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Tarek M. Harchaoui, Pierre Lasserre

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Testing the Option Value Theory of Irreversible Investment*

Tarek M. Harchaoui[†], *Pierre Lasserre*[‡]

Résumé / Abstract

En recourrant tour à tour à la programmation dynamique et à la méthode des actifs contigents, nous établissons la valeur de l'option d'effectuer des investissements irréversibles réels qui sont sensibles aux paramètres économiques prévalant au moment de la décision. Nous testons ensuite si des investissements en capacité de production effectués par des mines de cuivre canadiennes sont conformes aux implications de la théorie. Les résultats sont fortements en faveur de celle-ci; nos données rejettent le critère de la valeur actuelle nette et le modèle explique tant la taille que la date des investissements d'une manière statistiquement et économiquement satisfaisante.

This paper statistically tests the theory of irreversible investment under uncertainty. Using dynamic programming and contingent claims valuation alternatively, we derive the value of options to invest in capacity, where the projects are endogenous to the economic circumstances prevailing at the investment date. We then test whether capacity investment decisions made by Canadian copper mines are compatible with the theory. The result speak strongly in favor of option theory as a theory of real investment; in particular, we provide a test which rejects the Net Present Value criterion, and our model explains both investment size and timing satisfactorily from a statistical and from an economic point of view.

Mots clé : Investissement irréversible; Incertitude; Programmation dynamique; Actifs contingents; Valeur d'option; modèle «Putty Clay»; Investissement réel.

Keywords : Irreversible investment; Uncertainty; Dynamic programming; Contingent claims; Option value; Putty Clay; Real investment.

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[†] Industry Organization and Finance Division, Statistics Canada

[‡] Université du Québec à Montréal, GREQAM and CIRANO

1 Introduction

Much of the economic literature on investment has focused on incremental investment in a neoclassical framework. With notable exceptions such as the seminal works of Arrow (1968) and Henri (1974), and the strand of papers in the environment literature following Arrow and Fisher (1974), this literature generally ignores irreversibilities. More recently, the theory of option pricing has been brought to bear in the area of real investment. It was shown that irreversible investment opportunities may be viewed as options and valued accordingly, and that similar rules as govern the exercise of options may be applied to real investment decisions. While this new literature has been successful and innovative in modeling real investment decisions, and while many simulations have contributed to illustrate its implications, it has not been tested in a statistical sense, and it still has to make a dent in the econometrics of investment. Our paper is a step in that direction. We provide a test of option theory applied to real investment, and, in the process, we actually estimate econometrically both the magnitude and the timing of irreversible investments.

Our work follows, and often relies on, an impressive series of theoretical papers applied to real investment decisions or real asset valuation. Since irreversibility and uncertainty are basic ingredients of option theory, and since irreversibilities are nowhere as obvious as in the area of natural resources, resources and extraction are recurrent topics in that literature. Tourinho (1979) modeled the value of natural resource reserves under uncertainty as an option to extract the resource in the future. Using the contingent claim approach, Brennan and Schwartz (1985) showed how to value an option to invest in a mine and established the companion investment rule. Mackie-Mason (1990) cast the option pricing model into a framework involving the non linear taxation of mining firms. Paddock, Siegel and Smith (1988) used option valuation theory to value leases for offshore petroleum. But, as the survey by Pindyck (1991) and the book by Dixit and Pindyck (1994) make clear, even more papers are unrelated to natural resource investments. To mention some of the most relevant to our purpose, Pindyck (1988) investigated capacity investment as a compounded set of options while Majd and Pindyck (1987) modeled the 'time to build' as a process in which a firm invests continuously until the completion of a project, each expenditure buying it an option to spend the next dollar.

Besides illustrating the numerous implications of option theory to real investment, this literature cast a serious doubt on the validity of alternative, more traditional, theories. It showed that irreversibility affects investment rules in a fundamental way: by undertaking an irreversible investment, a firm gives up the possibility to use new information that might arrive later on (Bernanke, 1983). As stressed by Ingersoll and Ross (1992) and McDonald and Siegel (1986), the combination of irreversibility and uncertainty invalidates the Net Present Value (NPV) rule of investment. Clearly, the old 'Putty Clay' model had addressed the irreversibility issue. However, it had not followed up its implications under uncertainty, so that, in effect, it does not imply a divergence from the NPV rule.

The strong implications of option theory to the realm of real investment underline the urgency of empirical tests. Here, however, the literature is still in its infancy. According to Dixit and Pindyck (1994, p. 483), 'Given the difficulties (...), it is not surprising that there have been few attempts to statistically test the theory of irreversible investment under uncertainty.' They go on to survey 'the limited work that has been done to date.' There were a few attempts, by Rust (1987) and Pakes (1986), to estimate the optimality conditions of the full stochastic dynamic programming problem. Here, generality implies great conceptual complexity and considerable computational difficulties, which often obscure the meaning of empirical findings (see Pakes, 1993). Other papers focus on aggregate qualitative implications of the theory: as verified by Pindyck and Solimano (1993), aggregate investment should be affected by movements in return volatility. An alternative approach is to focus on the threshold that triggers investment; we use that approach here, overcoming the obstacle encountered by Caballero and Pindyck (1992) that the threshold cannot be observed directly. A disadvantage of studies using that approach is their partial nature: they focus on one, among several, characteristic of the investment process. This is also true of studies that focus on the waiting period, such as Hurn and Wright (1994), or on the identification of an option premium in (project or investment) prices, such as Quigg (1993). A related weakness of option models, even in their theoretical formulation, is the fact that they mostly treat project characteristics as exogenous. Our paper attempts to overcome this weakness by using a two-equation structural form, explaining both the threshold price and the magnitude of the investment simultaneously.

Thus the work presented here uses option theory to derive the value of capacity investment projects, where the projects are endogenous to the economic circumstances prevailing at the investment date. Knowing the value of the option to invest, firms compute a threshold value for the project, and decide on investment dates by comparing current project value with the threshold. Of course, a number of simplifying assumptions are made, which are described in details when appropriate. Perhaps the major one is that we focus on a single source of uncertainty: the price of output, copper in our application. While this is clearly a wild simplification, we believe that it preserves the essence of capacity investment decisions in the copper mining industry: once a deposit has been ascertained, the uncertainty surrounding output prices exceeds by far uncertainty on factor prices, the technology, or available ore reserves. Our results speak strongly in favor of option theory as a theory of real investment; in particular, we provide a test which rejects the NPV criterion, and our model explains investment behavior satisfactorily, both from a statistical, and from an economic, point of view.

The paper is organized as follows. In Section 2, we present the capacity investment model, taking the investment date as given; this model allows us to compute the project value. The option to invest is based on the underlying project; its valuation is presented in Section 3. In Section 4, we present the econometric model, the data set and its construction, and the results of our estimations and tests. Further discussion follows in the conclusion.

2 Capacity investment

2.1 A putty clay model of capacity investment

Capacity investments by mines provide a good example of irreversible investment under uncertainty. In fact, there is evidence that the 'Putty Clay' model of investment performs well in explaining capacity levels at Canadian mines (Lasserre, 1985; Harchaoui and Lasserre, 1995), although that model fails to explain the dates at which such investments are undertaken. The option model, on the other hand, focuses on timing, taking the characteristics of the project as given. We use a simple version of the 'Putty Clay' model in order to endogenize the characteristics of the investment project which will be used in the option model presented further below.

Let s be the date at which the capacity investment project is undertaken. We focus on the long term and neglect the construction period; thus s is also the start up date for the new capacity. At this stage, we treat s as exogenous and want to determine the value of the project at s, as a function of economic, technological, and geological conditions observed at s. In order to focus on what we consider the key element in the investment decision, uncertainty about future output price, we assume that, at s, the firm knows its mineral reserves $R(s)$, as well as the available technology, current and future factor prices, and the current and future tax systems. More specifically, we assume that real factor prices are non stochastic and rise at the common, constant, rate α_w; and we assume that the current tax system is expected to remain unchanged.

Once the project is operational, ore reserves are transformed into metal or concentrate, whose real price at date t, $p(t)$, is assumed to follow a Brownian geometric motion

$$\frac{dp}{p} = \alpha_p dt + \sigma dz \tag{1}$$

where dz is the increment of a Wiener process ($dz = \epsilon\left(t\right)\left(dt\right)^{\frac{1}{2}}$ where $\epsilon\left(t\right)$ is a serially uncorrelated and normally distributed random variable with zero expected value and unit variance); α_p, a constant, is the drift component of the price process; and σ, also constant, reflects the variance of the price process.

The 'Putty Clay' assumption implies that *ex ante*, before s, the firm has a choice among a wide array of technologies and scales; *ex post*, once the investment has been realized, the firm must use the particular technology selected at s: for any $t > s$, capacity is fixed at $Q\left(s\right)$ (a scalar), variable factors are fixed at $L\left(s\right)$ (a vector), capital is fixed at $K\left(s\right)$ (a scalar). In that sense, variable factors are just as fixed as capital. The distinction between variable factors and capital stems from the fact that variable factors are paid for at their rental rates w, as flows, over the active life of the mine, while capital is fully paid as a setup cost at s. Empirically, the distinction is also important because capital usually receives a different tax treatment than other factors of production. The technologies available *ex ante* may experience technological change over time; in contrast, *ex post*, any technology is fixed for the active life of the project. We assume that the productive capacity of capital is maintained throughout the operating life at a cost included in the definition of variable costs. Thus, $Q\left(t\right) = f\left(K\left(s\right), L\left(s\right) - L^M(K\left(s\right)), s\right)$ where the *ex post* production function f is defined over the quantities of inputs used in production *per se*, that is, net of the input quantities devoted to maintenance $L^M(K)$. Since $L^M(K)$ does not vary over the operating life of the project, there exists an alternative, equivalent, production function based on gross inputs $F(K, L, s) \equiv f\left(K(s), L(s) - L^M(K(s)), s\right)$.

Despite the maintenance of productive capacity during operating life, capital depreciates, first because of obsolescence (the *ex ante* technology set improves over time while the *ex post* technology is fixed at its level of date s); second because major mining investments are highly mine specific and costly to transfer and adapt to other sites. We assume that the project has no residual value at the end of its operating life.

2.2 The value of the investment project

Mining projects are highly capital intensive. As a result, once a particular capacity investment has been realized, revenues cover variable costs by far; furthermore, variable costs (as defined above) cannot be entirely suppressed by temporarily closing down because of maintenance costs, long-term contractual arrangements, etc.; finally, there are substantial costs associated with starting up again a mine that closed down temporarily. As a result, it is highly unusual to observe mines shut down temporarily, or even simply reduce output substantially, in periods where prices are low. For these reasons, we assume that mines produce at full capacity during their entire, uninterrupted, operating life T. Since the latter is constrained by available reserves, it follows (provided reserves are measured in the same units as output) that

$$T = \frac{R(s)}{Q(s)} \tag{2}$$

If a firm that discounts net real future revenues at rate r undertakes a capacity investment at s, it selects an *ex post* technology (input mix and corresponding capacity) so as to maximize expected net cumulative revenues over its endogenous operating life. Under the assumptions just enunciated, quantities ($Q,\ L,\ K$) are fixed over T, factor price paths are exponential functions, and so is the expected output price path implied by (1). As a result, expected net cumulative revenues may be integrated out, so that the net present value of the project is (see Harchaoui and Lasserre (1995) for a detailed derivation)

$$V = \max_{Q,T,L,K} \left\{ a(T,\delta) A_2 pQ - a(T,\rho) A_2 w' L - A_1 qK \right\} \tag{3}$$

where $a(T,i) = \frac{1-e^{-iT}}{i}$ is the capitalization function giving the present value of a constant flow of \$1 over a period of T when the rate of discount is i. All variables are evaluated at s. q is the asset price of capital. $\delta = r - \alpha_p$ is the discount rate applying to revenues, after correction for the drift in output price. $\rho = r - \alpha_w$ is the discount rate applying to variable costs, after correction for their rate of growth. A_1 and A_2 are tax parameters to be described shortly. As written, V is decomposed into three terms: expected cumulative discounted revenues over the life of the project; cumulative discounted variable costs over the life of the project; and the initial capital investment cost.

The tax parameters reflect the tax regime applying at s. A_1 is a parameter that reflects the extent to which the tax system alters the cost of capital: if tax rates and the various deductions from taxable income (among them capital depreciation allowances) combine in such a way that taxation leaves the cost of capital unchanged, then $A_1 = 1$; if the tax system favors capital, as is usually the case, then A_1 is smaller than 1, possibly negative.[1] Similarly, A_2 represents the after-tax value of one dollar of revenue or variable factor expenditure. In practice, for several of the observations in our sample, the taxation regime is also characterized by a tax holiday. This is a period, usually three years, over which a new mine is subject to a reduced tax load. The results presented in Harchaoui and Lasserre (1995) are inconclusive as to the impact of a tax holiday on investment decisions. This, and a quest for simplicity, is our justification for assuming it away.

2.3 Further properties and assumptions

Capacity investment projects may involve specific characteristics not taken into account in the foregoing analysis. We control for them by introducing the variables X for geological characteristics; G for ore grade G; D for location; and s for the state of technology at the time or the investment. Furthermore, it is immediate to show that V is homogeneous of degree one in after-tax prices (A_2p, A_2w, A_1q), so that it can be rewritten, expanding L and dividing

[1]For calculations of effective marginal tax rates in the Canadian mining industry, see Boadway et al. (1987).

all prices by the after-tax price of materials

$$\begin{aligned} V &= A_2 w_M \max\left\{a\left(T,\delta\right)PQ - a\left(T,\rho\right)\textstyle\sum_i W_i L_i - W_K K\right\} \\ &\equiv A_2 w_M \tilde{V}\left(P, W_L, W_E, W_K, \delta, \rho, R, X, G, D_j, s\right) \end{aligned} \tag{4}$$

where w_M is the price of materials, and $A_2 w_M$ is the after-tax price of materials; $P = \frac{p}{w_M}$; $W_i = \frac{w_i}{w_M}$, for $i = L$ (labor), E (energy); and $W_k = \frac{A_1 q}{A_2 w_M}$.

We assume that technological change is neutral and takes place at a constant rate γ, where neutrality means that $V\left(., s\right) = e^{\gamma[s-s']}V\left(., s'\right)$, where all variables other than s' are evaluated at s. By (4), this is $A_2 w_M\left(s\right)\tilde{V}\left(., s'\right)$ or, substituting $w_M\left(s\right) = e^{\alpha_w s}w_M\left(0\right)$, and choosing $s' = 0$, $V\left(., s\right) = e^{[\alpha_w+\gamma]s}A_2 w_M\left(0\right)\tilde{V}\left(., 0\right)$. Consequently, the optimal expected project value at s may be decomposed into the product of an exponential function of s by a time-autonomous function of P and Z, $v\left(P, Z\right) \equiv A_2 w_M\left(0\right)\tilde{V}\left(P, Z., 0\right)$, where $Z = W_L, W_E, W_K, \delta, \rho, R, X, G, D_j$:

$$V\left(p, w_L, w_E, w_M, q, A_1, A_1, A2, \delta, \rho, R, X, G, D_j, s\right) = e^{[\alpha_w+\gamma]s}v\left(P, Z\right) \tag{5}$$

The next section is devoted to the evaluation of the option to undertake such a project for the owner of the site characterized by (R, X, G, D_j).

3 The option model applied to capacity investment

3.1 The value of the option to invest

Prior to the date s of a capacity investment, a firm may be viewed as holding an option to invest into a project whose value V evolves over time as prices change. If the theory is correct, the firm holds on to the option in the years prior to s, and the option is actually exercised at s.The value of the underlying project (5) determines the value of the option. The latter may be determined by dynamic programming or by contingent claims analysis. Both approaches are presented in parallel by Dixit and Pindyck (1994, e.g. pp. 140-52). They yield similar, but not identical, results, and will be distinguishable in the empirical analysis.

Consider *dynamic programming* first. The value at t, $t < s$, of the option is the expected present value of the project obtained by optimally choosing the date of exercise

$$\Pi\left(P, Z, t\right) = \max_{s} E_t \left\{ e^{-rs} e^{[\alpha_w + \gamma]s} v\left(P, Z\right) \right\} \tag{6}$$

This equals $e^{-[\rho-\gamma]t} \max_s E_t \left\{ v\left(P, Z\right) e^{-[\rho-\gamma][s-t]} \right\}$ since $r - \alpha_w = \rho$. Thus evaluating $\Pi\left(P, Z, t\right)$ requires solving

$$\pi\left(P, Z\right) = \max_{s} E_t \left\{ v\left(P, Z\right) e^{-[\rho-\gamma][s-t]} \right\} \tag{7}$$

In order to distinguish the option Π from π and the project V from v, we call π and v pseudo option and pseudo project respectively. We note that Z is constant: relative factor prices are constant because factor prices are assumed to rise at the same rate; δ and ρ are constant rates; R, X, G, and D_j are geological and technological data which remain unchanged in the period preceding the investment. Consequently, for $t < s$, Bellman's equation corresponding to (7) is

$$[\rho - \gamma]\,\pi = \frac{1}{dt} E_t \left\{ \pi_P dP + \frac{1}{2} \pi_{PP} dP^2 \right\} \tag{8}$$

From (1), the fact that $P = \frac{p}{w_M}$, and the fact that w_M rises at rate α_w, we have

$$dP = \alpha P dt + \sigma P dz \tag{9}$$

where $\alpha = \alpha_p - \alpha_w$ so that

$$E_t \{dP\} = \alpha P\,(t)\,dt \tag{10}$$

$$E_t \left\{dP^2\right\} = P^2 \sigma^2 dt \tag{11}$$

Substituting into (8), we obtain

$$[\rho - \gamma]\,\pi = \alpha P \pi_P + \frac{1}{2} \sigma^2 P^2 \pi_{PP} \tag{12}$$

Solving this differential equation requires invoking appropriate boundary conditions.

Before studying this aspect of the problem, let us turn to the evaluation of the option by *contingent claims* analysis. This approach requires the assumption that stochastic changes in the value of the project may be spanned by existing assets, which means that there exists a portfolio whose price x is perfectly correlated with the value of the project.

The analog to problem (6) is then to evaluate an option $\Pi(P, Z, t)$ giving right to invest, at a date s to be chosen, into a project whose net current value at s will be $x(s) = v(P, Z) e^{[\alpha_w + \gamma]s}$. The coefficient $e^{[\alpha_w + \gamma]s}$ plays the role of a scaling factor for the option so that, for given P and Z, the change of Π over time is, in current value[2]

$$\Pi_t = [\alpha_w + \gamma] \Pi$$

To evaluate Π, we construct a portfolio consisting of the option and a short position consisting of n units of x, where n is selected in such a way that the portfolio is riskless. By the CAPM, in order to accept holding one unit of x, the party entering the long side of the transaction will expect a return of $\tilde{r}$.[3] A unit of x yields an expected rate of capital gain of

$$\frac{1}{dt}\frac{dx}{x} = \alpha_w + \gamma + \frac{\alpha P v_P + \frac{1}{2}\sigma^2 P^2 v_{PP}}{v}$$

Thus the short position will require that the expected rate of capital gain

[2]The option is worth $\Pi(P, Z, t) = E_t \left\{ v(P, Z) e^{[\alpha_w + \gamma]s} e^{-r[s-t]} \right\}$. If, over an interval dt, P and Z do not change, then the probability that the option to invest will be exercised in τ periods from t is the same over the whole interval dt, for any τ. This implies that $ds = dt$. Differentiating $E_t \left\{ v(P, Z) e^{[\alpha_w + \gamma]s} e^{-r[s-t]} \right\}$ under that constraint gives the result.

[3]$\tilde{r} = r_f + \left[\frac{r_m - r_f}{\sigma_m}\right] \rho_{xm}\sigma_m$, where r_m is the market rate of return, σ_m is the standard deviation of that return, r_f is the risk-free rate of return, and ρ_{xm} is the coefficient of correlation between the return on x and the market return.

be completed by a rate of dividend payment of

$$\mu = \tilde{r} - \left[\alpha_w + \gamma + \frac{\alpha P v_P + \frac{1}{2}\sigma^2 P^2 v_{PP}}{v}\right]$$

implying that the portfolio requires a dividend payment of $n\mu x dt$ per interval of time dt. On the other hand, the portfolio is worth $\Pi - nx$, so that it yields, in total, over dt

$$d\Pi - ndx - n\mu x dt$$

Evaluating $d\Pi$ by Ito's lemma, and evaluating dx at $t = 0$ without loss of generality, this yield becomes

$$\begin{aligned} &\Pi_P\left[\alpha P dt + \sigma P dz\right] + \frac{1}{2}\Pi_{PP}\sigma^2 P^2 dt + \Pi_t dt \\ &-n\left[v_P\left[\alpha P dt + \sigma P dz\right] + \frac{1}{2}v_{PP}\sigma^2 P^2 dt\right] - n\left[\tilde{r} - \frac{\alpha P v_P + \frac{1}{2}\sigma^2 P^2 v_{PP}}{v}\right] v dt \end{aligned}$$

where (9) was used to substitute for dP and dP^2. This may be seen to be riskless if $n = \frac{\Pi_P}{v_P}$; then, non arbitrage requires the yield to equal the riskless return on the value of the portfolio

$$\left\{\frac{1}{2}\sigma^2 P^2 \Pi_{PP} - \left[\tilde{r}\frac{v}{v_P} - \alpha P\right]\Pi_P + \Pi_t\right\} dt = r_f\left[\Pi - \frac{v}{v_P}\Pi_P\right] dt$$

where r_f is the riskless rate of return. Dividing by dt and rearranging we obtain

$$r_f \Pi = \left[\alpha - \frac{\frac{v}{P}}{v_P}\left[\tilde{r} - r_f\right]\right] P\Pi_P + \frac{1}{2}\sigma^2 P^2 \Pi_{PP} + \Pi_t \tag{13}$$

This is a partial differential equation. However, consider the corresponding pseudo-option defined as

$$\pi(P,Z) = \Pi(P,Z,t)\, e^{-[\alpha_w+\gamma]t}$$

This pseudo-option is time autonomous because $\Pi_t = [\alpha_w + \gamma]\,\Pi$; also, $\pi_P = \Pi_P e^{-[\alpha_w+\gamma]t}$ and $\pi_{PP} = \Pi_{PP} e^{-[\alpha_w+\gamma]t}$. Substituting into (13), and defining $\rho_f \equiv r_f - \gamma$, we obtain

$$[\rho_f - \gamma]\,\pi = \left[\alpha - \frac{\frac{v}{P}}{v_P}\left[\tilde{r} - r_f\right]\right] P\pi_P + \frac{1}{2}\sigma^2 P^2 \pi_{PP} \tag{14}$$

a homogeneous, second degree, differential equation in π similar to (12) but with different coefficients. Since v is not observable, an interesting special case of (14) is when v is proportional to P

$$[\rho_f - \gamma]\,\pi = [\alpha - [\tilde{r} - r_f]]\, P\pi_P + \frac{1}{2}\sigma^2 P^2 \pi_{PP} \tag{15}$$

In that case, comparing (15) with (12), one notes that the contingent claims analysis solution is identical to what had been found by dynamic programming if the discount rate had been r_f rather than r and if α had been reduced by the difference between the market rate and the risk free rate. Thus, in general, dynamic programming and contingent claims analysis give different evaluations of the option. It should be noted, however, that these versions become identical under risk neutrality, when the proper discount rate is r_f.

Whatever the evaluation procedure – dynamic programming of contingent claims – the differential equations must satisfy the same boundary conditions.

First, the value of the pseudo option collapses to zero if P approaches zero

$$\pi(0, Z) = 0 \tag{16}$$

Second, the value of the pseudo option, which is normally higher than the current value of the pseudo project, equals $v(P^*, Z)$ if the price equals P^*, the exercise price

$$\pi(P^*, Z) = v(P^*, Z) \tag{17}$$

This means that $\pi - v$ reaches a minimum with respect to P at P^*. Third, by the smooth pasting condition, this minimum is interior

$$\pi_P(P^*, Z) = v_P(P^*, Z) \tag{18}$$

3.2 Solution

The solution to (12) or (15) that satisfies (16)-(18) is

$$\pi(P, Z) = cP^b$$

where the parameters b and c differ depending on whether the option is evaluated by dynamic programming or by contingent claims analysis. In the case of dynamic programming

$$b = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \left[\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + 2\frac{[\rho - \gamma]}{\sigma^2}\right]^{\frac{1}{2}} \tag{19}$$

b must be greater than one, which is the case if

$$\rho - \gamma > \alpha \text{ i.e. } r - \alpha_w - \gamma > \alpha_p - \alpha_w \text{ or } r > \alpha_p + \gamma$$

Indeed, if the discount rate did not more than offset the combined effect of the growth in output price and technological change, it would always be preferable to wait, without investing, leaving the project rise in present value forever. In the case of contingent claims analysis

$$b = \frac{1}{2} - \frac{\alpha - [\tilde{r} - r_f]}{\sigma^2} + \left[\left[\frac{\alpha - [\tilde{r} - r_f]}{\sigma^2} - \frac{1}{2}\right]^2 + 2\frac{\rho_f - \gamma}{\sigma^2}\right]^{\frac{1}{2}} \tag{20}$$

b will be greater than one, as required, if

$$\rho_f - \gamma > \alpha - [\tilde{r} - r_f] \text{ i.e. } r_f - \alpha_w - \gamma > \alpha_p - \alpha_w - [\tilde{r} - r_f] \text{ or } \tilde{r} > \alpha_p + \gamma$$

c and P^* are obtained from (17) and (18): (17) implies

$$c = \frac{v(P^*, Z)}{P^{*^b}} \tag{21}$$

and, from (18)

$$cbP^{*^{b-1}} = v_P(P^*, Z) \tag{22}$$

The solution can now be written in an empirically useful form. From (5), $v_P = e^{-[\alpha_w + \gamma]s} V_p \frac{\partial p}{\partial P}$ with $P = \frac{p}{w_M}$; furthermore, since V in (3) is obtained by constrained maximization with respect to Q, the envelope theorem implies

$$V_p = a(T, \delta) A_2 Q$$

It follows that

$$v_P(P, Z) = e^{-[\alpha_w + \gamma]s} a(T, \delta) A_2 Q w_M \tag{23}$$

Substituting for c and v_P in (22)

$$\frac{v(P, Z)}{P^{*^b}} b P^{*^{b-1}} = e^{-[\alpha_w + \gamma]s} a(T, \delta) A_2 w_M Q$$

which, using (4) and (5), reduces to

$$\frac{b-1}{b} P^* = \frac{a(T, \rho) \sum_i W_i L_i + W_K K}{a(T, \delta) Q} \tag{24}$$

Whether in its dynamic programming version, or in its contingent claims version, (24) will be the basis of our econometric model. It deserves some comments. First, it is important to remember that (T, Q, L, K) solve the expected NPV maximization problem (3). T is the endogenous extraction period $\frac{R}{Q}$, reflecting the constraint imposed by the finiteness of reserves, L and K are factor demands, and Q is a supply (or capacity) function. As such, they are functions of (P, Z). It is useful to treat L and K as conditional factor demands, and Q as the supply function. In that case, the right-hand side of

(24) may be written as

$$U\left(Q\left(P,Z,s\right),Z,s\right)\equiv\frac{a\left(\frac{R}{Q(P,Z,s)},\rho\right)\sum_i W_iL_i\left(Q\left(P,Z,s\right),Z,s\right)+W_KK\left(Q\left(P,Z,s\right),Z,s\right)}{a\left(\frac{R}{Q(P,Z,s)},\delta\right)Q\left(P,Z,s\right)} \tag{25}$$

The numerator gives total cumulative costs as evaluated at s. The denominator is an output aggregator which, considering the definition of $a\left(T,\delta\right)$, gives more weight to units produced at the beginning of the operating period than it gives to units produced toward the end of the operating period.

Consequently, the right-hand side of (24) is a unit-cost function evaluated at the optimal level of Q. Thus (24) expresses the relationship between price and average total cost at the price P^* which triggers the investment

$$BP^* = U\left(Q\left(P,Z,s\right),Z,s\right) \tag{26}$$

where $B\equiv\frac{b-1}{b}$ is the inverse of a mark-up coefficient. Since $b>1$, $0<B<1$, so that the trigger price must exceed average cost, which means that the option of waiting has a positive value: invest only at some strictly positive NPV. If $B\rightarrow1$, as is the case if $\sigma\rightarrow0$, we have the standard NPV condition: invest if the NPV of the project is zero or higher, i.e. if price equals or exceeds average cost.

The assumption of a neutral form of technological change has led to a system of two equations (21) and (22) which define c and P^* implicitely as functions of P and Z only; consequently, the right-hand side of (26) must also be independent of s. This requires

$$\frac{\partial U}{\partial Q}\frac{\partial Q}{\partial s} = -\frac{\partial U}{\partial s} \tag{27}$$

Also, when v is proportional to P, the restriction giving rise to (15) under contingent claims evaluation, it can be shown that U is linear in P

$$U\left(Q\left(P, Z, s\right), Z, s\right) = \Gamma\left(Z, s\right) P$$

so that (26) reduces to

$$B = \Gamma\left(Z, s\right) \tag{28}$$

(27) and (28) correspond to testable restrictions in the econometric work whose description follows.

4 The empirical estimation

4.1 Data: output price

Crucial to the theoretical model described earlier is the assumption that the real output price follows a geometric random walk. Since the model will be applied to Canadian copper mines this implies that the Canadian price of copper, relative to materials, should verify (9): $dP = \alpha P dt + \sigma P dz$. This assumption must be tested; if it is not rejected, we also need to estimate α and σ. The random walk hypothesis for U.S. copper prices is known to be rejected when it is tested over the very long period. Our point of view is different, as we are interested in testing the hypothesis over periods that are relevant for capacity investment decisions, i.e. usually less than 20 years, and as the decisions under investigation were all made during the post-war period. For these reasons, we limit our investigation to the 1940-1980 period (see the Appendix for data description and sources).

Visual inspection of the graph of P (not provided) does not suggest that any decisive change in the slope or the level occurred during the period 1940-1980. Consequently the Dickey-Fuller test is appropriate. Consider the model

$$\Delta P(t) = \eta_0 + \eta_t t + [\eta_P - 1] P_{t-1} + \sum_{j=1}^{4} \eta_j \Delta P_{t-j} + \zeta(t)$$

where $\Delta P(t) = P(t) - P(t-1)$. Testing the random walk hypothesis[4]

[4] We also carried out the test on a discrete version of (9): $\ln P(t) - \ln P(t-1) = [\alpha - \sigma] + \zeta(t)$ where $\zeta(t) \sim n(0, \sigma)$. The augmented model was

$$\ln P(t) - \ln P(t-1) = [\alpha - \sigma] + \eta_t t + [\eta_P - 1] \ln P(t-1) + \sum \eta_j \Delta \ln P(t-j) + \zeta(t)$$

involves the joint test that $\eta_t = \eta_P - 1 = 0$. Since we want to test whether this is a good assumption from the point of view of a firm considering an investment at $s \in [1955 - 1980]$, we carry out the test for each of the 26 sub-periods $1940 - s$, $s \in [1955 - 1980]$. There is no period over which the geometric random walk is rejected[5].

Having failed to reject the random walk hypothesis, we proceed to estimate parameters α and σ under that assumption. Following Slade (1988), we use the following discrete approximation

$$\Delta P(t) = \alpha P(t) + \xi(t) \tag{29}$$

where $\xi(t)$ is an heteroscedastic error term such that $\xi(t) = \sigma P(t)\epsilon$. Again, since we want to consider firms making investment decisions, under the geometric random walk hypothesis, at various dates, we estimate (29) over each of the 26 possible $1940 - s$ periods so that, in each case, the estimates of α and σ reflect the information available at s and only that information.[6]

As shown in Table 1, α is a small, positive, number. From the point of view of testing the option theory of real investment, the magnitude and sign of α are of little interest, with the important proviso, however, that the resulting value of b be greater than one as mentioned in the theoretical

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where $\Delta \ln P(t) = \ln P(t) - \ln P(t-1)$. Testing the geometric random walk hypothesis involved the joint test that $\eta_t = \eta_P - 1 = 0$. Results were similar.

[5] The highest (least favourable to the random walk hypothesis) F statistic is 4.17 (for the regression 1940-1970); a level of at least 5.61 would be required to reject the hypothesis at a 90% significance level.

[6] Sampson (1995) studies the implications on the option model of assuming that the drift parameter, rather than being known with certainty, is learnt progressively as new observations arrive.

presentation of the model (otherwise P^* does not exist); this condition is satisfied at each investment date.

Variable (%)	Mean	Standard Deviation	Minimum	Maximum
α	1.93	.072	.066	4.23
σ	13.70	1.46	11.10	16.40
$\tilde{r}$	8.32	1.39	4.37	10.11
r_f	5.79	1.39	1.84	7.58

Table 1: the determinants of the inverse mark-up B

(355 capacity investment decisions at 20 different dates)

Since the investment rule of the option model coincides with the NPV criterion when $\sigma = 0$, it is important that the measured value of σ be different from zero. Its mean value of 13% indicates that the drift in the price, although low on average at less than 2%, is subject to fluctuations whose standard error exceeds six times its mean.

4.2 Other data

The data set is made of individual observations on Canadian copper mines. Details and sources are given in the Data Appendix. The sample period covers the period 1955-1980, corresponding to major capacity investments which actually occurred between 1961 and 1980. A few other investments involving copper extraction took place after 1980 but were not included in the sample because their major purpose was the mining of zinc. 38 different firms, some of which were observed more than once, made a total of 60 capacity investments, of which 20 in British Columbia, 9 in Ontario, 8 in Québec, and 1 in the Yukon. An observation is a vector of variables corresponding to the occurrence of a major capacity investment (which we define as either the creation of a new operation, or any capacity increase exceeding 20 percent of existing capacity[7]), or to the failure of any major investment to occur. The data set is neither a time series, nor a cross section or a panel: there may be several observations, or only one observation, in any given year. In fact the data set is organized as a succession of short time series whose dates may overlap. Each series corresponds to the 'history' of a particular investment. The last observation in each series corresponds to the actual investment, and the preceding observations correspond to a 'waiting period' over which the firm was – this is the hypothesis to be tested – holding on to the option to invest. The data indicate only a few instances of minor corrections or validations of reserve figures made in the period immediately preceding the investment, allowing us to assume that (R, X, G, D_j) was known and constant at $(R(s), X(s), G(s), D_j(s))$ during the 'waiting period'. The latter is set at six year, unless a firm made major capacity investments at intervals of less than six years, in which case we reduce the period in such a way that the time series for any two investments by the same firm do not overlap. Of course the time series corresponding to investments by different firms may have common years. Some of the variables in an observation vector, prices typically, are common to all firms and vary only according to the date, while others, such as mineral reserves, are mine specific. Although output and

[7]In Harchaoui and Lasserre (1995) it was found that the capacity chosen was not affected by whether it was a creation, or an expansion.

factor prices are common at any given date, after-tax prices are mine specific because the tax parameters entering their definition may differ according to mine location.

Thus we have 355 observations corresponding to the stacking of 60 time series of 7 or less observations each containing the variables necessary to estimate the two equation econometric model (30)-(31) presented further below. These variables, which are either directly observed, or constructed, are to recap:

B: inverse mark-up coefficient at s; $B = \frac{b-1}{b}$ with b given by (19) or (20) alternatively;

D_j: dummy variables for location (Ontario, Québec, Yukon);

G: ore grade, in percentage, at investment date (constant over the waiting period);

P, (P^*): after-tax flow price of copper relative to materials (unobserved exercise price) at s; P and P^* coincide by definition when an investment occurs;

Q^*: capacity chosen at s ($Q^* = 0$ if the investment did not occur at s);

r: real discount rate for long-term projects at s (includes a risk premium under the assumption of risk aversion; equals the risk-free rate otherwise);

r_f: risk free interest rate at s;

$\tilde{r}$: risk adjusted rate of return at s;

R: mineral reserves at investment date (constant over the waiting period);

s: year of a major capacity investment decision (wait, or invest);

W_L, W_E: after-tax flow prices of labor and energy relative to materials at s;

W_K: after-tax asset price of capital equipment relative to materials at s;

X: dummy variable (= 1 for open-pit operations; = 0 for underground operations);

α and σ: drift ($= \alpha_p - \alpha_w$) in the after-tax price of copper relative to materials, and its variance parameter, as estimated at the investment date;

δ ($= r - \alpha_p$): discount rate applying to revenues at s, after correction for the drift α_p in real output price;

γ: rate of neutral technological change;

ρ ($= r - \alpha_w$): discount rate applying to variable costs at s, after correction for the real rate of growth α_w common to the four factor of production prices (in practice $\alpha_w = 0$);

Except for the parameters of the price process whose estimation was described above, most data on that list are derived in a straightforward fashion from available sources, as described in the Appendix. However, r deserves some further comments.

Under risk neutrality, the discount rate equals the risk free rate: $r = r_f$. Otherwise, if its conditions of validity are satisfied, the CAPM provides a way to compute the risk premium. It should be noted that, when the CAPM applies, the valuation of the investment option by contingent assets analysis gives the maximized market value of the option, which implies that an evaluation by dynamic programming, either gives the same result, or undervalues the option. Thus it appears that contingent asset valuation should be preferred when the CAPM applies, and dynamic programming is the alternative otherwise. However, when the CAPM is not valid, we do not have a clear theory for the choice of a discount rate. Despite this, we use the risk premium from the CAPM to compute the discount rate for the dynamic programming valuation[8]: $r = \tilde{r}$.

We constructed three alternative 355 observation data sets, differing by the discount rate and by the variables involving the discount rate, B in particular. Each data set corresponds to one of the following alternative hypotheses:

- dynamic programming with risk aversion: B is computed according to (19), with the discount rate set at $r = \tilde{r}$;
- contingent claims valuation with risk aversion: B is computed according to (20); the discount rate is set at $r = \tilde{r}$;
- risk neutrality: $\tilde{r}$ and r are set equal to r_f and B is computed according to (19) or, which is equivalent under risk neutrality, (20).

It can be seen in Graph 1 that B averages between around .5 and .7 depending on the underlying hypothesis and varies within reasonable bounds. This implies an average mark-up over average total cost of between around 100% and 43%.

PLEASE INSERT GRAPH 1 HERE

[8] Since risks are likely to be less efficiently diversified when the CAPM is not valid, that evaluation should be viewed as a lower bound.

4.3 Econometric model and estimation procedure

Assume that $\ln U$ is linear in $\ln Q^*$ and in the Z_i's.[9] Then (26) may be written as

$$\ln P^* = a_0 - a_B \ln B + a_Q \ln Q^* + \sum_i a_i Z_i + a_s s + \epsilon \qquad (30)$$

where ϵ is a (full-sample) error term, the a_j's are parameters and, if (26) is true, $a_B = 1$. The notation Q^* is a reminder that Q is endogenous.

Assume that the supply function may be represented by the log-linear form

$$\ln Q^* = b_0 + b_P P + \sum_i b_i Z_i + b_s s + \nu \qquad (31)$$

where the b_i's are parameters and ν is a (full-sample) error term. Note that, in that equation, P is the market price, and not the trigger price, although these two prices coincide at the dates when investments are observed.

Equations (30) and (31) represent a simultaneous-equation model with truncation. The structural form may be estimated using the two-stage least square method suggested by Lee, Maddala and Trost (1980) for such truncated models. Since only (30) includes an endogenous variable $\ln Q^*$ on its right-hand side, the simultaneity problem is dealt with by first estimating (31) and then use its predicted value in (30).

In our initial estimation of (31) by Heckman (1979)'s two-stage method, the ratio of Mills recovered from the first-stage probit estimation was not

[9] We considered alternative specifications for U, ranging from the Cobb Douglas form to the standard log-linear form. The adopted specification dominates them in J tests based on the actual estimating equation (32) presented further below; the contribution of $\ln B$, which is crucial to our analysis, is not very sensitive to these alternative specifications.

significant in the second-stage equation, estimated from the positive truncation of the sample. If the residuals are normally distributed, this indicates that the truncation of the sample does not systematically bias the residuals of equation (31) away from zero. Consequently, (31) was estimated again by Ordinary Least Squares from the truncated sample without including Mills' ratio in the explanatory variables. The residuals were submitted to a Bera-Jarque test which confirmed the absence of any significant departure from normality.[10]

Next we substituted $\ln \hat{Q}^*$, the predicted value of $\ln Q^*$ from (31) into (30) which was in turn estimated by Heckman's method. Let $\hat{h} = \frac{\hat{\phi}}{\hat{\Phi}}$ be the ratio of Mills recovered from the first-stage probit estimation based on (30), where $\hat{\phi}$ and $\hat{\Phi}$ are the density and cumulative functions of the standard normal distribution evaluated at each predicted value of the probit model. Since $E(\epsilon \,|\, P^* > 0) = -\sigma_{1,\epsilon}\hat{h}$, where $\sigma_{1,\epsilon} = Cov(\epsilon, \epsilon \,|\, P^* > 0)$, the second-stage equation, estimated from the positive truncation of the sample, is

$$\ln P^* = a_0 - a_B \ln B + a_Q \ln \hat{Q}^* + \sum_i a_i Z_i + a_s s - \sigma_{1,\epsilon}\hat{h} + \epsilon_1 \qquad (32)$$

where ϵ_1 is a normally distributed residual whose expected value is zero.

While the estimation of (32) by ordinary least squares yields unbiased parameter estimates, Lee, Maddala, and Trost have shown that the associated asymptotic covariance matrice must be corrected to account for the fact that

[10] We corrected for heteroscedasticity using White (1980)'s consistent covariance matrix method.

$\hat{h}$ is estimated. The appropriate estimated covariance matrix is

$$\hat{\Sigma}(a, \sigma_{1,\epsilon}) = \hat{\sigma}_1^2 (X_1^{'} X_1)^{-1} - \hat{\sigma}_{1,\epsilon}^2 (X_1^{'} X_1)^{-1} X_1^{'} \left[D_1 - D_1 H_1 (H^{'} \Lambda H)^{-1} H_1^{'} D_1 \right] X_1 (X_1^{'} X_1)^{-1} \tag{33}$$

where

$X_1 = \left[1, \ln B, \ln \hat{Q}^*, Z, s, \hat{h}\right]$;

D_1 is a $(N_1 \times N_1)$ diagonal matrix, (N_1 being the number of observations for which P^* is observed) whose i^{th} diagonal term is the value at observation i of $\hat{h}\left(\hat{h} + \hat{\eta}^{'} Y\right)$ where Y and $\hat{\eta}$ are, respectively, the vectors of explanatory variables and the estimated parameters of the first-stage probit model associated with (30));

$H = \begin{bmatrix} Y_1^{'} \\ . \\ . \\ Y_N^{'} \end{bmatrix}$ is partitioned as $H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}$ where H_1 corresponds to the N_1 instances where P^* is observed while H_2 corresponds to the zeros;

Λ is a $(N \times N)$ diagonal matrix whose i^{th} diagonal term is the value at i of $\hat{h}\hat{g}$ where $\hat{g} = \frac{\hat{\phi}}{1 - \hat{\Phi}}$;

$\hat{\sigma}_1^2 = \frac{1}{N_1} \sum_{i=1}^{N_1} \left[\hat{\epsilon}_1^2 + \hat{\sigma}_{1,\epsilon}^2 \left(\hat{\eta}^{'} Y \right) \hat{h}_i + \hat{\sigma}_{1,\epsilon}^2 \hat{h}_i^2 \right]$;[11]

(33) takes account of the fact that $\hat{h}$ is an estimated variable in (32); however, it overlooks the fact that $\ln \hat{Q}^*$ is used as an instrument for $\ln Q^*$ in that equation. Murphy and Topel (1985) devised a procedure which corrects the resulting bias in the estimated covariance matrix of the second step

[11]This is actually the formula suggested by by Lee and Trost (1978, 362) to ensure that $\hat{\sigma}_1^2$ be positive.

estimator. Their corrected covariance matrix is

$$\tilde{\Sigma}(a,\sigma_{1,\epsilon}) = \hat{\Sigma}(a,\sigma_{1,\epsilon}) + (X_1^{'}X_1)^{-1}X_1^{'}F^{*}\hat{\Psi}F^{*^{'}}X_1(X_1^{'}X_1)^{-1} \tag{34}$$

where $\hat{\Psi}$ is the estimated covariance matrix of the parameters of (31) and F^{*} is a matrix whose elements are of the form $f_{i,j}^{*} = \hat{a}_Q\,[1, P, Z_i, s]$. (34) is the covariance matrix used to establish the t statistics for the parameters of the price equations displayed in Table 2 below.

4.4 Model validation and results

As the econometric version of (26), (32) is the key equation necessary to test option theory applied to real investment. If a_B is not a significant parameter, then there is no option value to waiting, and the model may be interpreted as a standard NPV model of irreversible investment. If a_B is a significant parameter, then option theory further implies that it should not be significantly different from 1. These two tests may be applied to the various versions of the option model discussed above, which differ by the data sets used in the estimation of both (32) and (31). If the theory passes these two tests, then both (32) and (31) become key components in the option model of real investment: (32) explains the timing of investment decisions (invest when P reaches P^{*} from below); (31) explains the magnitude of the investment.

Of course the results are worth what the estimation procedure and the data are worth. Heckman's two-stage procedure has been criticized by Nawata (1993): since the hazard ratio used in that procedure is closely approximated by a linear function of $\hat{\eta}' Y$, he argued that estimators based on that procedure are likely to perform poorly when there is a high degree of multicollinearity between $\hat{\eta}' Y$ and the explanatory variables used in the second-stage model. Such is not the case with our estimations, where correlation coefficients are not higher than 10%. Nawata and others also argued that the reliability of Heckman's procedure is uncertain when the residuals are not normally distributed. A Bera-Jarque test of normality is reported for all results presented below. The least favorable value of the statistics is .8, while rejection of normality at a significance level of 80% would require a level of at least 3.22. With respect to omitted variables, or errors in variables, we introduced dummy variables when we suspected a possible problem; the results are reported only when the variable is at least marginally significant, as in the case of X. In particular, dummies for location (provinces) and type of reserves (proven versus probable) did not make any difference.

Results are reported in Table 2. For all equations, the vector Z of the theoretical model was redefined as $W_L, W_E, W_K, \frac{\delta}{\rho}, R, X, G$, i.e. δ and ρ were introduced as a ratio rather than as separate variables, in an attempt to alleviate colinearity problems and to pick up some explanatory power from these rates. Also, $\frac{\delta}{\rho}$ was eliminated from the price equations because it was not significant, and the regional dummies were eliminated from all equations for the same reason. The table presents three (second-stage) estimations of the trigger price equation (32), one for each hypothesis; there are only two corresponding estimations of (31) because, B not being one of their explanatory variables, the capacity equations differ only by the discount rate and the same discount rate is used under both the dynamic programming and the contingent claims hypotheses. The explanatory variables $\ln \hat{Q}^*$ used in the price equations are the predicted values from the appropriate capacity equations.

PLEASE INSERT TABLE 2 AROUND HERE

The coefficient of the variable $\ln B$, crucial in testing the theory of options applied to real investment, is significant at a better than 95% level of significance under all three hypotheses. The t-test of the hypothesis that a_B is not different from 1 is passed at a better (i.e. lower) than 90% level of significance by the dynamic programming version of the risk aversion model (*t=-1.40*) and by the risk neutrality model (*t=-1.58*); however, the contingent claims version of the model fails that test (*t=-2.85*).

The fact that a_B is not equal to -1 in the contingent claims version of the option model may mean that the underlying spanning hypothesis does not hold for capacity investments in Canadian copper mines. However, as mentioned earlier, our test is based on a special case of the model where it is assumed that v is proportional to P, implying (28), i.e. that B is independent of P. This may be tested directly by writing a linear version of (28), extended to include P as explanatory variable

$$\ln B = c_0 + c_P \ln P + \sum_{i=1}^{m} c_i Z_i + c_s s - \sigma_{1,\omega} \hat{h}_B + \omega_1$$

where, in a two-stage estimation by the same procedure as was applied to estimate (32), $\hat{h}_B$ represents the ratio of Mills recovered from the first-stage probit and the rest of the notation is self-explanatory. Restriction (28) implies that c_P is not significantly different from zero. It is rejected[12], making the earlier test of the contingent claim version of the model inconclusive.

The three versions of the model must also satisfy (27), the assumption of neutral technological change, which implies the cross-equation restriction $a_Q\beta_s = -a_s$. This may be tested from the reduced form of the model obtained by substituting (31) into (30)

$$-a_Q b_P P + \ln P^* = a_0 + a_Q b_0 - a_B \ln B + \sum_{i=1}^{m} \left[a_Q b_i + a_i\right] Z_i - \left[a_Q\beta_s - a_s\right] s + \epsilon + \nu$$

On investment years, linearizing $\ln P^*$, the left-hand side may be approximated as $[1 - a_Q b_P]\, P$ which gives, after dividing by $[1 - a_Q b_P]$

$$P = d_0 - d_B \ln B + \sum_{i=1}^{m} d_i Z_i - d_s s - \sigma_{1,\xi}\hat{h}_P + \xi_1$$

where $d_s = \frac{a_Q\beta_s - a_s}{1 - a_Q b_P}$ and the rest of the notation is by now self-explanatory. Restriction (27) requires d_s not to be different from zero; it is rejected for all three versions of the model. However, with d_s estimated at around .04, s has little impact on the predicted values of P. Furthermore, the estimated value and significance of d_B is robust to the omission of s from the model. This suggests that the test based on a_B presented earlier is little affected by the assumption that (27) holds.

The three price equations may be compared on other grounds. We note that they have good explanatory powers, with R^2's ranging from .91 for the dynamic programming model under both risk aversion or risk neutrality to

[12] Detailed results pertaining to the test of (28) and (27) are available on request.

.86 for the contingent claims model under risk aversion. In an attempt to discriminate between the three models, since they do not involve the same data, we carried out J-tests. These tests (not reported) do not allow the rejection of any of the three price equations in favor of any of the remaining ones.

4.5 Economic analysis

To our knowledge, our work provides one of the first statistical test of option theory, as a theory of real investment. The tests just presented strongly support that theory, at least in its dynamic programming version. This result is most welcome, as alternative investment theories have a relatively poor record of explaining observed behaviors in many areas of economic activity. In particular, as documented by Chirinko (1993), alternative models imply that prices (especially output price and the user cost of capital) should explain a higher proportion of variations in investment than is actually found in empirical work. In contrast the option model, while not ruling out such an influence, implies that the influence of prices may also be manifest in the timing of investments. Similarly, empirical neo-classical supply models, especially in resource sectors, often find that output price explains supply poorly, as indeed one must expect in models such as the option model where supply is constrained by capacity, and capacity is subject to hysteresis.

To examine our results more closely, starting with the capacity choice equations (columns 2 and 4), we note that capacity is significantly positively related to output price, and that the corresponding elasticity (2.44 under risk aversion; see Table 3) is fairly high. In a neo-classical supply model this elasticity would probably have been lower, perhaps not significant, because instances where capacity was not adjusted would have been treated the same way as instances where it was increased. It appears nonetheless that, once the decision to invest has been taken, geological and physical considerations play the key role in the choice of the scale of operations. Thus higher reserves, as well as a geology amenable to open-pit mining, call for a higher production rate. In contrast, a high ore grade is associated with lower scale, in accordance with the observation that high-grade deposits are often small. Factor prices do not appear to affect capacity in any statistically significant way, but the ratio of adjusted discount rates $\frac{\delta}{\rho}$ affects it positively. As explained in the resource literature, the discount rate has an ambiguous effect on the rate of extraction of an exhaustible resource when capital is used as an input: on one hand, a low discount rate implies a low user cost of capital, allowing for a higher capacity; on the other hand, the same low discount rate implies a high resource rent, calling for slow extraction. In the specification adopted here, W_K picks up the input price effect, while $\frac{\delta}{\rho}$ picks up the resource rent effect. A low value of δ means that copper prices have a rising trend and (or) that the discount rate is low so that future revenues are weighted heavily: there is a gain involved in economizing on the exhaustible resource by adopting a slow extraction path. Similarly, a high value of ρ means that future costs are discounted heavily: again one may save on discounted future costs by choosing to extract slowly.

Turning to the threshold price equations, one should remember that a good prediction of that price amounts to a good prediction of investment timing. With R^2's ranging from .86 to .91, our model performs rather well in that respect. Because of their particular role in the option theory of real investment, it is natural to devote some attention to the impact of the variables that determine the mark-up of the trigger price over average cost, in particular the risk associated with the randomness of copper prices, as measured by σ, the drift in the relative price of copper α, and the risk adjusted rate of interest $\tilde{r}$. These variables do not enter the econometric model directly, but determine the value of B, so that they have a statistically significant impact on P^* (the t statistics associated with B range from -3.33 for the dynamic programming version of the model under risk aversion to -6.44 under risk neutrality). While their qualitative impact has been discussed extensively in the literature, this is, to our knowledge, the first instance in which the magnitude of that impact is measured econometrically. The elasticities[13] presented in Table 3 indicate that it is modest, except perhaps in the case of σ where the elasticity is .21 under risk aversion for the dynamic programming

[13]the impact of σ on the trigger price,via B, is, in elasticity form, $\epsilon_{P,\sigma}(\sigma,\alpha,\rho,\gamma) = \frac{\partial P^*}{\partial\sigma}\frac{\sigma}{P^*}$ with

$$\frac{\partial P^*}{\partial \sigma} = -\frac{\partial \ln P^*}{\partial \ln B}\frac{P^*}{B}\frac{\partial B(b)}{\partial b}\frac{\partial b(\sigma,\alpha,\rho,\gamma)}{\partial \sigma}$$

and

$$B = \frac{b(\sigma,\alpha,\rho,\gamma)-1}{b(\sigma,\alpha,\rho,\gamma)}$$

In the dynamic programming version of the model, b is given by (19) so that

$$\frac{\partial b(\sigma,\alpha,\rho,\gamma)}{\partial \sigma} = 2\frac{\alpha}{\sigma^3} + \frac{1}{2\sqrt{\left(\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2 + 2\frac{\rho-\gamma}{\sigma^2}\right)}}\left(-4\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)\frac{\alpha}{\sigma^3} - 4\frac{\rho-\gamma}{\sigma^3}\right)$$

The elasticity of P^* with respect to σ under risk aversion given in Table 3 is computed according to these formula, where the relevant variables are evaluated at their mean sample values. Other elasticities with respect to σ, $\tilde{r}$, and α are computed in a similar way.

model. The dimension of the project, as represented by Q^*, is an important and highly significant determinant of the trigger price. To some extent, this has been ignored in the theoretical option-value literature where projects are typically treated as exogenous. The endogeneity of the investment project implies that the effect of factor prices on the threshold price may be separated out into a direct effect on P^* and an indirect effect occurring via Q^*. It is clear from Table 3 that, once the combined effect is taken into account, factor prices have little measurable effect on P^*.[14]

PLEASE INSERT TABLE 3 AROUND HERE

[14]For $i = L,\ E,\ K$, the total effect on P^* of a change in W_i is, in elasticity form

$$E_{P,W_i} = \epsilon_{P,W_i} + \epsilon_{P,Q}\epsilon_{Q,W_i}$$

where $\epsilon_{P,x}$ ($\epsilon_{Q,x}$) represents the partial elasticity of P^* (Q^*) with respect to any variable x. In the case of W_L, for the dynamic programming model under risk aversion, this gives, according to Table 3, $E_{P,W_L} = -1.482 + .461 * 2.582 = -.2917$ which is not significantly different from zero since ϵ_{Q,W_L} is itself not significantly different from zero.

5 Discussion and conclusion

Certainly, variants of, or alternatives to, the standard neoclassical theory, such as the *Putty Clay* model of investment, have been designed to deal with irreversibilities and indivisibilities. However, even when they explain capacity levels, these alternative theories have practically nothing to say about the timing of investment decisions. The one exception is the NPV criterion, which, applied in a stochastic context that is foreign to it, calls for a go ahead whenever the project value is high enough to cover costs. It has been shown that such a decision rule is suboptimal and may be costly whenever a project's value is uncertain and new, valuable, information about it arrives over time. Our results indicate that firms behave in a way compatible with that remark. Furthermore the option value model of real, irreversible, investment appears to explain both capacity, and timing, decisions in a satisfactory way, both from a statistical and from an economic point of view.

This allows us to be hopeful that option theory, as applied to real investment, may become a useful tool of empirical investigation. Until now, option theory has been more an illustrative device than a tool of investigation. In particular, it has been much used in simulations but little as a basis for statistical inference. Of course, many gaps remain to be filled. Many features that have been studied in the theoretical literature were left out of our model. In particular, we did not consider the possibility of further capacity expansions; we did not consider the duration of the investment process; we ignored, although on good empirical grounds, the option to shut down during the mine's life; we focused on output price as the single source of uncertainty, using a procedure which remains conditional on the validity of the Brownian-motion assumption. These are serious restrictions which beg to be eliminated in further work, if the theory is to become applicable in wider a range of applications. As to the application presented in this paper, the statistical results suggest that its validity was not affected by these restrictions.

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Data Appendix

This appendix lists the major capacity investments investigated in the paper, and describes the variables. Sources are reported in Roman numbers between brackets and listed at the end of the appendix.

1. Firm name, location, and investment dates [II]

- Copper Rand Mine, Québec, 1960, 1968, 1970
- Craigmont Mines, British Columbia, 1961, 1962,
- Vauze Mines, Québec, 1961,
- Bethlehem Copper, British Columbia, 1962, 1964, 1966, 1967, 1971, 1976,
- Coast Copper Co. Ltd., British Columbia, 1962,
- Phoenix Copper, British Columbia, 1962, 1963, 1969,
- Solbec Mines, Québec, 1962,
- Sunro Mines, British Columbia, 1963,
- Kam Kotia Mines Ltd., Ontario, 1963, 1968,
- Manitouadge Mines, Ontario, 1963,
- Lake Dufault Mines, Québec, 1964,
- Copper Corp. Ltd., Ontario, 1965,
- Lorraine Mining Corp., Québec, 1965,
- Minoca Mines, British Columbia, 1965,
- Endako Mines, British Columbia, 1965, 1967, 1978,
- Kidd Creek Mines, Ontario, 1966, 1978,
- Westmin Resource Ltd., British Columbia, 1966, 1968
- Kidd Copper Mines, Ontario, 1966,
- Granisle Mines, British Columbia, 1966, 1972,
- Whitehorse Copper Mines, Yukon, 1967,
- Prace Mining Corp., Ontario, 1967,
- Munro Copper, Ontario, 1967,
- Mines Gaspe, Québec, 1968, 1973
- Madeleine Mines Ltd., Québec,1969,
- Cons. Churchill Copper Corp., British Columbia, 1970,
- Renzy Mines Ltd., Québec, 1970,
- Geco Mines, British Columbia, 1970,
- Brenda Mines, British Columbia, 1970,
- Island Copper Mines, British Columbia, 1971,
- Opemiska Mines, Québec, 1971,
- Lornex Mining Corp., British Columbia, 1972, 1974, 1979,
- Gilbraltar Mines, British Columbia, 1972,
- Similkameen Mining, British Columbia, 1972, 1975
- Bell Copper Mines, British Columbia, 1972,
- Maybrun Mines Ltd., Ontario, 1973,

- Sturgeon Lake Mines, Ontario, 1975,
- Thierry Mines, Ontario, 1976,
- Afton Mines, British Columbia, 1978,
- Equity Silver Mining, British Columbia, 1980,
- Highmont Mines, British Columbia, 1980.

2. Variables

Except where stated otherwise, all prices are Canadian prices expressed as index numbers (1971=100).

1. OUTPUT PRICE [1940-1980]

   The New York Stock Exchange nominal price index of one metric ton of copper, converted into Canadian dollars [(XII) and (VI), Series B3400].

2. ESTIMATED PARAMETERS OF THE OUTPUT PRICE PROCESS: DRIFT AND VARIANCE

   The estimation of the drift parameter of the Brownian geometric process is explained in Section 4.1. The variance parameter $\sigma(s)$ was obtained by regressing $\hat{\xi}(t)^2$ on $P(t)^2$, for each 1940-s period, where $\hat{\xi}(t)$ is the residual from the OLS estimation of (29) over 1940-s. t statistics ranged from 1.85 to 4.85.

3. VARIABLE INPUT PRICES [1940-1980]

   1. Nominal wage rate: *Index number of average wage rates for mining* [(VIII), Series E201 rebased for the period 1940-1975, and (IX) for the period 1976-1980];
   2. Nominal energy price: Törnqvist price index based on natural gas, electricity, and crude oil for the manufacturing, mining, and electric power industries. Shares are defined as value shares; prices are defined as $value \div quantity$. Data come from (VIII, Series Q31and Q32 for gas, Series Q104 and Q109 for electricity, and Series Q19 and Q20 for oil), and the corresponding series from (VII);
   3. Nominal price of materials: *general wholesale price index excluding gold* [(VIII) for the period 1940-1975, Series K33-43 (rebased) and (I) for the period 1976-1980, Series D500000].

4. NOMINAL ASSET PRICE OF CAPITAL [1940-1980]

   Implicit price indexes of Gross National Expenditures: new machinery and equipment [(VIII), Series K181, for the period 1940-1975, and (I), Series D40639, for the period 1976-1980].

5. RATE OF GROWTH OF FACTOR PRICES

   The rate of growth in the share-weighted index of factor prices was so close to that of the general price index that we set $\alpha_w = 0$.

6. TAX PARAMETERS [1960-1980]

   All tax parameters correspond to the post tax holiday period [(III) for the formulas and (IV) for the the computations].

7. RATES OF RETURN [1957-1980]

   The risk-adjusted rate of return is based on an unconditional CAPM. It is defined as the sum of i) the Canadian 90-day Treasury Bill real rate, ii) the market risk premium, and iii) an additional premium accounting for the long term nature of the investment, and measured as the difference between the *Long term Canada bond rate* [(VIII), Series J475, for the period 1954-1977; (VI), Series B14013, for the period 1971-1980]), and the *Canadian 90-day Treasury bill rate* [(VIII), Series J471, for the period 1954-1977; (VI), Series B14001, for the period 1971-1980]. Since data on the metal mining industry rates of return are available only since 1967 in Canada and given the empirical evidence of (at least partial) integration of Canadian and US stock exchanges (Koutoulas and Kryzanowski, 1994), we used US data to estimate

Canadian metal mines market beta's (risk-free rate: *US 3-Months Treasury Bills* [(X), Series X451, for the period 1957-1970; (XI), for the period 1971-1980)]; rate of return in the metal mining industry and market rate of return: based on Share prices and dividends from Standard & Poors [(V), average of high and low yearly values]). Where necessary, nominal rates were corrected for the Canadian rate of inflation (growth rate of the *Implicit Deflator of GDP* [(VI)] net of a quality change rate fixed at 3 percent per annum since 1950 [see Gordon, 1992]).

8. RATE OF TECHNICAL CHANGE

   We used the rates of regress estimated by Stollery (1985) over the period 1957-1979 for Canadian copper mines: -1.40% over 1957-65; -3% over 1966-70; and -3.6% over 1971-79.

9. RESERVES, ORE GRADE, CAPACITY, AND EXTRACTION MODE: [II]

   Reserves are proven or probable and expressed in thousands of metric tons; grade is a percentage rate by weight; capacity is in metric tons of ore per day; the extraction mode is $X = 1$ for open-pit mining and $X = 0$ for underground extraction.

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1. Data Sources:

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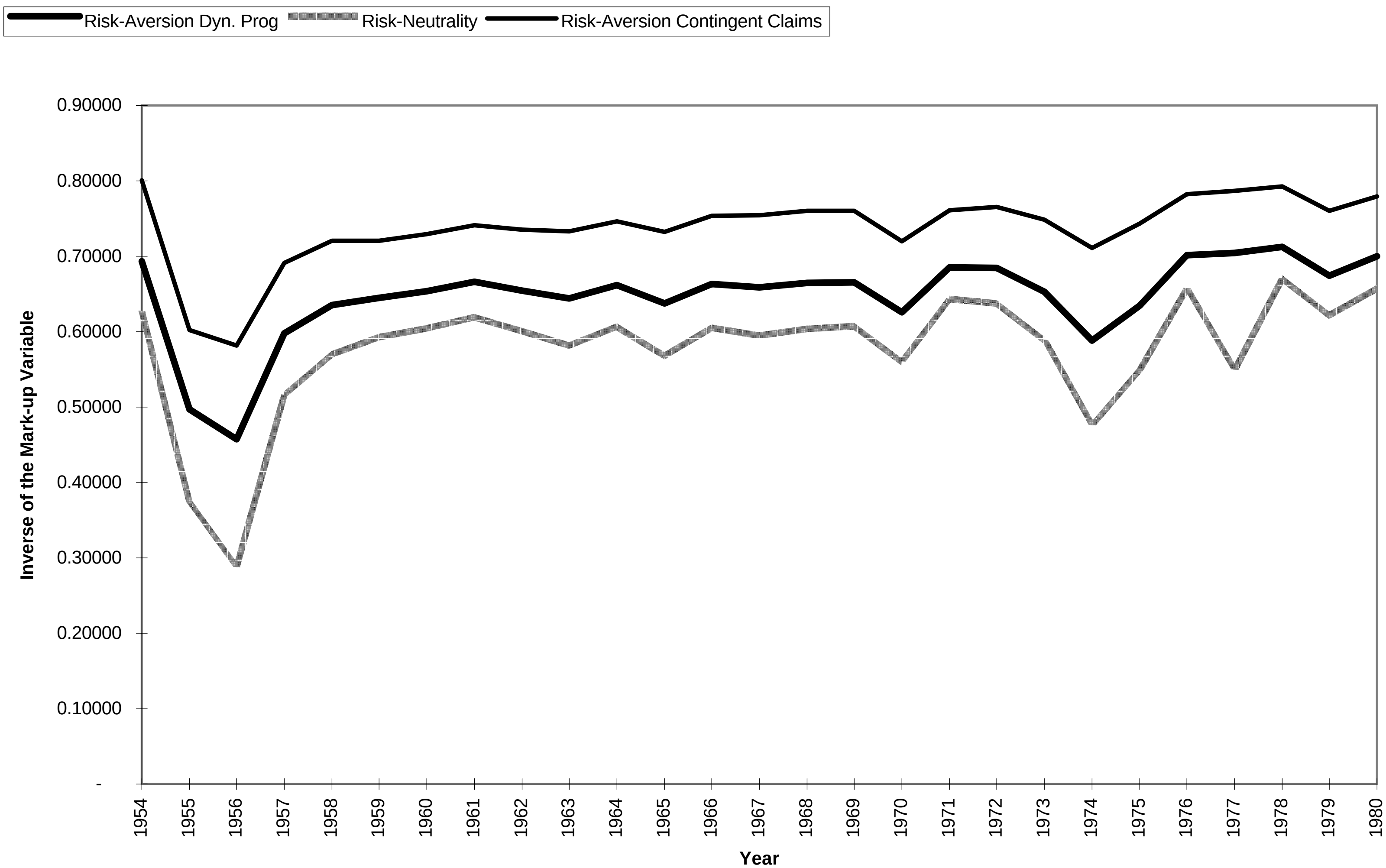

Risk-Aversion Dyn. Prog
Risk-Neutrality
Risk-Aversion Contingent Claims
Inverse of the Mark-up Variable
0.90000
0.80000
0.70000
0.60000
0.50000
0.40000
0.30000
0.20000
0.10000
-
1954
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Year

Table 2: Capacity and Exercise Price Models[a]

	Risk Aversion			Risk-Neutrality	
	*Ln Q**	*Ln P**		*Ln Q**	*Ln P**
		Dynamic Programming	Contingent Claims		
Constant	-0.426 (-0.217)	0.856 (1.454)	2.727 (6.886)	2.719 (1.854)	1.799 (4.439)
Ln B	-	-1.731 (-3.325)	-2.246 (-5.128)	-	-1.326 (-6.440)
R	0.757×10^{-8} (8.103)	-0.374×10^{-8} (-7.260)	-0.244×10^{-8} (-6.064)	0.759×10^{-8} (8.201)	-0.368×10^{-8} (-7.047)
*Ln Q**	-	0.460 (7.768)	0.317 (6.216)	-	0.484 (9.017)
h	-	0.220 (1.722)	-0.083 (-1.361)	-	-0.100 (-1.799)
P	2.749 (1.947)	-	-	1.596 (1.332)	-
W_L	3.016 (1.409)	-2.384 (-4.242)	-1.503 (-4.447)	3.369 (1.591)	-1.691 (-5.305)
W_E	1.157 (1.386)	-0.661 (-3.606)	-0.846 (-7.169)	1.191 (1.366)	-0.968 (-8.516)
W_K	0.531 (1.342)	0-.098 (-0.654)	-0.314 (-3.783)	0.519 (1.257)	-0.411 (-5.169)
δ / ρ	4.294 (2.243)	-	-	1.353 (2.269)	-
X	0.449 (1.842)	-0.221 (-6.655)	-0.157 (-5.514)	0.457 (1.848)	-0.226 (-8.004)
G	-36.9 (-3.688)	15.944 (1.897)	10.896 (4.933)	-36.5 (-3.564)	16.906 (7.870)
s	-0.104 (-1.077)	0.119 (5.203)	0.079 (6.077)	-0.109 (-1.105)	0.078 (6.449)
R^2	0.81	0.91	0.86	0.81	0.91
R^2 *adj.*	0.77	0.89	0.83	0.77	0.89
B-J[b]	0.80	0.14	0.13	0.71	0.22

Notes: [a] The *t*-statistics are between parentheses. The price equations were estimated by the two-stage least square method with truncation. The capacity equations were estimated by the OLS method (in using the two-stage method, the ratio of Mills was not significant). Standard errors of the price equations parameter estimates are based on (34). [b] Bera-Jarque asymptotic Lagrange multiplier normality test.

Table 3: Capacity Elasticities and Threshold Price Elasticities at Mean Values

	Risk Aversion			Risk-Neutrality	
Variable	Capacity	Threshold Price		Capacity	Threshold Price
		Dynamic Programming	Contingent Claims		
P	2.444	-	-	1.419[a]	-
R	0.503	-0.249	-0.162	0.504	-0.245
Q^*	-	0.461	0.317	-	0.484
W_L	2.582[a]	-1.482	-1.287	2.884[a]	-1.447
W_E	1.369[a]	-0.783	-1.002	1.369[a]	-1.146
W_K	0.459[a]	-0.085[a]	-0.272	0.449[a]	-0.356
δ / ρ	3.251	-	-	0.862	-
G	-0.517	0.223	0.153	-0.511	0.237
σ	-	0.209	0.090	-	0.168
r	-	-0.058	-0.018	-	-0.034
α	-	0.043	0.034	-	0.063

Note: [a] All elasticities are different from zero at a better than 5% level of significance, except where otherwise indicated by a 'a' superscript.

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