CIRANO
Knowledge into action

ENERGY TRANSITION UNDER MINERAL CONSTRAINTS AND RECYCLING: A LOW-CARBON SUPPLY PEAK

SIMON **CHAZEL**
SOPHIE **BERNARD**
HASSAN **BENCHEKROUN**

2023s-09
WORKING PAPER
CS

ISSN 2292-0838 (online version)

Energy transition under mineral constraints and recycling: A low-carbon supply peak

Simon Chazel,[*] Sophie Bernard[†], Hassan Benchekroun[‡]

January 2023

Abstract/Résumé

What are the implications of primary mineral constraints for the energy transition? Low-carbon energy production uses green capital, which requires primary minerals. We build on the seminal framework for the transition from a dirty to a clean energy in Golosov et al. (2014) to incorporate the role played by primary minerals and their potential recycling. We characterize the optimal paths of the energy transition under various mineral constraint scenarios. Mineral constraints limit the development of green energy in the long run: low-carbon energy production eventually reaches a plateau. We run our simulations using copper as the limiting mineral and we allow for its full recycling. Even in the limiting case of a 100% recycling rate, after five to six decades green energy production is 50% lower than in the scenario with unlimited primary copper, and after 30 decades, GDP is 3–8% lower. In extension scenarios, we confirm that a longer life duration of green capital delays copper extraction and the green energy peak, whereas reduced recycling caps moves the peak in green energy production forward.

Quelles sont les implications des contraintes liées aux minéraux primaires pour la transition énergétique ? La production d'énergie à faible teneur en carbone fait appel au capital vert, qui nécessite des minéraux primaires. Nous nous appuyons sur le cadre fondateur de la transition d'une énergie sale à une énergie propre de Golosov et al. (2014) pour intégrer le rôle joué par les minéraux primaires et leur recyclage potentiel. Nous caractérisons les voies optimales de la transition énergétique dans divers scénarios de contraintes minérales. Les contraintes minérales limitent le développement des énergies vertes à long terme : la production d'énergie à faible teneur en carbone finit par atteindre un plateau. Nous effectuons nos simulations en utilisant le cuivre comme minéral limitant et nous permettons son recyclage complet. Même dans le cas limite d'un taux de recyclage de 100 %, après cinq à six décennies, la production d'énergie verte est inférieure de 50 % à celle du scénario où le cuivre primaire est illimité, et après 30 décennies, le PIB est inférieur de 3 à 8 %. Dans les scénarios d'extension, nous confirmons qu'une durée de vie plus longue du capital vert retarde l'extraction du cuivre et le pic de production d'énergie verte, tandis qu'une réduction des plafonds de recyclage avance le pic de production d'énergie verte.

---

[*] Polytechnique Montreal, department of Mathematics and Industrial Engineering, Canada and Ecole Polytechnique, France
[†] Polytechnique Montreal, department of Mathematics and Industrial Engineering, and CIRANO, Canada
[‡] McGill University, Department of Economics, and CIREQ, Canada

We thank the two generous reviewers, and participants at the Monte Verità Conference on Sustainable Resource Use and Economic Dynamics (SURED) 2020, Canadian Resource and Environmental Economics Association (CREEA) 2020, the workshop" Matières premières critiques dans la transition énergétique", France Stratégie 2020, and the Société Canadienne de Sciences Economiques (SCSE) 2021 for helpful comments. In particular, we are thankful to Aude Pommeret and Francesco Ricci. Hassan Benchekroun and Simon Chazel thank the Canadian Social Sciences and Humanities Research Council (SSHRC) for financial support.

Keywords/Mots-clés: Energy Transition, Green Capital, Recycling, Circular Economy, Mineral Constraint, Dynamic General-Equilibrium Model / Transition énergétique, capital vert, recyclage, économie circulaire, contrainte minérale, modèle d'équilibre général dynamique

Pour citer ce document / To quote this document

Chazel, S., Bernard, S., & Benchekroun, H. (2023). Energy transition under mineral constraints and recycling: A low-carbon supply peak (2023s-09, Working Papers, CIRANO.) https://doi.org/10.54932/EZHR6690

1 Introduction

The necessity of an energy transition toward low-carbon energy is well documented. However many actors warn that low-carbon energies are far more material intensive than fossil energies, and that the energy transition will require huge amounts of raw materials.[2] Indeed, low-carbon energy production requires large infrastructures that are made of huge quantities of base metals. Vidal, Goffé, and Arndt [31] estimate that "for an equivalent installed capacity, solar and wind facilities require up to [. . .] 90 times more aluminium, and 50 times more iron, copper and glass than fossil fuels or nuclear energy." Focusing on copper, Hertwich *et al.* [11] show that wind and solar energy production technologies are 8 times more copper intensive than coal and oil energy production. Vidal, Goffé, and Arndt [31] estimate that the World Wide Fund for Nature (WWF) energy transition scenario requires 40Mt of copper (2 times current annual production) and 310Mt of aluminium (almost 5 times current annual production) by 2050, which is considerable since renewable energy infrastructure is only a small fraction of those metal use worldwide. Indeed, copper is not only used for clean energy production, but also for other energy transition key uses, such as electrical road transport and electricity distribution. Thus, copper uses related to the energy transition account for 50% of total uses in 2050 in Seck *et al.* (2020) scenarios [25], and for 30-45% of total uses in International Energy Agency (IEA) scenarios [14]. Vidal *et al.* [32] highlight that production of those metals is already highly solicited by many country's current industrialization, and that energy transition additional demand could increase this growth to critical levels.

Base metals have a very high recycling potential, and the recycling industry is already developed and cost efficient. For example, recycled copper has the exact same physical properties and value as primary copper, and one fifth of world copper production already comes from recycling. Metal recycling is also in general 50–90% more energy-efficient than primary production. However, secondary mineral production is limited by the total amount of mineral that has already been mined, so that recycling in itself cannot sustain a growing demand. Even with high recycling potential, we rely on primary mineral production to satisfy the booming demand for minerals.

[2] See, for instance, [29, 28, 2, 5, 6]

In this paper, we examine the impact of limited mineral availability on the energy transition. We build on the seminal framework developed in Golosov, Hassler, Krussel and Tsyvinski [9] (GHKT). In their model, carbon-free energy production only depends on labour inputs; they abstract from green capital production, and hence from mineral resource constraints. In our general model, we incorporate a mineral (copper) extraction sector, which depends on both scarcity and labour costs. This allows us to treat the infinite mineral scenario as a special case, and to create a benchmark scenario that replicates GHKT. In a dynamic general equilibrium model, calibrations from various expected scenarios on copper availability are compared to the benchmark.

Our results show how limited access to mineral resources affects GHKT core results on the energy transition. In all scenarios, we find that the mineral constraint limits the development of green energy. In our model, green energy eventually reaches a plateau, in contrast with GHKT where it grows at a positive rate in the long-run. Our simulations also provide insights into the "mineral transition," from primary extraction to recycling, in the composition of green capital. An increasing mineral recycling rate allows delaying by 40–60 years the green capital plateau compared to a situation without recycling. When recycling reaches full capacity, labour productivity gains compensate this plateau, thus maintaining a positive growth in green energy production. After 6–8 decades, green energy production reaches a peak, and then falls and converges towards a plateau. Other results show that slower capital depreciation postpones the peak of green energy production, while lower recycling rate caps reduce green energy production capacity and move peak production forward.

While there is a relatively rich literature on the energy transition, or growth under resource scarcity,[3] there are very few studies that explicitly incorporate the role of minerals, and their recycling, in green capital and low-carbon energy production. Fabre, Fodha and Ricci [7] and Pommeret, Ricci and Schubert [22] use dynamic models of energy transition with green capital. They explore the impact of finite mineral stocks on optimal timing in energy production and resource use. Our paper contributes to this literature in three ways: (1) by offering results from an energy transition model calibrated with real data, (2) by capturing the impact of a specific mineral, namely copper, and (3) by quantifying the deviation from a benchmark model induced by a mineral constraint. The model used in our paper is calibrated with real data. This allows providing meaning-

[3] See, for instance, [1, 23, 26, 27, 10]

ful quantitative measures of the transition phases. By focusing on copper, we can provide a specific quantitative analysis of the exhaustion of an important critical mineral for the energy transition. The adoption of GHKT's model as a starting point is motivated by the fact that it has provided a well-established benchmark that examines energy transition in a fairly general continuous time framework, and which was used to provide a quantitative analysis of the energy transition using real life data.

Fabre *et al.* study the optimal energy mix, when both mineral and fossil resources are scarce, but when minerals are recyclable. Green capital stock accumulation is fed by nonrenewable resources extraction and by the exogenous recycling of depreciated capital. They characterized analytically the possible optimal transition paths in a simplified version of their model. The addition of important features such as technological change, convex extraction costs and environmental damages is done in a two-period version model, resorting to numerical illustration of the qualitative results. The numerical examples provided were using illustrative parameter values not related to any real-life parameter values of a particular critical mineral. While Fabre *et al.* (2020) consider a recycling process that is exogenous, constant and costless, in our framework, we allow for technological change, endogenous and costly recycling. They take renewable energy to be simply proportional to the amount of green capital whereas in our case renewable energy is produced using labour and green capital as inputs which allows us to also track the transition of the labour force during the energy transition. Fabre *et al.* (2020) show that more CRM results in more investments in green capital. This is confirmed in our framework as well. However, our analysis reveals that the path of the stock of green capital reaches a peak before declining. Our simulations also reveal that extraction of minerals can be nonmonotonic (inverted U shaped) when the initial stock of minerals is large enough. They also show that an increase in the recycling rate of minerals results in an increase of the share of renewable energy and an acceleration of the investment in renewable energy capacity. While these intuitive qualitative results still hold in our framework, our analysis provides a quantitative analysis of the impact of recycling. Indeed, our simulations reveal that the paths are almost identical for an initial period of time (70 years) and the difference between the paths only become 'significantly' different after 80 years.

Pommeret *et al.* (2021) also examines the effect of the scarcity of minerals on energy transition. A first important difference with our framework is the objective: while we examine a social optimum that can be decentralized in a market economy as in GHKT through a tax on carbon, they abstract from the modelling of pollution damages and, instead, consider a carbon budget and examine different policies when a carbon tax alone is not feasible. They allow for the carbon tax revenue to subsidize the production of renewable energy. The other two important differences lie in the model assumptions: they assume a backstop clean technology that requires no Critical Raw Materials (CRM), and perfect substitutability between fossil fuel and clean energy.

In contrast with Pommeret *et al.* (2021) and Fabre *et al.* (2020) and given the interest in simulating the model with real life data, we have adopted the assumption in GHKT regarding the imperfect substitutability of the energy sources, as well as the absence of a backstop technology. This facilitates the comparison of the outcome with scarce CRM to the case where minerals are abundant (provided in GHKT). While a backstop technology is likely to mainly influence the long-run outcome of the economy, the substitutability is likely to play an essential role both in the short-run and long-run effects of the scarcity of CRM. The degree of substitutability of the different sources of energy is clearly crucial for the energy transition. A natural and promising extension of our work (and of Pommeret *et al.* (2021)) would be to allow for the substitution between energy sources to be endogenous. This extension is beyond the scope of the present paper. Moreover, in our model we have two sources of fossil fuels: oil and coal. Those are two important but distinct fossil fuel sources of energy with differentiated use and characteristics: coal is assumed abundant, but labour intensive, whereas oil is scarce. Their extractions during the transition towards clean energy and their responses to the severity of the scarcity of CRM are not identical. Thus, our framework allows us to keep track of the impact of the scarcity of CRM on the damage from pollution as well as identify the specific impacts on coal versus oil extraction. Indeed, our simulations reveal that initial extraction of oil is a decreasing function of the initial stock of CRM whereas the opposite is true for coal with the overall impact of a slower accumulation of carbon in the atmosphere.

An important point raised in our paper relates to the availability of a primary copper stock for the energy transition. In addition to the scarcity cost, environmental damages of metal mining are substantial—one of the top polluting and energy intensive industries [21] [32]—and increase over time when mine depths increase and deposit ore grade declines [20, 13]. Hence, although there is no consensus on the probability of base metal peak production due to resource exhaustion in the next centuries,[4] growing environmental constraints call for a limit on primary resource production. We do not model the environmental damages from copper extraction, but our scenarios' initial primary copper stock can be interpreted as a *copper budget* for energy transition. Analogous to a carbon budget (see Pommeret *et al.* (2021) for example), mineral constraints can represent the political desire to lower pollution accumulation derived from extraction.

Section 2 covers the model. In Section 3 we characterize the optimal paths for labour allocation across sectors, as well as resource scarcity rents. Section 4 presents the simulation method and model calibration. Section 5 highlights the results. In Section 6 we extend the model to look at various depreciation rates and recycling caps. Section 7 concludes.

2 The model

We first offer an overview of the benchmark framework of GHKT where the energy transition is examined in the absence of mineral constraints. We then build on that framework to include green capital and the dynamics of the mineral sector to the production of green energy.

2.1 Energy transition without mineral constraints: GHKT (2014)

The simulation lasts T periods and the discount factor is β. We denote by C_t consumption at time t and $U(C_t)$ the instantaneous utility of a representative household, with

$$U(C_t) = \ln(C_t). \tag{1}$$

Y_t is total output of the economy, and K_t the amount of capital at time t. In each period, the production of final goods is shared between immediate consumption C_t and savings K_{t+1} (i.e. capital for the next period). A total depreciation of capital over the course of one period is assumed. Therefore,

$$K_{t+1} + C_t = Y_t \tag{2}$$

[4] See, for instance, [20, 18, 24] for a discussion on this topic.

$$0 \leq C_t \leq Y_t. \tag{3}$$

Final goods are produced from capital K_t, labour $N_{0,t}$ (where 0 stands for the final good sector), and energy E_t. The production function is a standard Cobb-Douglas function:

$$Y_t = A_{0,t}\ e^{-\gamma(S_t-\bar{S})}\ K_t^{\alpha}\ N_{0t}^{1-\alpha-\nu}\ E_t^{\nu}. \tag{4}$$

$A_{0,t}$ is total factor productivity, α and ν are the output elasticities of capital and energy. Production of final goods also depends on climate (described here by the amount of carbon in the atmosphere S_t), with a damage factor γ. $\bar{S}$ is the pre-industrial level of atmospheric carbon.

Energy comes from three sources: oil ($E_{1,t}$), coal ($E_{2,t}$) and a low-carbon energy ($E_{3,t}$).[5] Those energy sources are imperfect substitutes. The parameter ρ characterizes interfuel substitution, whereas κ_1, κ_2 and κ_3 designate the relative efficiency of each energy source:

$$E_t = \left(\kappa_1 E_{1,t}^{\rho} + \kappa_2 E_{2,t}^{\rho} + \kappa_3 E_{3,t}^{\rho}\right)^{\frac{1}{\rho}}. \tag{5}$$

Oil is extracted from the oil stock $R_{1,t}$. The cost of extraction, in terms of labour, capital and energy is assumed to be negligible with respect to the cost of scarcity. Therefore, $E_{1,t}$ can be chosen freely in the range of admissible values, without any labour, capital or energy involved.

$$E_{1,t} = R_{1,t} - R_{1,t+1} \tag{6}$$

$$0 \leq E_{1,t} \leq R_{1,t}. \tag{7}$$

As an abundant resource, coal's extraction cost dominates its scarcity cost. Therefore, GHKT assumes an extraction cost of:

$$E_{2,t} = A_{2,t} N_{2,t}, \tag{8}$$

where $A_{2,t}$ is an exogenous labour productivity variable and $N_{2,t}$ is labour in the coal extraction sector.

[5] Thus in contrast with Fabre *et al.* (2020) and Pommeret *et al.* (2021), we have two sources of energy from fossil fuel. While oil reserves are finite, coal is assumed abundant.

Carbon emissions accumulate in the atmospheric carbon stock during each period. Part of the stock also depreciates; $(d_s)_{s=0}^{\infty}$ denotes the proportion of CO_2 emitted s periods ago that have been removed from the atmosphere. GHKT specifies the sequence $(d_s)_{s=0}^{\infty}$.[6] Then,

$$S_t = \bar{S} + \sum_{s=0}^{t} (1 - d_s)\ (E_{1,t-s} + E_{2,t-s}). \tag{9}$$

2.2 Green Capital and Minerals

We now depart from the benchmark framework which assumes that low carbon energy production only uses labour. In our model, low carbon energy production requires, in addition to labour, green capital denoted by G_t. We use a standard CES production function with a negative parameter of substitution $\ddot{\rho}$.[7] $A_{3,t}$ exogenously measures labour productivity. Parameter ψ characterizes the energy that can be obtained from a given amount of green capital over the course of one period. The variable $N_{3,t}$ stands for labour directly involved in low-carbon energy production. Hence, green energy production is:

$$E_{3,t} = \left[\kappa_L (A_{3,t} N_{3,t})^{\ddot{\rho}} + \kappa_G (\psi G_t)^{\ddot{\rho}}\right]^{\frac{1}{\ddot{\rho}}}. \tag{10}$$

Green capital G_t is constituted from a flow of primary and secondary mineral resources, and does not require investment of savings. We denote $m_{p,t}$ (resp. $m_{s,t}$) the flow of primary (resp. secondary) minerals. The green capital production function is a standard CES function. The substitution parameter of primary for secondary mineral resources is $\tilde{\rho}$, and the share parameters are κ_s and κ_p. As for regular capital in GHKT, a total depreciation of green capital is assumed over the course of one period.[8] Thus, its stock depends on the current flow of primary and secondary minerals, and stays independent from past amounts of green capital:

$$G_t = \left(\kappa_s m_{s,t}^{\tilde{\rho}} + \kappa_p m_{p,t}^{\tilde{\rho}}\right)^{\frac{1}{\tilde{\rho}}}. \tag{11}$$

[6] See Table 2 or GHKT 2014 for details.

[7] Knoblach *et al.* [17] metastudy estimates an elasticity of substitution between labour and capital smaller than one, which makes the substitution parameter $\ddot{\rho}$ negative. See Appendix D for more details on the choice of parameter values.

[8] In this reference case, we replicate GHKT and assume full depreciation after 10 years. See Section 6 for a 20-year depreciation rate.

As for coal in GHKT, primary mineral extraction $m_{p,t}$ requires labour $N_{p,t}$, whose productivity is $A_{p,t}$. Primary mineral extraction is bounded by the remaining primary mineral reserves $M_{p,t}$.

$$m_{p,t} = A_{p,t} N_{p,t} \tag{12}$$

$$m_{p,t} \leq M_{p,t} \tag{13}$$

$$M_{p,t+1} = M_{p,t} - m_{p,t}. \tag{14}$$

Similarly, secondary mineral extraction $m_{s,t}$ requires a labour input $N_{s,t}$, whose productivity is denoted by the exogenous variable $A_{s,t}$. It is bounded by the total reserve of secondary mineral available at this time $M_{s,t}$.

$$m_{s,t} = A_{s,t} N_{s,t} \tag{15}$$

$$0 \leq m_{s,t} \leq M_{s,t}. \tag{16}$$

Since green capital depreciates entirely over the course of one period, secondary mineral stock evolution is:

$$M_{s,t+1} = M_{s,t} - m_{s,t} + (m_{s,t} + m_{p,t})$$

which gives

$$M_{s,t+1} = M_{s,t} + m_{p,t}. \tag{17}$$

It is hence assumed that 100% of minerals accumulated in the stock $M_{s,t}$ are available for recycling without any loss.[9] Nonetheless, recycling comes at the opportunity cost of labour as per (15) and recycling rates $m_{s,t}/M_{s,t}$ will vary over time.

An underlying assumption in (17) is that there is a known and finite stock of mineral allocated to the energy transition ($M_{p,0}$), and that the input to the secondary material is solely from that original stock.[10] Because of its dependency on non-renewable mineral resources, green energy can also be interpreted as

[9] This reference case stays close to GHKT optimistic approach. This assumption is relaxed in Section 6, where recycling caps are introduced.

[10] We hence avoid the conjecture where, as we transition towards greener energy, a larger share of the original stock moves to green capital production, leaving a smaller share to other sectors in the economy. This would imply, for instance, that copper from communication networks is recycled into green capital.

non-renewable. Total energy production (5) hence combines three primary non-renewable energy sources (oil, coal and minerals) that differ in their abundance, labour intensity and carbon emissions. With resource availability constraints and carbon stock accumulation, we can anticipate an inevitable declining trajectory in the long-run.

It is assumed, as in GHKT, that labour can move freely between all sectors at all times. The feasibility constraint on labour allocation is:

$$N_{0,t} + N_{2,t} + N_{3,t} + N_{p,t} + N_{s,t} = N_t \tag{18}$$

with N_t the total labour supply, which is exogenous.

3 The planner's problem

The social planner's problem is to maximize total discounted utility,

$$\max_{\{C_t, K_{t+1}, N_{0,t}, E_{1,t}, N_{2,t}, N_{3,t}, N_{p,t}, N_{s,t}\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t U(C_t)$$

under the feasibility constraints (2), (3), (7), (13), (16), (18). The constraints are linear, and the objective function is strictly concave, so that there is a unique solution to the optimization problem. Note that the problem's number of decision variables was reduced to eight, i.e., C_t, K_{t+1}, $N_{0,t}$, $E_{1,t}$, $N_{2,t}$, $N_{3,t}$, $N_{p,t}$ and $N_{s,t}$. All the other variables can be obtained as a combination of these eight variables, using Equations (4)–(6), (8)–(12), (14)–(15), (17).

3.1 Lagrangian

We write the Lagrangian, using Greek letters for the Lagrange multipliers. We intentionally omit the Karush-Kuhn-Tucker multipliers here to enhance readability.

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{T} \beta^t U(C_t) &+ \sum_{t=0}^{T} \beta^t \pi_{K,t}\ (K_{t+1} - Y_t + C_t) \\
&+ \sum_{t=0}^{T} \beta^t \lambda_{0,t}\ (Y_t - A_{0,t}\ e^{-\gamma_t (S_t - \bar{S})}\ K_t^{\alpha}\ N_{0,t}^{1-\alpha-\nu}\ E_t^{\nu}) \\
&+ \sum_{t=0}^{T} \beta^t \xi_t\ (E_t - (\kappa_1 E_{1,t}^{\rho} + \kappa_2 E_{2,t}^{\rho} + \kappa_3 E_{3,t}^{\rho})^{\frac{1}{\rho}}) \\
&+ \sum_{t=0}^{T} \beta^t \mu_{1,t}\ (E_{1,t} - R_{1,t} + R_{1,t+1}) \\
&+ \sum_{t=0}^{T} \beta^t \lambda_{2,t}\ (E_{2,t} - A_{2,t} N_{2,t}) \\
&+ \sum_{t=0}^{T} \beta^t \zeta_t\ (S_t - \bar{S} - \sum_{s=0}^{t+T_0} (1 - d_s)\ (E_{1,t-s} + E_{2,t-s})) \\
&+ \sum_{t=0}^{T} \beta^t \lambda_{3,t}\ (E_{3,t} - (\kappa_L (A_{3,t} N_{3,t})^{\ddot{\rho}} + \kappa_G (\psi G_t)^{\ddot{\rho}})^{\frac{1}{\ddot{\rho}}}) \\
&+ \sum_{t=0}^{T} \beta^t \pi_{G,t}\ (G_t - (\kappa_s m_{s,t}^{\tilde{\rho}} + \kappa_p m_{p,t}^{\tilde{\rho}})^{\frac{1}{\tilde{\rho}}}) \\
&+ \sum_{t=0}^{T} \beta^t \lambda_{p,t}\ (m_{p,t} - A_{p,t} N_{p,t}) \\
&+ \sum_{t=0}^{T} \beta^t \mu_{p,t}\ (M_{p,t+1} - M_{p,t} + m_{p,t}) \\
&+ \sum_{t=0}^{T} \beta^t \lambda_{s,t}\ (m_{s,t} - A_{s,t} N_{s,t}) \\
&+ \sum_{t=0}^{T} \beta^t \mu_{s,t}\ (M_{s,t+1} - M_{s,t} - m_{p,t}) \\
&+ \sum_{t=0}^{T} \beta^t \chi_t^N\ (N_{0,t} + N_{2,t} + N_{3,t} + N_{p,t} + N_{s,t} - N_t).
\end{aligned}
$$

In Section 3.2 we analytically derive the optimal path of labour allocation across sectors. Section 3.3 presents a set of conditions that characterize the optimal paths of resource scarcity rents. Section 3.4 gives a necessary condition on labour productivity's initial values $A_{s,0},\ A_{p,0},\ A_{3,0}$ for which, under infinite mineral stock, the planner's solution in our model corresponds to the planner's solution in GHKT.

3.2 Labour distribution in optimal paths

The labour force is shared by five sectors: final goods, coal extraction, low-carbon energy, primary and secondary minerals. In each sector, it contributes to final goods production, either directly ($N_{0,t}$), or indirectly via energy production ($N_{2,t}$, $N_{3,t}$, $N_{s,t}$, $N_{p,t}$). Using the Lagrangian, we prove that the optimal distribution of labour, when the feasibility constraint (16) is not binding, is achieved when the marginal benefit of labour in each sector is equal.

Proposition 1 *On the optimal paths, the marginal benefit of labour in each sector is equal at all times. More specifically, coal labour's marginal benefit (correcting the marginal product by the marginal damage) is equal to the final good's labour marginal product:*

$$\frac{\partial Y_t}{\partial N_{0,t}} = \frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{2,t}}\frac{\partial E_{2,t}}{\partial N_{2,t}} - \Lambda_t \frac{\partial E_{2,t}}{\partial N_{2,t}} \tag{19}$$

where Λ_t is the marginal externality damage as defined in GHKT.
The marginal product of direct labour in low-carbon energy production is equal to the marginal product of labour in the final goods sector:

$$\frac{\partial Y_t}{\partial N_{0,t}} = \frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{3,t}}\frac{\partial E_{3,t}}{\partial N_{3,t}}. \tag{20}$$

When the feasibility constraint (16) *is not binding, the marginal product of labour in mineral recycling and the final goods sectors are equal:*

$$\frac{\partial Y_t}{\partial N_{0,t}} = \frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{3,t}}\frac{\partial E_{3,t}}{\partial G_t}\frac{\partial G_t}{\partial m_{s,t}}\frac{\partial m_{s,t}}{\partial N_{s,t}}. \tag{21}$$

A proof of Proposition 1 is presented in Appendix A.

The marginal contribution of the primary mineral sector's labour $N_{p,t}$ to total welfare cannot be written explicitly. This is because it contributes to both present and future welfare, directly (through the primary mineral flow $m_{p,t}$) and indirectly (through the composition of the secondary mineral stock $M_{s,t}$ and depletion of the primary mineral stock $M_{p,t}$). Coal production also has an indirect (and intertemporal) impact through carbon accumulation, but GHKT makes a series of assumptions to obtain an explicit form (Λ_t).[11]

Note that coal extraction impacts welfare through the production of final goods and the climate externality. The positive contribution corresponds to the first term of the right-hand side in Equation (19). The negative contribution, via the marginal externality Λ_t, corresponds to the second term. GHKT shows that Λ_t is exactly equal to the optimal Pigouvian tax.

When the recycling feasibility constraint (16) is binding, that is

$$M_{s,t} = A_{s,t} N_{s,t},$$

the Karush-Kuhn-Tucker multiplier associated with this equation in the Lagrangian is non-null. Therefore, a supplementary term appears when writing the first order condition with respect to $N_{s,t}$, and Equation (21) does not hold anymore. However, in this situation, $N_{s,t}$ can be directly obtained from :

$$N_{s,t} = \frac{M_{s,t}}{A_{s,t}}.$$

3.3 Resource scarcity rent in optimal paths

We derive two more optimality conditions from the Lagrangian. For both oil and mineral resources:

- the extraction level is chosen so that marginal costs of extraction (labor, increasing scarcity of primary mineral resource) exactly match the marginal benefits of extraction (final goods production, decreasing scarcity of secondary mineral resource);
- the Hotelling's rule applies to the resource scarcity rents of the three stocks (oil $\mu_{1,t}$, primary mineral $\mu_{p,t}$ and secondary mineral $\mu_{s,t}$), which increase at a rate equals to the discount rate.

Proposition 2 presents this result and a proof is given in appendix B.

[11] An explicit form is not needed for the calibration. The general form is given through Equations (31) and (32a) in Appendix B.2. Note that Proposition 2 gives conditions for the optimal paths of primary and secondary mineral stocks, and this links $N_{p,t}$ to $N_{p,t+1}$.

Proposition 2 *On the optimal paths, resource scarcity rents increase at rate $\frac{1}{\beta}$. For the oil stock, we have:*

$$\left(\frac{\nu\kappa_1}{E_{t+1}^{\rho}E_{1,t+1}^{1-\rho}} - \widehat{\Lambda}_{t+1}\right) = \frac{1}{\beta}\left(\frac{\nu\kappa_1}{E_t^{\rho}E_{1,t}^{1-\rho}} - \widehat{\Lambda}_t\right). \tag{22}$$

For primary and secondary mineral stocks, we have:

$$\left[\kappa_p \pi_{G,t+1}\left(\frac{G_{t+1}}{m_{p,t+1}}\right)^{1-\tilde{\rho}} + \frac{Y_{t+1}}{C_{t+1}}\frac{1-\alpha-\nu}{A_{p,t+1}N_{0,t+1}}\right] = \frac{1}{\beta}\left[\kappa_p \pi_{G,t}\left(\frac{G_t}{m_{p,t}}\right)^{1-\tilde{\rho}} + \frac{Y_t}{C_t}\frac{1-\alpha-\nu}{A_{p,t}N_{0,t}}\right] \tag{23}$$

where $\pi_{G,t}$ is the marginal contribution of green capital to welfare

$$\begin{aligned}\pi_{G,t} &= -U'(C_t)\frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{3,t}}\frac{\partial E_{3,t}}{\partial G_t}\\ &= -\frac{Y_t}{C_t}\frac{\nu\kappa_3\kappa_G\psi}{E_t}\left(\frac{E_t}{E_{3,t}}\right)^{1-\rho}\left(\frac{E_{3,t}}{\psi G_t}\right)^{1-\ddot{\rho}}\end{aligned}$$

and $\widehat{\Lambda}_t = \frac{\Lambda_t}{Y_t}$.

Equation (22) gives a condition that links $E_{1,t}$ to $E_{1,t+1}$. In equation (23), the second term in each bracket is the shadow price of primary mineral production $\lambda_{p,t}$, it links $N_{p,t}$ to $N_{p,t+1}$. This is specific to our model.

3.4 Relationship to the benchmark model (GHKT)

Below we provide the conditions under which calibration of parameters in our model leads to optimal extraction and labour allocation paths that coincide, in the limiting case of an infinite initial stock of mineral, with those obtained in GHKT. We identify GHKT variables with a $\sim$. Moreover, growth rates of labour productivity A_i are denoted g_{Ai} where $i = 1, 2, 3$.

In GHKT, the renewable energy production function is

$$\widetilde{E}_{3,t} = \widetilde{A}_{3,t}\widetilde{N}_{3,t}$$

where labour productivity $\widetilde{A}_{3,t}$, follows an exogenous path.[12]

Proposition 3 below clarifies the conditions under which we generate GHKT's optimal paths as a special case of our model with an infinite amount of minerals.

[12] See Table 3 in Appendix D for details.

To economize on notation, we also denote these optimal paths of low-carbon energy and labour allocated to low-carbon energy by $\widetilde{E}_{3,t}$ and $\widetilde{N}_{3,t}$.

Proposition 3 *Let $M_{p,0} = M_{s,0} = +\infty$, and let the aggregated low-carbon energy labour productivity be $\widehat{A}_{3,t} \equiv F(A_{3,t},\ A_{s,t},\ A_{p,t})$ where the function F is given in Appendix C. When $g_{A3} = g_{As} = g_{Ap} = \widetilde{g}_{A3}$ and $\widehat{A}_{3,0} = \widetilde{A}_{3,0}$,*

$$E_{3,t} = \widehat{A}_{3,t}(N_{3,t} + N_{s,t} + N_{p,t}) = \widetilde{E}_{3,t} \tag{24}$$

for all $t \geq 0$, where $N_{3,t}, N_{s,t}$ and $N_{p,t}$, solve

$$(N_{3,t} + N_{s,t} + N_{p,t}) = \widetilde{N}_{3,t} \tag{25}$$

$$\frac{N_{p,t}}{N_{s,t}} = \alpha_1 \tag{26}$$

$$\frac{N_{3,t}}{N_{s,t} + N_{p,t}} = \alpha_2 \tag{27}$$

and where α_1 and α_2 are constants defined in Appendix D.

Appendix C provides a proof of Proposition 3, and detailed expressions of the function F, aggregated labour productivity $\widehat{A}_{3,0}$ and labour share ratio expressions α_1 and α_2.

Propositions 1 and 2 are used for numerical resolution of the model while Proposition 3 is used for calibration of the model.
Remark: The optimal solution in our model shares the features of GHKT's optimal solution that (i) the optimal savings' rate is constant and therefore consumption is a constant ratio of total output and (ii) that the marginal damages from emissions is a constant proportion of the GDP. This greatly simplifies the numerical approach to complete characterization of the optimal paths of energy production and allocations of labour, and the simulation exercise.

4 Simulations

Section 4.1 details the method used for the simulations. Section 4.2 provides the details on the calibration and the choice of mineral resource constraint scenarios.

4.1 Algorithm

The simulation is run on Matlab where we compute an approximate finite horizon solution. As noted in the remark above, the consumption level is a constant ratio of total output Y_t. Constraints (2) and (18) allow the elimination of 2 more decision variables, for instance K_{t+1} and $N_{0,t}$. At this point, 5 decisions remain in each period: labour distribution N_2, N_3, N_s, N_p, and oil extraction level E_1. Since the simulation lasts 30 periods, there are a total of 150 decisions.

We use Propositions 1 and 2 to reduce the problem to a two variable optimization problem. First, Proposition 1 allows $N_{2,t}$, $N_{3,t}$ and $N_{s,t}$ to be computed when $N_{p,t}$ and $E_{1,t}$ are given, by solving a nonlinear 3 equation system. Then, Proposition 2 yields the optimal $N_{p,t+1}$ and $E_{1,t+1}$ when $N_{p,t}$ and $E_{1,t}$ are given, by solving a nonlinear 2 equation system. Both of those systems are solved using *fsolve* algorithm. In the end, for any couplet of initial values ($E_{1,0}$, $N_{p,0}$), we are able to compute the paths that:

- start with the given initial values for $E_{1,t}$ and $N_{p,t}$
- respect optimality conditions given in Propositions 1 and 2
- respect all feasibility conditions.

Finally, we optimize over those two initial values to maximize the total discounted value, using *fminsearch* algorithm.

4.2 Calibration and mineral resource constraint scenarios

We use the parameter values in GHKT for the parameters that are not related to green capital and minerals. Then, we calibrate the parameters that are specific to our model using the case of copper in wind energy production.[13] Full details are provided in Appendix D.

The main exercise was to determine a value for the initial stock of primary minerals $M_{p,0}$. To give an order of magnitude, 40MtCu would be required to build wind and solar facilities that correspond to a WWF scenario of energy transition on the 2050 horizon [31]. 330MtCu would be needed for a full adaptation of infrastructure, including the electrification of transportation [8]. Current

[13]This can be interpreted as if the energy transition relied on wind energy exclusively, or, alternatively, as if the mineral and labour intensity of the transition (with the development of the network, electric vehicles, PV solar, storage batteries, etc.) was analog to copper in wind energy production.

global production of copper is around 20MtCu/year while for wind energy production only, the amount is approximately 2.5MtCu/year. Finally, total copper resources—identified and undiscovered—are estimated at 5600MtCu [16, 30].

There is no consensus in the literature on estimates of total copper resources ultimately available for mining. Resource estimates depend on many economic parameters, and can change if extraction costs are reduced, or if technological innovations grant (economically viable) access to further deposits. However, it is unclear whether technological innovations and productivity gains can maintain copper extraction costs constant in the long-run while deposits inevitably become poorer and more remote. In other words, as Meinert *et al.* [18] highlight, it is hard to provide an estimate of ultimate recoverable resources. Moreover, our model only considers one specific use of copper, that is manufacturing green capital for low-carbon energy production. However, copper production is dedicated to many different uses that are not accounted for in this model. Finally, governments could limit primary copper extraction because of the growing pollution intensity of that industry. Therefore, only a small ratio of total copper resources will be used for green capital manufacturing.

We model four copper budget scenarios for wind energy production, ranging from $M_{p,0} = 50MtCu$ to $M_{p,0} = 2000MtCu$.

5 Results

This section presents the results of the set of the simulations we ran. They are the first best solution to the social planner's problem from Section 3. These results are compared to those of GHKT, which disregard green capital and recycling. As in GHKT, we have a 300 year horizon: 30 periods of 10 years. It means for instance that $t = 10$ stands for the year 2110 (time starts in 2010).

In the infinite mineral scenario, there is no scarcity rent for primary and secondary minerals. We have shown in Proposition 3 that low-carbon energy production function can be written as the product of a labour share times an aggregated labour productivity. We calibrate this aggregated labour productivity $A_{3,0}$ so that it matches GHKT low-carbon energy labour's productivity. We check numerically whether our simulated low-carbon energy paths match

GHKT's simulation. We obtain less than 1% relative error. This is an important consistency check, since we intend to measure the impact of a mineral constraint on the optimal energy transition paths computed by GHKT.

5.1 Low-carbon energy production

When there is a finite amount of mineral in the model, primary and secondary mineral flows are bounded. Therefore, green capital is also bounded. However low-carbon energy is produced with labour and green capital inputs. Since the parameter capturing labour-capital substitution is negative, the upper limit on green capital induces an upper limit on low-carbon energy production. Therefore, the mineral constraint leads to a limitation on low-carbon energy production.

Figure 1 shows low-carbon energy production paths under various mineral stock scenarios. In the infinite mineral scenario, production growth is unlimited at a yearly rate of 1.9%. At the end of the time horizon, when the mineral stock is finite, low-carbon energy production is just a fraction of its value under resource abundance: less than 10% even in the scenario with $\boldsymbol{M_{p,0}} = 2000$MtCu. For the two scenarios with initial stocks of copper $M_{p,0} = 50,\ 200$MtCu, the paths of low-carbon energy production are markedly lower than the low-carbon energy path under unconstrained mineral availability, from the initial period. The impact of the mineral constraint on green capital production is presented in Figure 2.

Result 1 *Under primary mineral constraint scenarios, low-carbon energy production reaches a peak whose date and value depend on total mineral amount.*

5.2 Primary copper production and recycling

Figure 3 shows primary copper extraction paths in all scenarios and the copper recycling rate. In the infinite mineral scenario, primary copper extraction grows by 1.9% annually. In all other scenarios, primary copper production peaks in the next century. In the two scenarios with the tightest primary mineral constraint ($M_{p,0} = 50,\ 200$ MtCu), the peak occurs during the first period of the simulation.

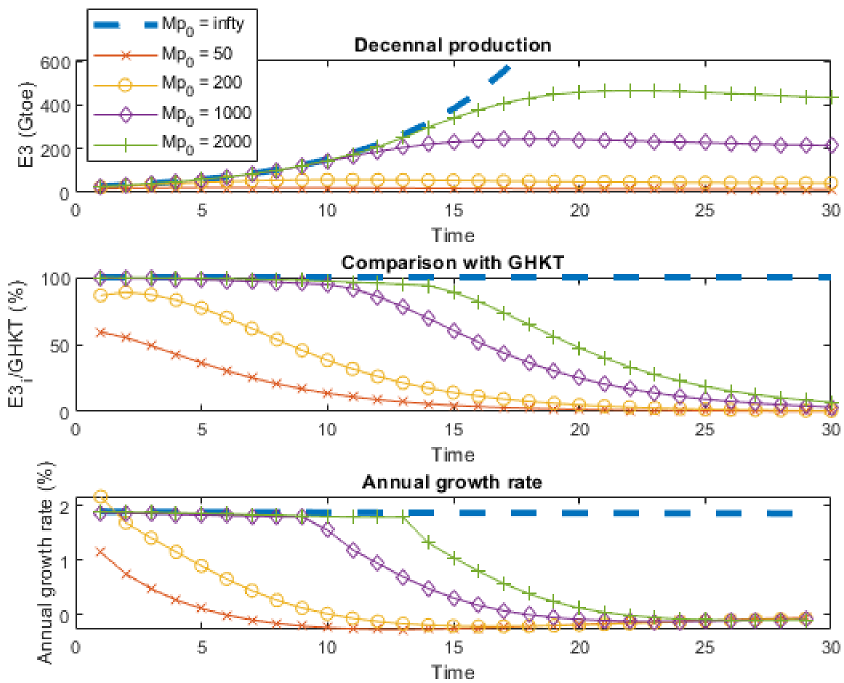

Figure 1: Low-carbon energy production in various mineral stock scenarios

After primary copper production peaks, recycled copper production growth compensates for the decline in primary copper. The recycling rate (defined as the ratio of the secondary mineral stock that is recycled in a given period $m_{s,t}/M_{s,t}$) increases, until it reaches its maximum of 100%, which takes 4–6 decades. Before hitting the feasibility limit, there is some slack on scarcity because we can partially cope with increased scarcity of the primary resource by increasing the recycling rate. Once this maximum is reached, green capital can not increase anymore. This moment coincides with the decline in low-carbon energy production growth. At that point, there is in fact only one decision variable to determine the evolution of green capital, instead of two, as it was previously the case. We conclude that mineral recycling delays the peak of green energy production by 40–60 years. Note that the initial stock of secondary mineral $M_{s,0}$ is relatively small, which leads to high recycling rates even for more abundant mineral constraints. This explains the early U-shape in recycling rates in Figure 3.

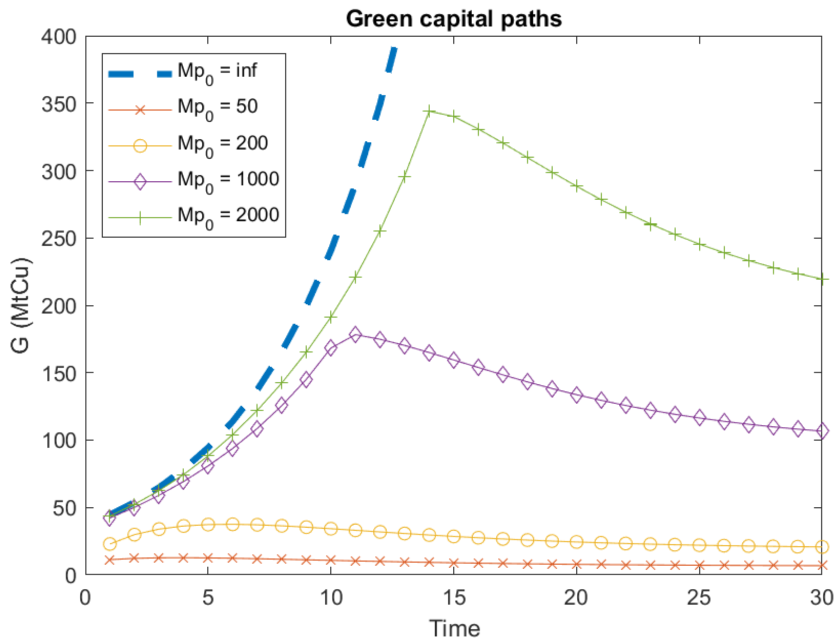

Figure 2: Green capital production in various mineral stock scenarios

Result 2 *Green capital production peaks when the recycling rate reaches 100%. Mineral recycling allows dissociating green capital production from primary mineral extraction, and the peak of green capital production occurs 40 to 60 years after the peak of primary mineral extraction.*

Note that from Figures 2 and 3, we have that the path of green capital is uniformly increasing with respect to the initial stock of minerals, which corroborates the finding of Fabre *et al.* (2020). However, our analysis reveals that the path of the stock of green capital reaches a peak before declining whereas in the case of Fabre *et al.* (2020) these paths are declining over time.

5.3 Green capital recycled mineral content

Green capital is made of primary and secondary minerals. Figure 4 shows the evolution of the recycled mineral ratio in green capital manufacturing. In the infinite mineral scenario, this ratio is constant at 15%. In all other scenarios, recycled mineral gradually substitutes to primary mineral in green capital manufacturing. Within 10 to 12 decades, the share of recycled mineral in green capital increases from 20% to 90%. In the long run, green capital is made of

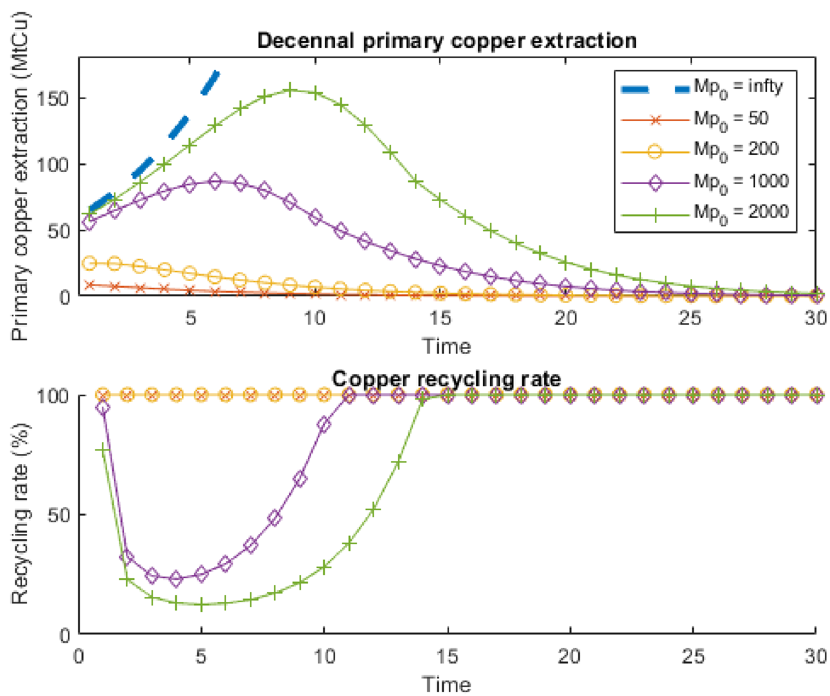

Figure 3: Decennial primary copper extraction m_p, and copper recycling rate paths in all scenarios

100% recycled minerals. A comparison between Figure 3 and Figure 4 shows that the recycled mineral share in green capital starts to increase before the peak of primary mineral production, when primary mineral production growth starts to slow down and depart from the infinite mineral scenario. When the peak of primary mineral production occurs (Figure 3), the recycled mineral content of green capital has already reached 55% (± 5%) in all scenarios.

Result 3 *When the primary mineral is abundant, green capital's recycled mineral content is stable at 15%. The slowdown and decline of primary mineral production leads to a gradual increase of recycled mineral share in green capital, until it eventually reaches 100%.*

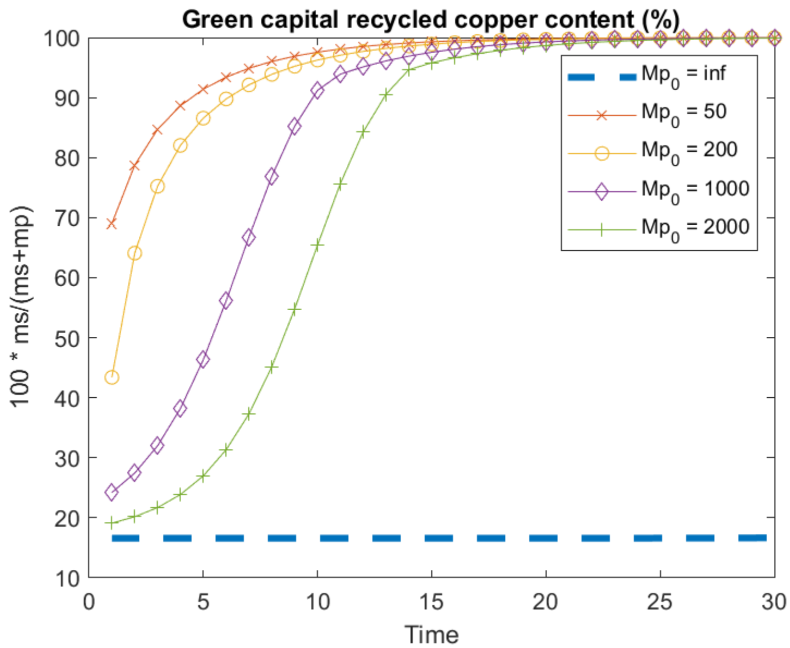

Figure 4: Share of recycled copper in green capital

5.4 Labour and green capital inputs to low-carbon energy production

Figure 5 shows the paths of green capital and labour inputs to low-carbon energy production in the $M_{p,0} = 1000$ MtCu scenario. Green capital input reaches a peak, which coincides with a drop in low-carbon energy labour share, but the labour input keeps increasing thanks to labour productivity gains. In the long run, the imperfect substitutability of green capital and labour input limits low-carbon energy production. Labour productivity gains delay by 6–8 decades the peak of low-carbon energy production.

Result 4 *After the green capital peak, labour productivity gains delay the peak of low-carbon energy production by 6–8 decades.*

5.5 Mineral reserves and oil and coal extraction paths

Figures 6 and 7 respectively give the oil and coal extraction paths under the different scenarios on copper reserves. While lower mineral reserves result in a decrease in coal extraction, the impact on oil extraction is ambiguous: a smaller

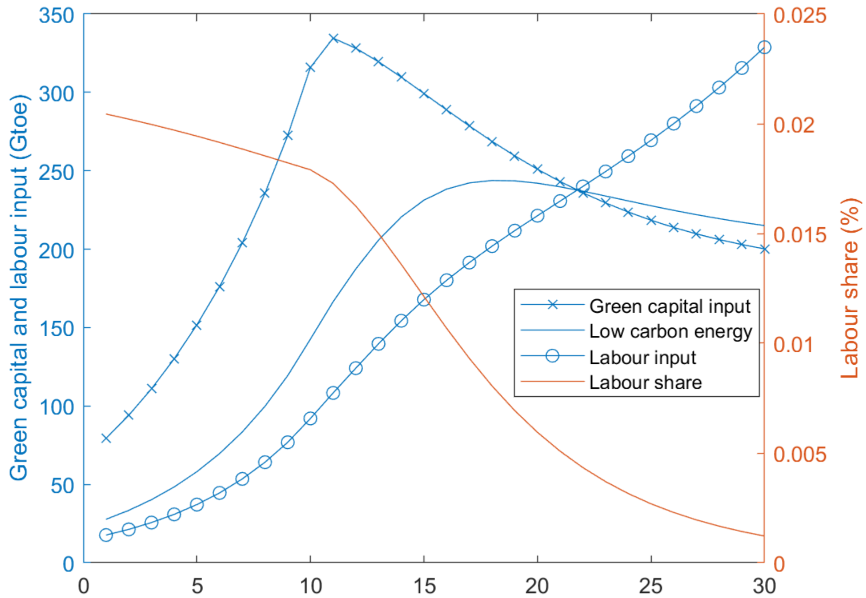

Figure 5: Green capital and labour inputs to low-carbon energy production, for a mineral constraint $M_{p,0} = 1000$ MtCu

initial stock on copper moves oil extraction forward. This result is driven by the imperfect substitutability of energy sources in production. Because the input from all energy sources is essential at all time, a lower mineral resource endowment reduces the potential value of both oil and coal. Oil resources are depleted after 300 years à la Hotelling, and hence lower value translates into early extraction.

Greater mineral reserves accelerate and increase low-carbon energy production. Nonetheless, the overall impact on the carbon stock is not clear *a priori* since the impact on fossil fuel sources differ. From Figure 8 we can see that, although the impact is discrete, a smaller stock of copper slows the accumulation of carbon in the atmosphere: the total atmospheric carbon stock is 1% lower after 16 decades with the $M_{p,0} = 50MtCu$ constraint than in the infinite mineral scenario.[14]

[14]This is not modelled here, but here are some intuitions on what occurs if we assume substitutability of energy sources. The result is that energy sources can be used more sequentially (and less simultaneously). In a first phase, oil extraction is anticipated because its production becomes nonessential in the future. As carbon accumulates, the environmental

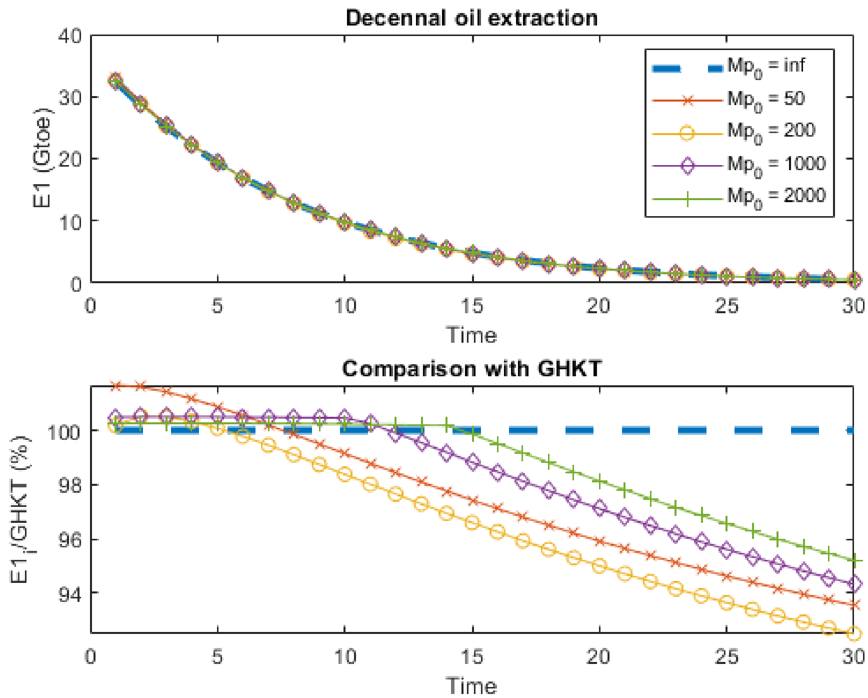

Figure 6: Oil extraction path, and relative oil extraction with respect to the infinite minerals scenario

Result 5 *A tighter minerals constraint results in a faster depletion of oil but a slower extraction of coal and ultimately in a slower accumulation of carbon in the atmosphere.*

5.6 Key dates in each scenario

The previous sections describe some key steps that occur in every finite mineral scenario, namely:

---

cost becomes greater. This spurs low-carbon energy production, which becomes dominant in a second phase. In a third phase, low-carbon energy peaks and coal extraction increases, despite the higher environmental cost of that energy source. Note that as we approach the end of the world (T = 300), carbon emissions affect fewer generations and hence come at lower environmental cost. The overall result is that we pollute more in the first and third phases, whereas the second phase is less polluting. With higher inter-fuel substitutability, one can expect a more advantageous environmental damage/energy trade-off in terms of total utility, but potentially at the expense of the environment, especially once the low-carbon source has reached its ceiling, which shall occur early with small mineral reserves.

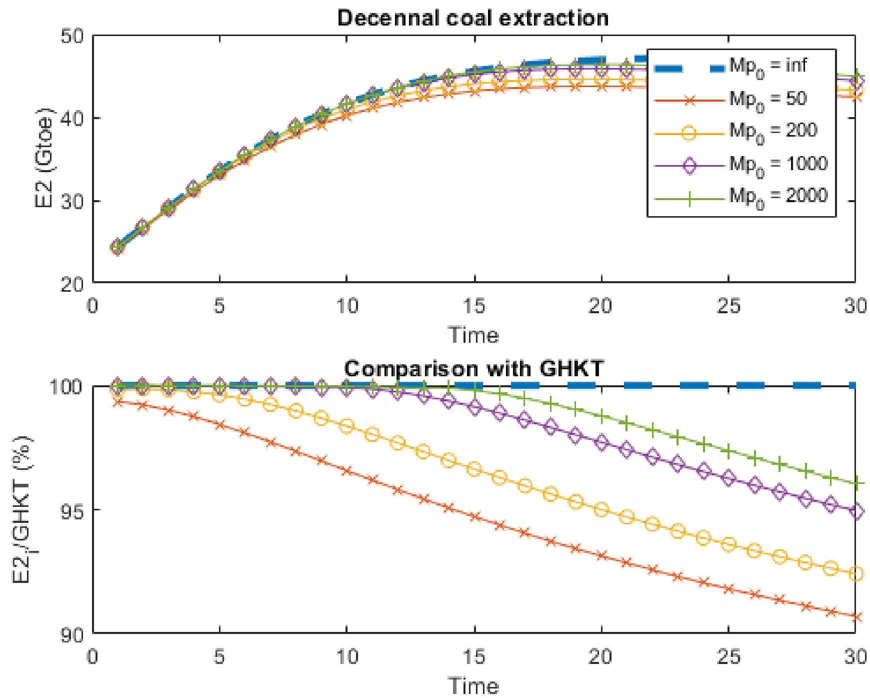

Figure 7: Coal extraction path, and relative coal extraction with respect to the infinite minerals scenario

- primary copper extraction peak
- 100% recycling rate
- low-carbon energy production peak

The dates of those events differ according to the magnitude of the mineral constraint. Thus, a higher $M_{p,0}$ delays the primary copper peak and the low-carbon energy peaks. This section gives the dates of those events in each scenario.

In order to highlight the growing difference between low-carbon energy production in finite and infinite mineral scenario, we also note the dates of key events summarized in Table 1. The time unit is 10 years, as in GHKT, and time starts in 2010: $t = 6$ refers to an event that happens in 2070. $10\% < infty$ refers to the moment when low-carbon energy production is 10% lower than

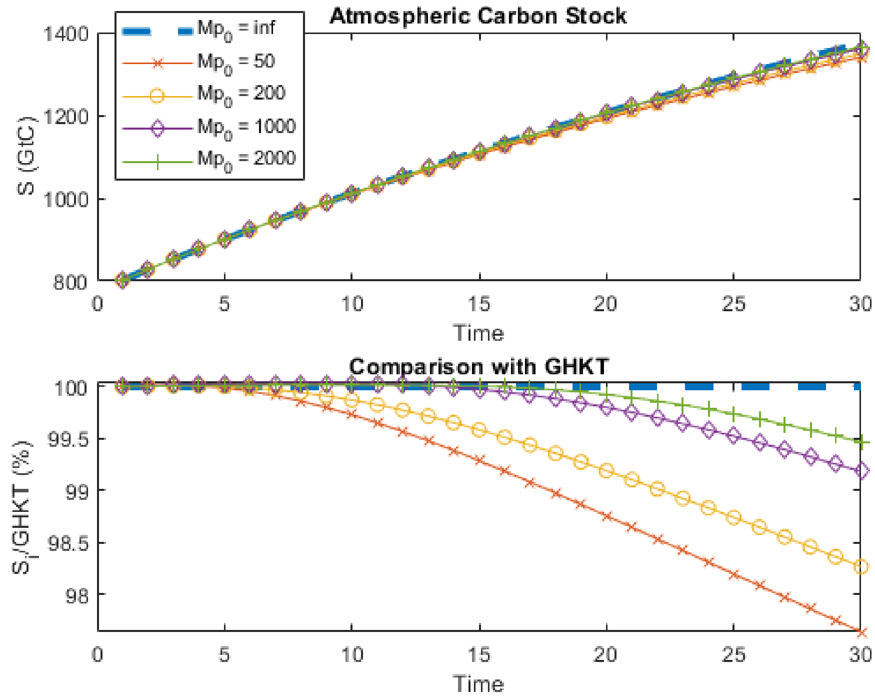

Figure 8: The carbon stock path

in the infinite mineral scenario, the same for 50% and 66%. Dates are given with a precision of ± 0.5 decade. Notice that in all scenarios, the low-carbon energy peak coincides with the moment when low-carbon energy production is approximately 2/3 lower than in the infinite mineral scenario.

Table 1: Dates of key events in each scenarios

Mineral constraint (MtCu)	50	200	1000	2000
Primary copper peak	< 1	1	6	9
100% recycling rate	< 1	1	11	15
10% $<$ infty	< 1	2	11.1	15
50% $<$ infty	3	8.7	16.2	19.6
66% $<$ infty	6	11	18	22
Low-carbon energy peak	6	11	18	22

5.7 Impact on GDP and optimal carbon tax

Figure 9 shows that in all constrained scenarios considered, GDP starts to depart from the infinite mineral scenario path at one point. When comparing with the key dates from Table 1, we observe that GDP drops from the moment at which the mineral recycling rate has reached its maximum level of 100%, a direct consequence of Result 2. That is, for instance, at $t = 11$ in the 1000Mt primary copper budget scenario. Nine decades after this date, at $t = 20$, GDP is 1% lower in the constrained scenario. It is 4% lower at the end of the simulation.

The decreasing phase of GDP in the scenarios with mineral constraints is due to the fact that low carbon energy reaches a peak and eventually has a decreasing phase. Since energy sources are imperfect substitute, total available energy for final good production, E_t, also goes through a decreasing phase, in sharp contrast with the scenario where the constraint of a limited stock of minerals is ignored. This results in a loss of GDP compared to the unconstrained scenario.

As in GHKT, the optimal carbon tax is a stationary ratio of the GDP. Therefore, plotting relative variations of GDP or optimal carbon tax paths is equivalent. Thus, Figure 9 also shows how the optimal carbon tax path is changed by the mineral constraint. In constrained scenarios, the optimal carbon tax is 3–8% lower than in the infinite mineral scenario after 30 decades, depending on the mineral constraint considered.

6 Extensions: depreciation and recycling cap

This section tests two alternative scenarios. We use $M_{p,0} = 1000$ MtCu as a benchmark. The first scenario assumes slower total depreciation of capital. In our model, as in GHKT, capital stock depreciates over 10 years. In our discrete-time model, this is done by using 10-year time steps, a strategy that avoids capital accumulation. One can argue that production capital and green capital depreciate at a lower pace. In line with the initial strategy, we hence test the model with time steps of 20 years, while keeping a 300-year time horizon. All the parameters stay unchanged. Figure 10 shows the primary and secondary production of copper when the 20 year scenario is expressed in 10 year periods for comparability. Copper extraction is delayed, with a copper peak approximately 30 years later. All primary copper is extracted at the end of 300 years. Copper

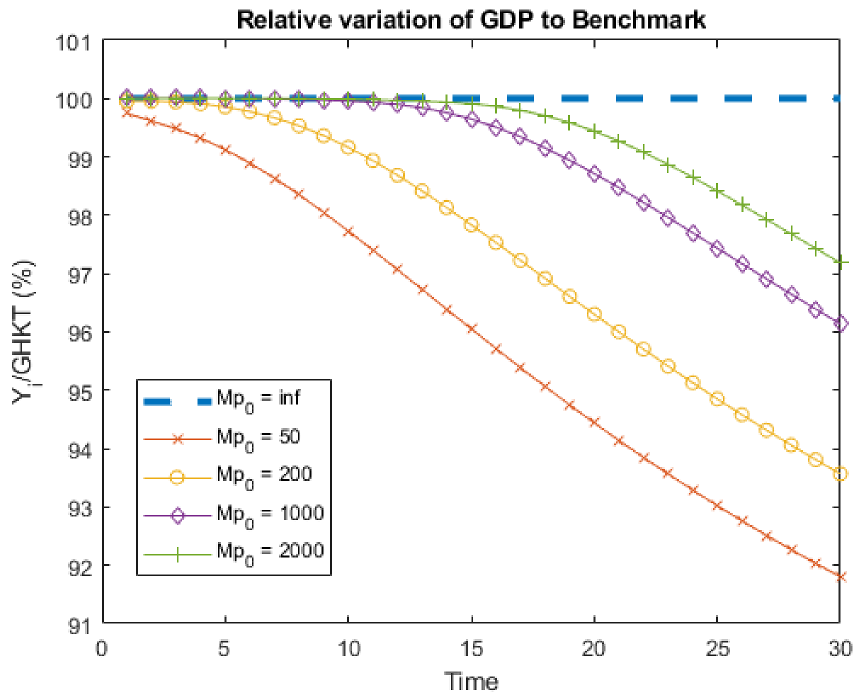

Figure 9: Comparison of GDP with the infinite mineral scenario for various mineral constraints

recycling is approximately twice as prevalent in the 10 year scenario, simply because recycling is needed twice as often. Figure 11 reveals that longer life duration ($Dep = 20$) allows the production of greater green capital stock with smaller copper input. With Equation (11) evaluated at $m_s = M_{p,0} = 1000$ and $m_p = 0$, everything is extracted and recycled — green capital stock converges to $G_{t=30+} = 95.2$ a little after 300 years. Figure 12 shows the impact on green energy production, which is greater and peaks later. We hence have the following result.

Result 6 *Longer life duration of green capital delays copper extraction and copper peak, and leads to greater green capital stock and green energy production. Green energy peak is also delayed.*

The second scenario acknowledges the fact that a 100% recycling rate comes at infinite recovery or collection costs, and proposes various recycling caps θ,

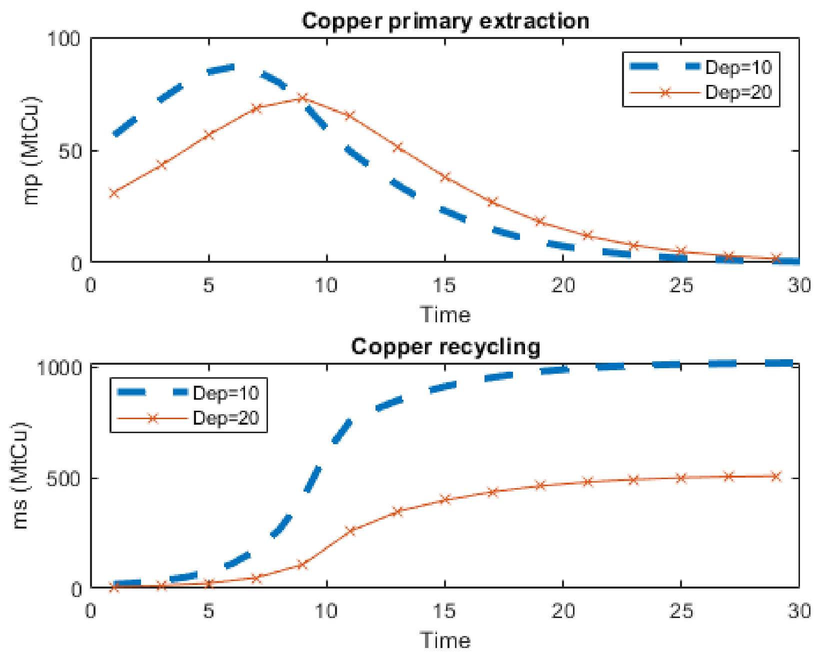

Figure 10: Comparison of primary and secondary production of copper for two depreciation scenarios, expressed in 10 year periods

with $\theta = 0.8,\ 0.5$, that represent technological, logistical or economic barriers to recycling. In terms of secondary mineral extraction, this means that some of the secondary mineral stock is in fact too dispersed to be repurposed. Therefore, secondary mineral production $m_{s,t}$ is bounded, and Equation (16) becomes:

$$0 \leq m_{s,t} \leq \theta M_{s,t}. \tag{28}$$

Figures 13 and 14 present the main impacts of a recycling cap on green energy production $E3$ and GDP Y. When full recycling capacity is reached, lower recycling caps reduce green energy production capacity and move peak production forward. After 300 years, green energy production is 22.8% lower when the cap is at 80%, whereas this number falls to 46% when the cap is 50%. This translates to GDP losses of 0.31% and 0.97%, respectively.

Result 7 *A lower recycling rate caps reduce green energy production capacity and moves peak production forward.*

The qualitative nature of this result is intuitive and was also obtained in Fabre

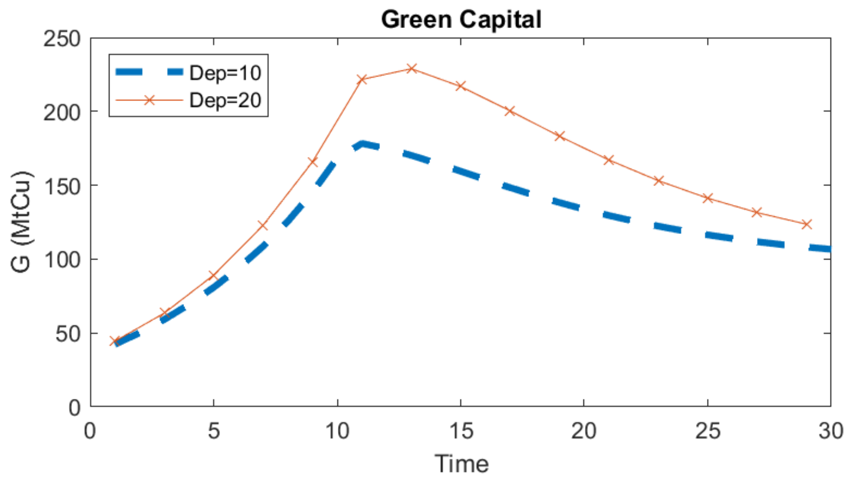

Figure 11: Comparison of green capital stocks for two depreciation scenarios

et al. (2020). Our analysis provides a quantitative analysis of the impact of recycling. Indeed, our simulations plotted in Figure 13 reveal that the paths are almost identical for an initial period of time (70 years) and the difference between the paths of green energy production only become 'significantly' different after 80 years.

7 Conclusion

Many actors emphasize the key role played by minerals in the energy transition. We model the primary and secondary mineral sectors' responses to a boom in demand caused by growth in low-carbon energy production. We observe the impact of a mineral constraint on green energy production paths in various mineral scarcity scenarios.

For this purpose, we expand a benchmark energy transition model (Golosov, Hassler, Krussel, and Tsyvinski, 2014) with a "green capital" input created from primary or recycled minerals. Following GHKT, our simulations draw on a three energy source case (oil, coal and low-carbon energy). We use the copper input to low-carbon energy production as a case study.

When there is an infinite amount of minerals, the result of our model simulation is identical to the benchmark case: our model matches GHKT. When there

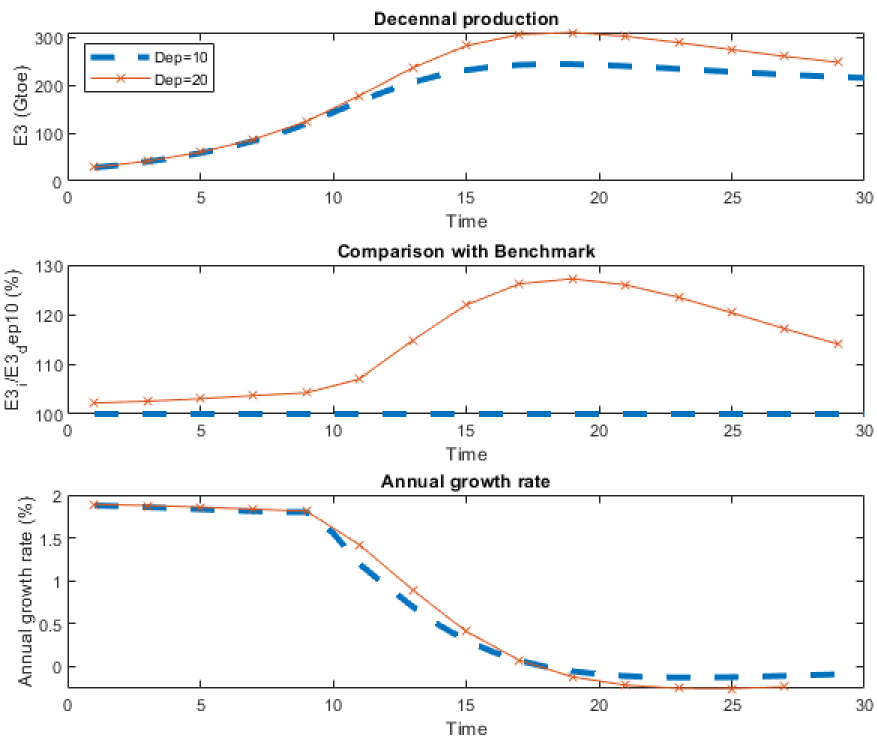

Figure 12: Comparison of green energy production for two depreciation scenarios

is a finite amount of minerals, the energy transition paths, and particularly low-carbon energy production, eventually depart from the infinite minerals scenario. We simulate the model's response to various levels of mineral constraint. If the primary copper budget for low-carbon energy green capital is 50 MtCu (which is 20 times the current amount of copper involved in wind energy production worldwide), the low-carbon energy production path will be 50% lower than in the infinite mineral scenario in three decades, and will peak in 6 decades. More generally, a doubling (resp. halving) of the primary copper budget postpones (resp. brings forward) this date by 30 years (±10 years). During the energy transition, the recycled mineral content of green capital grows, as a consequence of the combined decline of primary mineral extraction and an increase in mineral recycling. When the mineral recycling rate reaches 100%, which happens 4 to 6 decades after primary mineral extraction peaks, green capital also peaks. Once green capital's upper bound is reached, low-carbon energy production's growth rate declines. Labour productivity gains notably mitigate the slowdown of low-

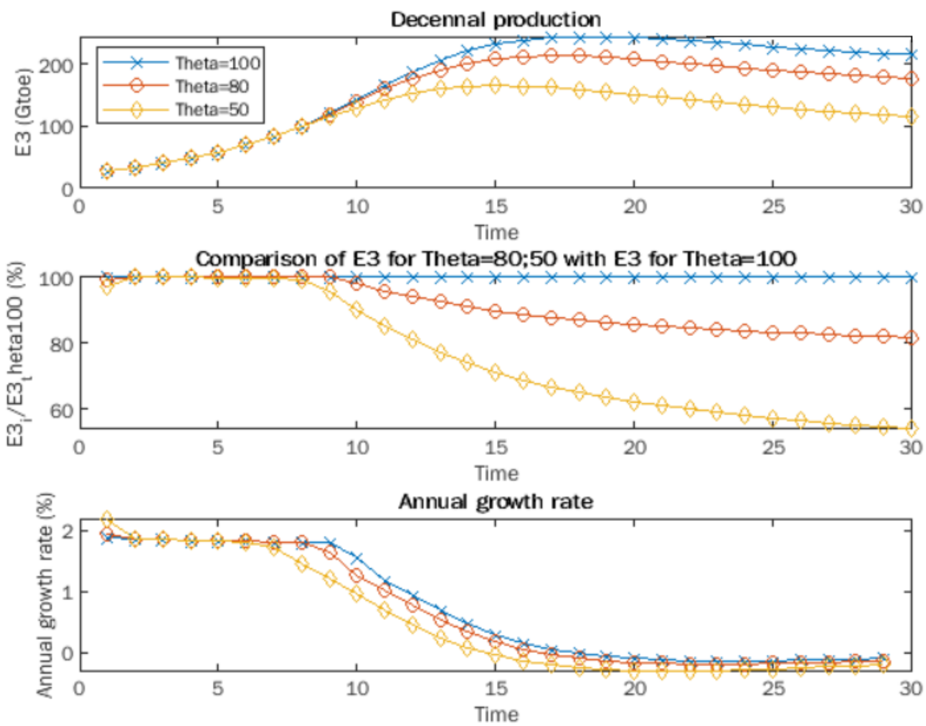

Figure 13: Comparison of low-carbon energy production and annual growth rate paths for various values of recycling caps

carbon energy production, but in the long run, low-carbon energy production stabilizes at a plateau, the value of which depends on the mineral constraint considered.

Our simulations show that the mineral constraint has to be taken into account in energy transition modelling, since it significantly impacts low-carbon energy production paths. It reveals the successive steps that ultimately limit low-carbon energy production and yields some insights into the date of these key events for many mineral constraint scenarios.

The impact of mineral constraints on oil extraction is different from its impact on coal extraction: scarcer minerals result in faster oil but a slower coal extraction. Tighter mineral constraints yield a slower accumulation of carbon in the atmosphere. It is important to note that our analysis offers quantitative

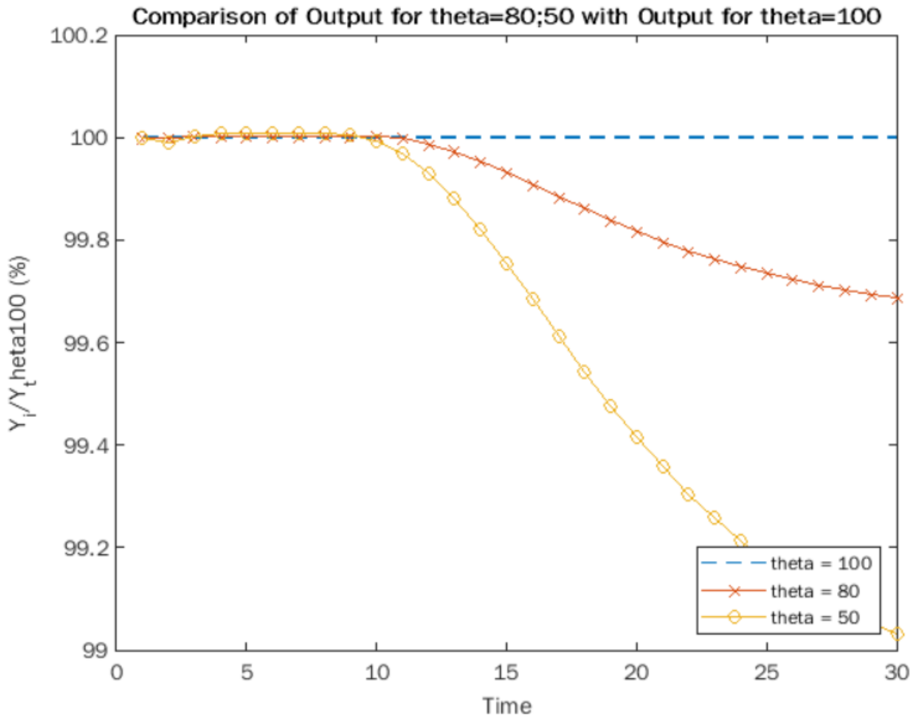

Figure 14: Comparison of GDP paths for various values of recycling caps

results only in the case of one mineral, which is assumed recyclable, adding other minerals to the analysis would likely complicate the results obtained in this paper in terms of the limitations of low-carbon energy growth. Measuring these effects would be interesting for future investigation. Clearly, the assumed substitutability between the different energy sources plays a major role on the optimal energy transition as well as on the design of any policy that takes into account the critical scarcity of minerals. This substitutability is likely to be itself the target of specific policies and could be an endogenous variable. This is an ambitious and undoubtedly fruitful exercise for future research.

References

[1] J.-P. Amigues, A. Ayong Le Kama, and M. Moreaux, "Equilibrium transitions from nonrenewable energy to renewable energy under capacity con-

straints," *Journal of Economic Dynamics & Control*, vol. 55, pp. 89–112, 2015.

[2] D. L. P. Arrobas, K. L. Hund, M. S. Mccormick, J. Ningthoujam, and J. R. Drexhage, *The growing role of minerals and metals for a low carbon future.* World Bank Group, 2017.

[3] Copper Development Association. (2018) Copper in wind power 2. [En ligne]. Disponible: https://copperalliance.org.uk/knowledge-base/education/education-resources/copper-wind-power-2/

[4] Copper Development Association Inc. (2020) Copper in the USA: Bright future glorious past. [En ligne]. Disponible: https://www.copper.org/education/history/us-history/g_fact_future.html

[5] European Commission, *Study on the review of the list of critical raw materials*, 2017.

[6] ——, *Report on critical raw materials and the circular economy*, 2018.

[7] A. Fabre, M. Fodha, and F. Ricci, "Mineral resources for renewable energy: optimal timing of energy production," *Resource and Energy Economics*, vol. 59, p. 101131, 2020.

[8] A. García-Olivares, J. Ballabrera-Poy, E. García-Ladona, and A. Turiel, "A global renewable mix with proven technologies and common materials," *Energy Policy*, vol. 41, pp. 561–574, 2012.

[9] M. Golosov, J. Hassler, P. Krusell, and A. Tsyvinski, "Optimal taxes on fossil fuel in general equilibrium," *Econometrica*, vol. 82, no. 1, pp. 41–88, 2014.

[10] R. Hart, "To everything there is a season: Carbon pricing, research subsidies, and the transition to fossil-free energy," *Journal of the Association of Environmental and Resource Economists*, vol. 6, no. 2, pp. 349–389, 2019.

[11] E. G. Hertwich, T. Gibon, E. A. Bouman, A. Arvesen, S. Suh, G. A. Heath, J. D. Bergesen, A. Ramirez, M. I. Vega, and L. Shi, "Integrated life-cycle assessment of electricity-supply scenarios confirms global environmental benefit of low-carbon technologies," *Proceedings of the National Academy of Sciences*, vol. 112, no. 20, pp. 6277–6282, 2015.

[12] International Copper Study Group. (2020) Copper: Preliminary data for october 2019. [En ligne]. Disponible: https://www.icsg.org/index.php/component/jdownloads/finish/114/2993?Itemid=

[13] International Council on Mining and Metals, *The role of mining in national economies*, 2014.

[14] International Energy Agency, *The Role of Critical Minerals in Clean Energy Transitions*, 2021.

[15] International Renewable Energy Agency, *Renewable Energy Cost Analysis - Wind Power*, 2012.

[16] K. M. Johnson, J. M. Hammarstrom, M. L. Zientek, and C. L. Dicken, "Estimate of undiscovered copper resources of the world, 2013," *US Geological Survey Fact Sheet*, vol. 3004, p. 2014, 2014.

[17] M. Knoblach, M. Roessler, and P. Zwerschke, "The elasticity of substitution between capital and labour in the US economy: A meta-regression analysis," *Oxford Bulletin of Economics and Statistics*, vol. 82, no. 1, pp. 62–82, 2020.

[18] L. D. Meinert, G. R. Robinson, and N. T. Nassar, "Mineral resources: Reserves, peak production and the future," *Resources*, vol. 5, no. 1, p. 14, 2016.

[19] National Resources Canada. (2019) Copper facts. [En ligne]. Disponible: https://www.nrcan.gc.ca/our-natural-resources/minerals-mining/minerals-metals-facts/copper-facts/20506

[20] S. Northey, S. Mohr, G. M. Mudd, Z. Weng, and D. Giurco, "Modelling future copper ore grade decline based on a detailed assessment of copper resources and mining," *Resources, Conservation and Recycling*, vol. 83, pp. 190–201, 2014.

[21] G. Pitron, *La guerre des métaux rares. La face cachée de la transition énergétique et numérique.* Les Liens qui Libèrent, 2018.

[22] A. Pommeret, F. Ricci, and K. Schubert, "Critical raw materials for the energy transition," *European Economic Review*, vol. 103991, 2021.

[23] A. Pommeret and K. Schubert, "Optimal energy transition with variable and intermittent renewable electricity generation," *Journal of Economic Dynamics and Control*, vol. 134, no. 104273, 2022.

[24] N. Rötzer and M. Schmidt, "Decreasing metal ore grades—is the fear of resource depletion justified?" *Resources*, vol. 7, no. 4, p. 88, 2018.

[25] G. S. Seck, E. Hache, C. Bonnet, M. Simoën, and S. Carcanague, "Copper at the crossroads: Assessment of the interactions between low-carbon energy transition and supply limitations," *Resources, Conservation and Recycling*, vol. 163, no. 105072, 2020.

[26] O. Tahvonen and S. Salo, "Economic growth and transitions between renewable and nonrenewable energy resources," *European Economic Review*, vol. 45, no. 8, pp. 1379–1398, 2001.

[27] Y. Tsur and A. Zemel, "Scarcity, growth and R&D," *Journal of Environmental Economics and Management*, vol. 49, no. 3, pp. 484–499, 2005.

[28] United Nations Environmental Program, *Critical Metals for Future Sustainable Technologies and their Recycling Potential*, 2009.

[29] US Department of Energy, *Critical Materials Strategy*, 2011.

[30] US Geological Survey. (2021) Mineral commodity summaries 2021. [En ligne]. Disponible: https://pubs.usgs.gov/periodicals/mcs2021/mcs2021.pdf

[31] O. Vidal, B. Goffé, and N. Arndt, "Metals for a low-carbon society," *Nature Geoscience*, vol. 6, no. 11, p. 894, 2013.

[32] O. Vidal, F. Rostom, C. François, and G. Giraud, "Global trends in metal consumption and supply: the raw material–energy nexus," *Elements: An International Magazine of Mineralogy, Geochemistry, and Petrology*, vol. 13, no. 5, pp. 319–324, 2017.

A Proof of Proposition 1

A.1 Equation (19)

This result also holds in GHKT's model; we emphasize that the modifications and additions that are specific to our model keep this result and its proof unchanged.

We combine the first order conditions with respect to $N_{0,t}$ and $N_{2,t}$ to obtain the following equality:

$$\lambda_{0,t}\frac{\partial Y_t}{\partial N_{0,t}} = \lambda_{2,t}\frac{\partial E_{2,t}}{\partial N_{2,t}}.$$

Then, the first order conditions with respect to $E_{2,t}$, E_t and S_t give an expression of coal's marginal production cost $\lambda_{2,t}$:

$$\lambda_{2,t} = \lambda_{0,t}\frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{2,t}} - \lambda_{0,t}\Lambda_t,$$

where the expression for the marginal climate externality damage Λ_t is identical to GHKT:

$$\Lambda_t = Y_t \sum_{j=0}^{T-t} \beta^j \gamma(1-d_j).$$

We replace in the first equality $\lambda_{2,t}$ by its expression to obtain Result 19.

A.2 Equation (20)

Since we use a new production function for low-carbon energy production (that includes green capital), Equation (20) differs from GHKT's result, but the proof is analogous.

We combine the first order conditions with respect to $N_{0,t}$ and $N_{3,t}$ to obtain the following equality:

$$\lambda_{0,t}\frac{\partial Y_t}{\partial N_{0,t}} = \lambda_{3,t}\frac{\partial E_{3,t}}{\partial N_{3,t}}.$$

Then, we obtain from the first order conditions with respect to $E_{3,t}$ and E_t that

$$\lambda_{3,t} = \lambda_{0,t}\frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{3,t}},$$

which leads immediately to Result (20).

A.3 Equation (21)

Equation (21) is specific to our model, since there is no secondary mineral sector in GHKT.

We combine the first order conditions with respect to $N_{0,t}$ and $N_{s,t}$, and for a non-binding recycling feasibility constraint (16), we obtain:

$$\lambda_{0,t}\frac{\partial Y_t}{\partial N_{0,t}} = \lambda_{s,t}\frac{\partial m_{s,t}}{\partial N_{s,t}}.$$

Then, we get from the first order conditions with respect to $m_{s,t}$ and G_t that:

$$\lambda_{s,t} = \lambda_{3,t}\frac{\partial E_{3,t}}{\partial G_t}\frac{\partial G_t}{\partial m_{s,t}}.$$

Using the intermediate result from the proof of Equation (20), we obtain:

$$\lambda_{s,t} = \lambda_{0,t}\frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{3,t}}\frac{\partial E_{3,t}}{\partial G_t}\frac{\partial G_t}{\partial m_{s,t}},$$

which leads to result (21).

B Proof of Proposition 2

B.1 Equation (22)

Equation (22) and its proof are identical to GHKT.

The first order conditions with respect to $E_{1,t}$, E_t, and S_t give

$$\mu_{1,t} = U'(C_t)\frac{\partial Y_t}{\partial E_t}\frac{\partial E_t}{\partial E_{1,t}} + \frac{1}{\beta^t}\sum_{j=t}^{T} U'(C_j)\frac{\partial Y_j}{\partial S_j}\frac{\partial S_j}{\partial E_{1,t}} \tag{29}$$

To enhance readability, denote by A_t the first term in the right-hand side of equation (29), and B_t the second term. Then,

$$\mu_{1,t} = A_t + B_t,$$

where $\mu_{1,t}$ is the Lagrange multiplier that stands for oil's scarcity cost.

The first order condition with respect to $R_{1,t+1}$ gives a necessary condition for the evolution of the Lagrange multiplier over time:

$$\mu_{1,t} = \beta \mu_{1,t+1} \tag{30}$$

A_t is the contribution of $E_{1,t}$ to final good consumption. Using equations (1), (4) and (5), it becomes:

$$A_t = -\frac{Y_t}{C_t} \frac{\nu \kappa_1}{E_t^{\rho} E_{1,t}^{1-\rho}}$$

B_t accounts for all the present and future damages induced by a present carbon emission $E_{1,t}$.

Using equations (4) and (9), we obtain

$$\text{for all } j > t, \quad \frac{\partial Y_j}{\partial S_j} \frac{\partial S_j}{\partial E_{1,t}} = -\gamma Y_j \ (1 - d_{j-t}).$$

Therefore, after a variable change, B_t can be written:

$$B_t = -\sum_{j=0}^{T-t} \beta^j \gamma \frac{Y_{t+j}}{C_{t+j}} (1 - d_j).$$

Define

$$\hat{\Lambda}_t = \sum_{j=0}^{T-t} \beta^j \gamma (1 - d_j).$$

Using GHKT's assumption that the optimal savings rate is constant over time, *i.e.* C_t/Y_t constant, B_t becomes:

$$B_t = -\hat{\Lambda}_t \ \frac{Y_t}{C_t}.$$

Finally, replacing $\mu_{1,t}$ and $\mu_{1,t+1}$ by their expressions in equation (30) yields result (22).

B.2 Equation (23)

The first order condition with respect to $m_{p,t}$ is

$$\lambda_{p,t} + \mu_{p,t} - \mu_{s,t} - \pi_{G,t} \frac{\partial G_t}{\partial m_{p,t}} = 0. \tag{31}$$

The first order conditions with respect to $N_{p,t}$ and $N_{0,t}$ give:

$$\chi_t^N - \lambda_{p,t}\frac{\partial m_{p,t}}{\partial N_{p,t}} = 0 \tag{32a}$$

$$\chi_t^N - \lambda_{0,t}\frac{\partial Y_t}{\partial N_{0,t}} = 0. \tag{32b}$$

Therefore,

$$\begin{aligned}\lambda_{p,t} &= \lambda_{0,t}\frac{\partial Y_t/\partial N_{0,t}}{\partial m_{p,t}/\partial N_{p,t}} \\ &= -\frac{Y_t}{C_t}\frac{1-\alpha-\nu}{A_{p,t}N_{0,t}},\end{aligned}$$

where we use that the shadow price of production $\lambda_{0,t}$ is equal to the marginal utility of consumption $U'(C_t) = 1/C_t$. We now have an expression of $\lambda_{p,t}$. Next, consecutive substitutions of Lagrange multipliers by their expressions obtained from the first order conditions with respect to G_t, $E_{3,t}$, E_t yields

$$\pi_{G,t} = -U'(C_t)\cdot\frac{\partial Y_t}{\partial E_t}\cdot\frac{\partial E_t}{\partial E_{3,t}}\cdot\frac{\partial E_{3,t}}{\partial G_t}.$$

Then,

$$\frac{\partial Y_t}{\partial E_t} = \nu\frac{Y_t}{E_t}$$

and

$$\begin{aligned}\frac{\partial E_t}{\partial E_{3,t}} &= \frac{1}{\rho}(E_t^{\rho})^{\frac{1}{\rho}-1}\times\rho\kappa_3 E_{3,t}^{\rho-1} \\ &= \kappa_3\left(\frac{E_t}{E_{3,t}}\right)^{1-\rho}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial E_{3,t}}{\partial G_t} &= \frac{1}{\ddot{\rho}}(E_{3,t}^{\ddot{\rho}})^{\frac{1}{\ddot{\rho}}-1}\times\ddot{\rho}\kappa_G\psi(\psi G_t)^{\ddot{\rho}-1} \\ &= \kappa_G\psi\left(\frac{E_{3,t}}{\psi G_t}\right)^{1-\ddot{\rho}}\end{aligned}$$

so that :

$$\pi_{G,t} = -\frac{Y_t}{C_t}\frac{\nu\kappa_3\kappa_G\psi}{E_t}\left(\frac{E_t}{E_{3,t}}\right)^{1-\rho}\left(\frac{E_{3,t}}{\psi G_t}\right)^{1-\ddot{\rho}}$$

Finally, the first order conditions with respect to $M_{p,t+1}$ and $M_{s,t+1}$ is:

$$\mu_{p,t} - \beta\mu_{p,t+1} = 0$$
$$\mu_{s,t} - \beta\mu_{s,t+1} = 0$$

so that

$$\mu_{p,t} - \mu_{s,t} = \beta(\mu_{p,t+1} - \mu_{s,t+1}).$$

In the end, we obtain Equation (23).

C Proof of Proposition 3

This section provides details on the analytical solution to the infinite mineral case. We prove that under some conditions, the infinite mineral case in our modelling is equivalent to GHKT's model. We first aim to express green capital G_t as a function of labour shares $N_{p,t}$ and $N_{s,t}$.

Assume that $M_{p,0} = \infty$, $M_{s,0} = \infty$, and $g_{As} = g_{Ap} = g_{A3}$. Since the primary and secondary mineral stocks are infinite, mineral scarcity rents $\mu_{p,t}$ and $\mu_{s,t}$ are equal to zero. Hence, the first order condition with respect to $m_{p,t}$, Equation (31), can be written:

$$\lambda_{p,t} = \pi_{G,t}\frac{\partial G_t}{\partial m_{p,t}}. \tag{34}$$

With Equations (32a) and (32b), we have:

$$\lambda_{p,t}\frac{\partial m_{p,t}}{\partial N_{p,t}} = \lambda_{0,t}\frac{\partial Y_t}{\partial N_{0,t}}. \tag{35}$$

We combine Equations 34 and 35 to obtain:

$$\pi_{G,t}\frac{\partial G_t}{\partial m_{p,t}}\frac{\partial m_{p,t}}{\partial N_{p,t}} = \lambda_{0,t}\frac{\partial Y_t}{\partial N_{0,t}}. \tag{36}$$

We repeat the same process with the secondary mineral flow variables $m_{s,t}$ and $N_{s,t}$. The first order condition with respect to $m_{s,t}$ gives:

$$\lambda_{s,t} = \pi_{G,t}\frac{\partial G_t}{\partial m_{s,t}},$$

and the first order conditions with respect to $N_{s,t}$ and $N_{0,t}$ lead to:

$$\lambda_{s,t}\frac{\partial m_{s,t}}{\partial N_{s,t}} = \lambda_{0,t}\frac{\partial Y_t}{\partial N_{0,t}},$$

so that

$$\pi_{G,t}\frac{\partial G_t}{\partial m_{s,t}}\frac{\partial m_{s,t}}{\partial N_{s,t}} = \lambda_{0,t}\frac{\partial Y_t}{\partial N_{0,t}}. \tag{37}$$

We combine Equations 36 and 37:

$$\frac{\partial G_t}{\partial m_{s,t}}\frac{\partial m_{s,t}}{\partial N_{s,t}} = \frac{\partial G_t}{\partial m_{p,t}}\frac{\partial m_{p,t}}{\partial N_{p,t}}. \tag{38}$$

Equation 38 states that labour's marginal contribution to green capital manufacturing in primary and secondary mineral sectors are equal. It extends the result in Proposition 1 and is only valid when primary and secondary mineral stocks are infinite. Equation 38 can now be written using the formula (11) that define G_t as a function of $m_{p,t}$ and $m_{s,t}$:

$$\kappa_p A_{p,t}\left(\frac{G_t}{m_{p,t}}\right)^{1-\tilde{\rho}} = \kappa_s A_{s,t}\left(\frac{G_t}{m_{s,t}}\right)^{1-\tilde{\rho}},$$

which leads to

$$\frac{N_{p,t}}{N_{s,t}} = \frac{A_{s,t}}{A_{p,t}}\left(\frac{\kappa_p A_{p,t}}{\kappa_s A_{s,t}}\right)^{\frac{1}{1-\tilde{\rho}}}.$$

Denote by α_1 the primary to secondary mineral labour shares ratio:

$$\alpha_1 = \frac{A_{s,t}}{A_{p,t}}\left(\frac{\kappa_p A_{p,t}}{\kappa_s A_{s,t}}\right)^{\frac{1}{1-\tilde{\rho}}}.$$

Note that α_1 is constant since $A_{s,t}$ and $A_{p,t}$ are geometric sequences with equal growth rates, therefore

$$\alpha_1 = \frac{A_{s,0}}{A_{p,0}}\left(\frac{\kappa_p A_{p,0}}{\kappa_s A_{s,0}}\right)^{\frac{1}{1-\tilde{\rho}}}.$$

We write the expression of G_t using α_1:

$$G_t = \left[\kappa_s\left(A_{s,t}\frac{1}{1+\alpha_1}\right)^{\tilde{\rho}} + \kappa_p\left(A_{p,t}\frac{\alpha_1}{1+\alpha_1}\right)^{\tilde{\rho}}\right]^{\frac{1}{\tilde{\rho}}}(N_{p,t}+N_{s,t}),$$

and define the aggregated green capital labour productivity

$$\widehat{A}_{G,t} = \left[\kappa_s \left(A_{s,t}\frac{1}{1+\alpha_1}\right)^{\tilde{\rho}} + \kappa_p \left(A_{p,t}\frac{\alpha_1}{1+\alpha_1}\right)^{\tilde{\rho}}\right]^{\frac{1}{\tilde{\rho}}}, \tag{39}$$

so that

$$G_t = \widehat{A}_{G,t}\ (N_{p,t} + N_{s,t}). \tag{40}$$

Equation 40 expresses green capital as a function of primary and secondary labour shares, using aggregated green capital labour productivity $\widehat{A}_{G,t}$. This result is only valid when mineral stocks are infinite. Note that since $A_{s,t}$ and $A_{p,t}$ are both geometric sequences with growth rate $g_{As} = g_{Ap} = \widetilde{g}_{A3}$, $\widehat{A}_{G,t}$ is also a geometric sequence with growth rate $\widetilde{g}_{A3}$.

Using the intermediate result of Equation 40, the following steps aim to express low-carbon energy production $E_{3,t}$ as a function of labour shares $N_{p,t}$, $N_{s,t}$ and $N_{3,t}$. Combining Equations 20 and 21 reveals that low-carbon energy labour and secondary mineral labour's marginal contribution to low-carbon energy production are equal:

$$\frac{\partial E_{3,t}}{\partial N_{3,t}} = \frac{\partial E_{3,t}}{\partial G_t}\frac{\partial G_t}{\partial N_{s,t}}. \tag{41}$$

In Equation 41, we replace $E_{3,t}$ and G_t by their expression:

$$\kappa_L A_{3,t}\left(\frac{E_{3,t}}{A_{3,t}N_{3,t}}\right)^{1-\ddot{\rho}} = \kappa_G \psi \widehat{A}_{G,t}\left(\frac{E_{3,t}}{\psi G_t}\right)^{1-\ddot{\rho}},$$

which leads, using Equation 40, to:

$$\frac{N_{3,t}}{N_{s,t}+N_{p,t}} = \frac{\psi \widehat{A}_{G,t}}{A_{3,t}}\left(\frac{\kappa_L A_{3,t}}{\kappa_G \psi \widehat{A}_{G,t}}\right)^{\frac{1}{1-\ddot{\rho}}}$$

Denote by α_2 the direct to indirect low-carbon energy labour ratio:

$$\alpha_2 = \frac{\psi \widehat{A}_{G,t}}{A_{3,t}}\left(\frac{\kappa_L A_{3,t}}{\kappa_G \psi \widehat{A}_{G,t}}\right)^{\frac{1}{1-\ddot{\rho}}}.$$

Since $\widehat{A}_{G,t}, A_{3,t}$ are both geometric sequences with equal growth rates, α_2 is indeed constant with

$$\alpha_2 = \frac{\psi \widehat{A}_{G,0}}{A_{3,0}} \left(\frac{\kappa_L A_{3,t}}{\kappa_G \psi \widehat{A}_{G,0}} \right)^{\frac{1}{1-\ddot{\rho}}}.$$

We write the expression of $E_{3,t}$ using α_2:

$$E_{3,t} = \left[\kappa_L \left(A_{3,t} \frac{\alpha_2}{1+\alpha_2} \right)^{\ddot{\rho}} + \kappa_G \left(\psi \widehat{A}_{G,t} \frac{1}{1+\alpha_2} \right)^{\ddot{\rho}} \right]^{\frac{1}{\ddot{\rho}}} (N_{3,t} + N_{p,t} + N_{s,t})$$

and define the aggregated low-carbon energy labour productivity

$$\widehat{A}_{3,t} \equiv F(A_{3,t},\ A_{s,t},\ A_{p,t})$$

where $F(A_{3,t},\ A_{s,t},\ A_{p,t}) \equiv \left[\kappa_L \left(A_{3,t} \frac{\alpha_2}{1+\alpha_2} \right)^{\ddot{\rho}} + \kappa_G \left(\psi \widehat{A}_{G,t} \frac{1}{1+\alpha_2} \right)^{\ddot{\rho}} \right]^{\frac{1}{\ddot{\rho}}}$, and where $\widehat{A}_{G,t}$ is given by 39. Note that since $\widehat{A}_{G,t}$ and $A_{3,t}$ are both geometric sequences with growth rate $\widetilde{g}_{A3}$, $\widehat{A}_{G,t}$ is also a geometric sequence with growth rate $\widetilde{g}_{A3}$. Therefore $A_{3,t}$ and $\widetilde{A}_{3,t}$ are equal at all times if $A_{3,0} = \widetilde{A}_{3,0}$.
We then have

$$E_{3,t} = \widehat{A}_{3,t}\ (N_{s,t} + N_{p,t} + N_{3,t}) = \widetilde{E}_{3,t}. \tag{42}$$

Equation 42 expresses low-carbon energy as a function of labour shares of all sectors that are involved in low-carbon energy production. Thus, when primary and secondary mineral stocks are infinite, the low-carbon energy production function can be written in a form that is analogous to GHKT's low-carbon energy production function. The social optimum in our model coincides with the social optimum in GHKT's model.

D Calibration

As presented in Table 2, we use the values of GHKT for the parameters that are not related to green capital and minerals. Then, we calibrate the parameters that are specific to our model using the case of copper in wind energy production. Table 3 and the following paragraphs give additional details on the process of calibration used for the parameters that are specific to our model.

Table 2: Calibration summary for parameters from GHKT

Parameter	Value	Unit
ν	0.04	-
α	0.3	-
β	0.985	annual
g_{A2}, g_{A3}	2%	% per year
ρ	-0.058	-
κ_1	0.5429	-
κ_2	0.1015	-
κ_3	0.3556	-
$A_{2,0}$	7693	Gtoe/labour
$R_{1,0}$	253.8	GtC
N	1	-
$1 - d_s$	$\phi_L + (1-\phi_L)\phi_0(1-\phi)^S$	-
ϕ	0.0228	-
ϕ_L	0.2	-
ϕ_0	0.393	-
$\bar{S}$	581	GtC
S_0	802	GtC

Table 3: Summary of calibration for parameters that are specific to our model

Parameter	Interpretation	Calibration condition	Value	Unit
$\boldsymbol{\kappa}_s$, $\boldsymbol{\kappa}_p$, $\tilde{\boldsymbol{\rho}}$	Production function of green capital substitution and share parameters of primary and secondary mineral inputs	$\kappa_s + \kappa_p = 1$ $\frac{\kappa_s}{\kappa_p}(\frac{m_{s,t=0}}{m_{p,t=0}})^{\tilde{\rho}-1} = 1$ $\tilde{\rho} = 0.5$	$\kappa_s = 0.3085$ $\kappa_p = 0.6915$ $\tilde{\rho} = 0.5$	–
$A_{p,t=0}$, $A_{s,t=0}$	Initial labour productivity in primary and secondary mineral production sectors	$\frac{(1-\alpha-\nu)Y_{t=0}}{A_{p,t=0}} = 3.5e^9$ $\frac{(1-\alpha-\nu)Y_{t=0}}{A_{s,t=0}} = 3.5e^9$	$A_{p,t=0} = 132000$ $A_{s,t=0} = 132000$	$\frac{MtCu}{Labour}$
ψ	Energy produced (over the course of one period) for one unit of green capital	$\frac{G_{t=0}}{E_{3,t=0}} = 4tCu/MW$	$\psi = 1.877$	$\frac{Gtoe}{MtCu}$
$\boldsymbol{\kappa}_G$, $\boldsymbol{\kappa}_L$, $\ddot{\boldsymbol{\rho}}$	Green energy production function - substitution and share parameters of labour and green capital inputs	$\kappa_G + \kappa_L = 1$ $\frac{\kappa_L}{\kappa_G}(\frac{A_{3,t=0}N_{3,t=0}}{\psi G_{p,t=0}})^{\ddot{\rho}-1} = \frac{1}{3}$ $\ddot{\rho} = -3$	$\kappa_G = 0.75$ $\kappa_L = 0.25$ $\ddot{\rho} = -3$	–
$A_{3,t=0}$	Initial labour productivity in green energy sector	$F(A_{3,t=0}, A_{s,t=0}, A_{p,t=0}) = 1311$	$A_{3,t=0} = 865.14$	$\frac{Gtoe}{Labour}$
g_{A3}, g_{As}, g_{Ap}	Growth rate of labour productivities	$g_{A3}, g_{As}, g_{Ap} = 1.02^{10}$	$g_{A3}, g_{As}, g_{Ap} = 1.02^{10}$	–
$M_{s,t=0}, M_{p,t=0}$	Initial primary and secondary mineral stocks	$M_{s,t=0} = 19MtCu$	$M_{s,t=0} = 19$ $M_{p,t=0} = 50, 200, 1000, 2000$	MtCu

$\boldsymbol{\kappa_s}$, $\boldsymbol{\kappa_p}$, $\boldsymbol{\tilde{\rho}}$ are the parameters of the green capital CES production function. The relative price of primary to recycled mineral is one [4]. Current primary and recycled copper production is given in [12]. On the one hand, copper is 100% recyclable and does not degrade in the process, so that primary and secondary copper can equivalently be used [19], which suggests a high substitution parameter. On the other hand, if primary and secondary copper were perfect substitutes, the simulation would lead to situations where only the currently cheapest industry is active at a time. This is not what is observed in reality, where primary and secondary copper industries coexist. Therefore, we choose a very high substitution parameter, but strictly lower than one: $\tilde{\rho} = 0.5$. The choice of this parameter has an impact on the difference between the peak low-carbon energy production, and long-run plateau low-carbon energy production. The better substitutes primary and secondary mineral are, the lower the difference. In the limit, if primary and secondary mineral are perfect substitutes ($\tilde{\rho} = 1$), green energy production in the long run is equal to peak production.

$\boldsymbol{A_{p,0}}$, $\boldsymbol{A_{s,0}}$ Primary copper production cost is estimated at 3500\$/tCu [13]. We don't have an estimate for secondary copper production cost, but good quality scrap copper is currently worth 3400\$/tCu. For simplicity, we assume that the total production cost of secondary copper is 3500\$/tCu (including capital and labour cost).

$\boldsymbol{\psi}$ is the amount of wind energy produced over the course of one period for each unit of green capital. It depends on the technology, and especially the location of the wind turbine. Garcia-Olivares *et al.* (2012) [8] estimate that onshore wind turbines need 2 tons of copper per MW installed, whereas offshore wind turbines need 10 tons per MW. The Copper Development Association's estimate is 2.4–6.5 t/MW [3]. We use 4t/MW in order to take into account both onshore and offshore wind turbines, since offshore wind turbines are expected to have a growing role in the future of wind energy production. We also use an average capacity factor of 33% [8], meaning that for 3-5 MW turbines, each MW will produce approximately 87,000MW/h over the course of their lifetime. With a conversion rate of 1 toe=11.63MW/h, we obtain 1.877Gtoe/MtCu.

$\boldsymbol{\kappa_G}$, $\boldsymbol{\kappa_L}$, $\boldsymbol{\ddot{\rho}}$ characterize the substitution between labour and the green capital input in wind energy production. Labour cost share over capital cost share in wind energy production varies from one country to another, but it is around 1 to 3 on average: Labour represents 25% of wind energy production cost, and capital 75% [15]. The 2020 metastudy by Knoblach *et al.* estimates that the elasticity of substitution between labour and capital is between 0.45 and 0.87 at the most aggregated level, and that it is 0.2 points lower at the industry level [17]. We take the lowest value of the interval, assuming that green capital is not easily substituted by labour. Therefore, we get an elasticity of substitution of 0.25, which gives a substitution parameter of –3. Arguing that green capital and labour are weak substitutes, we approximate the labour input to the green capital input ratio with one. With this assumption, we simplify the Table 3 equation to $\frac{\kappa_L}{\kappa_G} = \frac{1}{3}$. This leads to $\kappa_L = 0.25$. This result states that wind energy production is more capital intensive than labour intensive.

$\boldsymbol{A_{3,0}}$. We use proposition (3) to calibrate $A_{3,0}$

$\boldsymbol{g_{A3}}$, $\boldsymbol{g_{As}}$, $\boldsymbol{g_{Ap}}$. Following GHKT, we assume an annual 2% labour productivity growth rate in each sector.

$\boldsymbol{M_{s,0}}$. We assume that the initial secondary mineral stock corresponds to one year of current primary copper production, that is 19Mt copper.