to the knowledge of the firm at the beginning of exploration and extraction with no correlation between the two realizations.

The typical "no disturbance at the bottom" result does not apply during exploration in this dynamic setup because decisions in each phase affect conditions in the other. The standard explanation given in the literature for "no disturbance at the bottom" is that, by forcing agents to produce below the efficient level, the principal reduces the rent that she has to concede in order agents not to mimic other types. This necessity disappears in the case of the most efficient type as no other type wants to mimic it. When information asymmetry affects both exploration and extraction, this is not the end of the story. Adverse selection in extraction generates a rent in favor of the firm. Firms, including the most efficient type in exploration, can increase that rent by generating more reserves in the exploration phase. Hence adverse selection in extraction increases the discoveries requested from the most efficient exploration types relative to the Pareto efficient symmetric information situation.

We have highlighted the implications of this mechanism in terms of the size and composition of economic reserves as well as the sophistication of the technologies used in exploration and extraction, as well as the forms of resource waste induced by adverse selection. As we have shown, the inefficiency induced by adverse selection not only may take the form of excessive reserve production; it may also cause reserves to be left unexploited while they would be used under symmetric information; it may cause an overreliance on the best types of exploration prospects; it may cause too much technological sophistication in extraction.

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